ESTABLISHMENT OF FLUX IN MAGNETIC CORES

J.Edwards* and T.K Saha**

* School of Electrical &Electronic Systems Engineering
Queensland University of Technology

** Department of Computer Science and Elect Engineering
University of Queensland.

Abstract

The same fundamental propagation and diffusion processes that establish current in conductors are also involved in establishing flux on the surface of magnetic cores and laminations but the spatial positions of the E and H fields are interchanged in the insulating medium. The E fields appear around the magnetic cores when the flux changes ($\int Edl = d\phi/dt$), and the H fields are normal to idealized core surfaces. In the case of conductors, the displacement current plays an important part in layering any current changes onto the conductor surface during each transition of the line. However, in the case of magnetic cores, it is the 'leakage' flux that builds up the surface core flux, and adds any changes in $d\phi/dt$ to the core surface, on each transition of the magnetic line. Once set up on the surface the flux, and hence the surface H field, slowly diffuses into the interior of the core until it is uniformly distributed throughout its cross sectional. Thus leakage flux is essential for the operation of magnetic circuits, inductors, and transformers and exists even in the case of idealized systems with materials of infinite permeability.

1. INTRODUCTION

The purpose of this work is to show that the equation *Flux = MMF/Reluctance* relies on electromagnetic propagation and diffusion to establish flux changes in magnetic circuits and cores, similar to the way current changes are established in conductors. In this case however the transmission line is magnetic rather than electric. Magnetic transmission lines are very similar to the familiar electrical lines except for the following:

- The E and H fields are spatially interchanged.
- A greater amount of energy is required to establish surface magnetic flux (ie. line up surface magnetic domains) on cores, compared to that required to establish surface currents (give electrons a net acceleration) on conductors.
- Internal fluxes are much higher in magnetic steels than copper, so the internal E & H fields move slower to produce similar back emf's. Thus the surface flux diffuses much slower into the interior of magnetic steels ($\rho \approx 5\times10^{-7}\ \Omega$m, $\mu_r \approx 10000$, diffusivity, $\alpha = \rho/\mu \approx 4\times10^{-5}$ m/sec^2), compared with the diffusion of surface currents into copper ($\rho \approx 2\times10^{-8}\Omega$m, $\alpha = 1.37\times10^{-2}$ m/sec^2).
- The higher resistivity of ferrites (eg. MnZn: $\rho \approx 0.5\Omega$m, $\mu_r \approx 2000$) give them a much higher diffusivity ($\alpha \approx 200$ m/sec^2) so the surface flux diffuses quickly into a ferrite material. At 100kHz it has 2.5μsec (ie. ¼ period) to move into the interior and will go in as far as 2.5cm (skin depth, $\delta = \sqrt{(2\alpha/\omega)} \approx 2.5$ cm) at 100kHz, corresponding to a phase velocity u = $\delta\omega \approx 16\times10^3$m/sec.

Apart from these magnetic and electric transmission lines are similar. Both act as guides for electrical power, that flows through the insulating material of the line, by means of the causal E and H fields in the insulating medium, and the associated Poynting vector. Magnetic lines have characteristic impedances similar to electrical lines. Inductors are essentially short-circuited magnetic lines.

2. MAGNETIC CORED INDUCTOR MODEL

Consider a magnetic core suddenly excited by a current flowing through a multi-turn coil as in Fig 1.

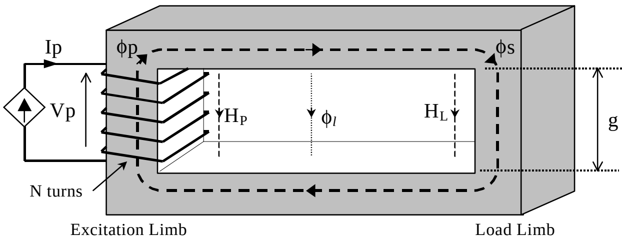

Fig .1. Basic Magnetic Core Inductor Model

The multi-turn coil can be approximated by a single turn current sheet carrying a current of $I_A = NI_P$ amps. Although we are only considering an inductor, the fluxes in the magnetic limbs need to be identified, and are labeled (ϕ_P,, ϕ_S) as if it were a transformer.

2.1. Propagating H Field due to Current Change

When the current is turned on the magnetising force $I_A = NI_P$ produces a magnetic field $H = H_1 + H_2$ that radiates from the coil conductors as E & H fields. H_1 moves away from the coil while H_2 moves into its interior.

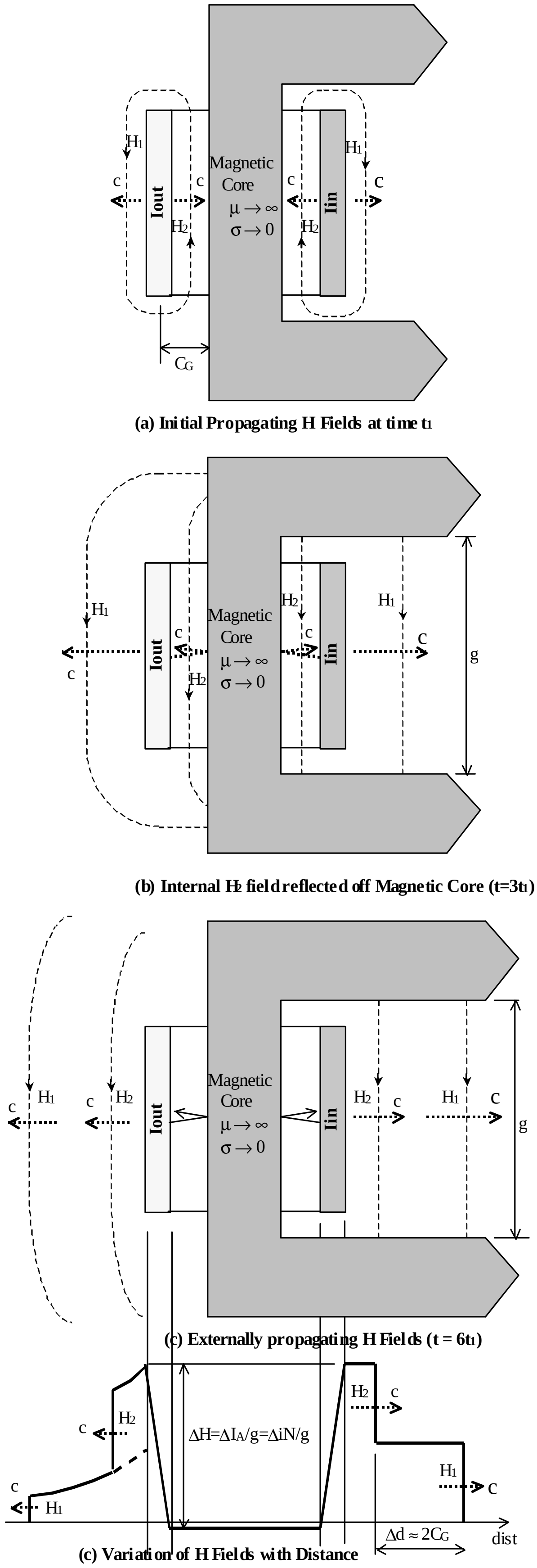

Fig 2. Propagating H Field with Idealised Core.

Fig 2(a),(b) and (c) show the propagating H_1 *&* H_2 fields, resulting from the step change $\Delta I_A = N\Delta I$ in coil current, at various times. Since the coil embraces a magnetic core, the inward propagating H_2 field, is reflected back off the surface of the core in an inverted form. This reflected H_2 field tends to cancel out the incident one in the interior of the coil. Outside the coil the reflected H_2 field reinforces the original H_1 field, as shown in Fig 2(c) and (d).

It can be seen that the field $\Delta H = H_1 + H_2 \approx \Delta I_A/g$, propagates in the insulating space between the horizontal limbs, that act as guides for the propagating waves. The distance $\Delta d \approx 2C_G$, between H_1 and the reflected H_2 fields, get smaller as the gap C_G, between the coil and the core, is reduced. From here on, it will be assumed that the coil is wound fairly close to the idealised magnetic core, so that the propagating H_1 and H_2 steps shown in Fig2(d) can be approximated by a single step/ramp function of magnitude $\Delta H \approx \Delta I_A/g$.

However, this is a simplification of reality since *E* and *H* are required to establish the changes in coil current. Even a single-turn coil itself behaves as a transmission line, in which current changes are not instantly established throughout the whole turn. The equivalent single turn coil, carrying a current $I_A = Ni$, is also only an approximation to the more practical *N*-turn coil carrying current *i*. The *N*-turn coil behaves as a tapered transmission line and it takes approx *N* times longer to establish the coil current. Thus the step functions in the *H* fields, shown in Fig.2(d), are really more gradual, because of the time required to establish currents in the coil.

A more significant aspect is the fact that we have assumed an ideal core ($\mu \to \infty$, $\sigma \to 0$). In practice, energy is required to reorientate the surface magnetic domains, to reduce the surface H field, during the reflection process. This causes the reflected waves to be further delayed and more gradual. In spite of this, the propagating stepped *H* fields illustated in Fig 2 are still useful, at least at a conceptual level, and will be assumed to exist.

2.2. Establishment of Initial Surface Core Flux

When the coil ampere-turns increases by an amount $\Delta I_A = N\Delta i$, the *H* field in the insulating gap between the horizontal limbs rises by $DH_g = \Delta I_A / g$, due to the increased coil current and reflection off the magnetic core inside the coil. This rise in the H_g field produces an increase in the leakage flux density $\Delta B_g = m_o \Delta H_g$, in the gap between the limbs. This change ΔB_g, in gap leakage flux density, produces an *E* field (*Curl E* = -¶*B*/¶ *t*) at the wavefront, and the *E&H* wavefront propagates down the insulating space between the horizontal limbs towards the secondary limb.

The horizontal limbs act as guides for the wavefront, and the coil current maintains the steady H_g field, and hence the leakage flux density, behind the wavefront via the Poynting vector. The propagating wave thus sets up flux on the surface of the magnetic core, as it moves along the magnetic line at velocity c, as shown in Fig 3. The situation is similar to electrical lines → but in this case it is the leakage flux density at the wavefront that sets up the surface flux. The E and H fields in the insulating medium are spatially interchanged from those in the electrical transmission line as indicated in Fig 3(b). Displacement current I_D exists at the propagating wavefront of these magnetic lines, as it does with electrical lines.

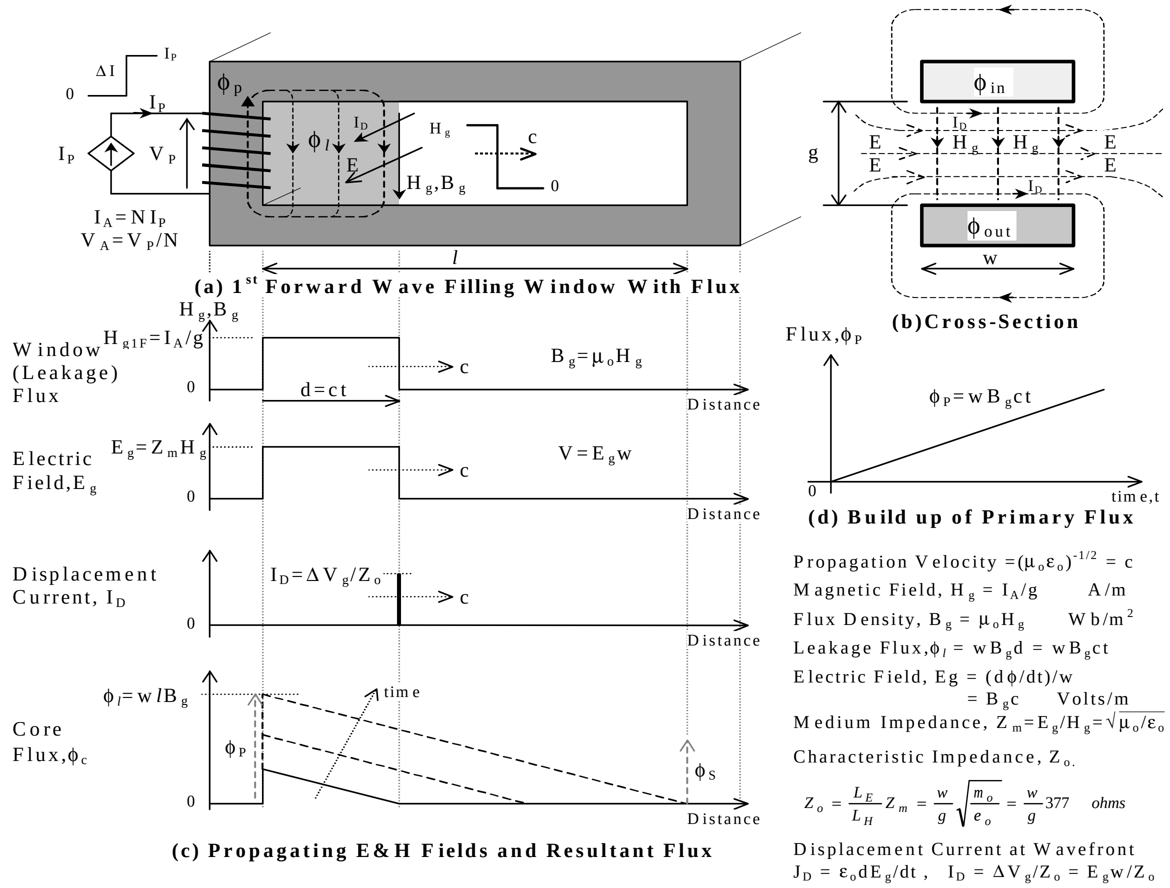

Fig. 3 Initial Forward Propagation Wave on Magnetic Line Due to Current Drive

In the following analysis it will be assumed that the width (w) of the core is much greater than the gap (g), so that fringing is negligible and the fields are concentrated in the insulating window space between the horizontal limbs as in Fig3(b). The propagating H field has an associated B field at the wavefront, that generates an E field as it moves down the magnetic line.

Magnetic Field, $H_g = I_A/g$ Amps/m

Leakage Flux Density, $B_g = m_o H_{g1F}$ Wb/m^2

$$\text{Now, } Curl\, E_g = \frac{\partial E_g}{\partial z} = \frac{\partial E_g}{v \partial t} = -\frac{\partial B_g}{\partial t}$$

∴ Electric Field, $E_g = \Delta B_g v$

$$\text{where, Velocity, } v = \frac{1}{\sqrt{m_o e_o}} = c$$

$$\text{Voltage, } V_g = E_g w = \Delta B_g w v = m_o \frac{w}{g} c \Delta I_A \quad \text{Volts}$$

This change in E at the wavefront, moves along the core window, producing a displacement current I_D in the space around the core, whose current density $J_D = dE/dt$, is concentrated in the gap between the horizontal limbs. These E and H fields, propagating in the insulating medium of the magnetic line, give rise to a medium impedance $Z_m = DE_g/DH_g$, and hence a characteristic impedance Z_o.

$$Z_o = \frac{\Delta(Voltage/Turn)}{\Delta(AmpereTurns)} = \frac{\Delta V}{\Delta I} = \frac{\Delta E_g w}{I_D} = \frac{\Delta E_g w}{\Delta H_g g} = Z_m \frac{w}{g}$$

$$\therefore Z_o = Z_m \frac{w}{g} = \sqrt{\frac{m_o}{e_o}} \frac{w}{g} = \frac{w}{g} 377 \; ohms$$

$$\text{Displacement Current at Wavefront, } I_D = \frac{\Delta V_A}{Z_o} = -\Delta I_A$$

This displacement current I_{D1F}, is equal and opposite to the excitation ampere-turns $\Delta I_{1F} = \Delta I_A = N \Delta I_P$ in the winding, and allows Amperes Law to be satisfied at points in front of the propagating wavefront. The E

field is maintained behind the wavefront by the changing flux in the core ($\oint E.dl = -df/dt$). Due to the opposing fluxes in the top and bottom limbs, this E field is not uniformly distributed around the core but concentrated in the window space between the horizontal magnetic limbs as indicated in Fig 3(b).

$$\text{Voltage/Turn} = \int_{core} E.dl \approx V_g = E_g w = -\frac{df}{dt} \quad \text{Volts}$$

The total leakage flux, and hence the flux in the primary limb, builds up linearly as the wavefront moves along the magnetic line. It leaves an electric field $E_g = B_g c$ in its wake, as indicated in Fig 2(d).

$$\text{Leakage Flux, } f_l = B_g wd = B_g wct \quad \text{Wb}$$

Rate of Increase of Leakage Flux, f_{1F}'

$$f_{1F}' = \frac{df_l}{dt} = \frac{df}{dt} = m_o H_g wc = I_A Z_o \quad \text{Wb/sec (V/turn)}$$

The total leakage flux flows though the interior space of the coil so:

$$\text{Flux in Primary Limb, } f_P = f_l = B_g Wd = B_g Wct \quad \text{Wb}$$

This core flux takes the path of lowest energy, and is initially established on the surface of the core. In electrical transmission lines the E field behind the front is maintained by the electric charge on surface of the conductors, but in this case it is the every increasing core flux that maintains the E field behind the wavefront. Apart from this, the magnetic line behind the wavefront contains steady E and H fields, with associated electrical and magnetic energies, similar to electrical lines. Power flows down the line from the source via the Poynting vector, to maintain these steady E and H fields behind the wavefront.

Energy Supplied to Line During Transition = $G(t)$

$$G(t) = (E \times H)\, Area \times time = E_g H_g gwt = E_g H_g gw\frac{d}{c} \quad \text{J}$$

When the wavefront reaches the secondary end of the magnetic line, $d = l$ and:

$$\text{Primary Flux, } f_{P1} = f_l = B_g wl \quad \text{Wb}$$

$$\text{Secondary Flux, } f_{S1} = 0$$

$$\text{Core Flux Rate of Change, } \frac{df_{1F}}{dt} = m_o H_g wc \quad \text{Wb/sec}$$

$$\text{Total Energy Supplied to Line, } G_T = E_g H_g gw\frac{l}{c} \quad \text{J}$$

$$G_T = Z_m H_g{}^2 gw\frac{L}{c} = \sqrt{\frac{m_o}{e_o}}\left(\frac{I_A}{g}\right)^2 gwl\sqrt{m_o e_o} = m_o\frac{w}{g} I_A{}^2 l$$

$$\text{Magnetic Energy of Line, } G_m = \frac{1}{2}\int_v B_g H_g\, dv \quad \text{Joules}$$

$$G_m = \frac{1}{2} m_o \int_v H_g{}^2\, dv = \frac{1}{2} m_o \left(\frac{I_A}{g}\right)^2 gwl = \frac{1}{2} m_o \frac{w}{g} I_A{}^2 l = \frac{G_T}{2}$$

$$\text{Electrical Energy of Line, } G_e = \frac{1}{2}\int_v D E_g\, dv \quad \text{Joules}$$

$$G_e = \frac{1}{2}\int_v D E_g\, dv = \frac{1}{2} e_o \int_v E_g{}^2\, dv = \frac{1}{2} e_o (Z_m H_g)^2 gwl$$

$$G_e = \frac{1}{2} e_o \frac{m_o}{e_o}\left(\frac{I_A}{g}\right)^2 gwl = \frac{1}{2} m_o \frac{w}{g} I_A{}^2 l = \frac{G_T}{2} \quad \text{Joules}$$

Thus at the end of this 1st forward transition:

- Magnetic and 'electrostatic' line energies are equal.
- Voltage around core ($V = E_g w$) = $I_A Z_o$ Volts/turn.
- Flux in the primary limb, $f_{P1} \approx f_l$.
- Leakage flux in window, $f_l = B_g wl$.

At this point the magnetic line has been energized for a voltage of $I_A Z_o$ Volts/turn (i.e. flux rate $df/dt = I_A Z_o$ Wb/sec), at a current of I_A Ampere-turns (i.e. $H = I_A/g$).

2.3 Line Reflection Off the Secondary Limb

When the wavefront reaches the secondary end of the line there are 3 sources of energy available, as is the case with electrical lines.

- Stored magnetic energy in medium due to H_g.
- Stored 'electrostatic' energy in medium due to E_g.
- Continuous power ($E_g \times H_g$) coming from the source.

The vertical secondary limb of the core acts as a magnetic short circuit, in which the resultant H is very small. Thus the reflected H field is almost equal and opposite to the incident field H_g, as indicated in Fig 4. The change $DH_g \approx -H_{g1F}$, so the resultant H_g during this return leg, is almost zero. As this reflected wavefront returns back up the line, the leakage flux, previously build up in the window, empties out into the secondary limb. Thus the stored magnetic energy falls to almost zero, during this backward transition.

The stored magnetic energy, and source power ($E \times H$), are converted to 'electrostatic' energy, during this backward transition of the line, so the E_g field almost doubles. The emptying out of the leakage flux, from the window into the secondary limb, increases the flux in the core behind this returning wavefront, as indicated in Fig 4. Thus the rate of change of core flux behind this returning wavefront f_{1B}', is almost twice that of what it was on the forward transition f_{1F}'.

At the end of this backward transition:

- There is negligible leakage flux in the window.
- Voltage around core ($V = E_g w$) has almost doubled, to approx $2I_A Z_o$ Volts/turn.
- Flux throughout the whole of the core has almost doubled, to $f_{P1} \approx f_{S1} \approx 2f_l$.
- Rate of change of core flux $f_{P1}' \approx 2I_A Z_o$ Wb/sec.

On reaching the primary winding the H_g field is increased back to I_A/g, and the change DH_g, propagates down the line for the second time, further increasing the E field, and hence the rate of increase in core flux, in its wake.

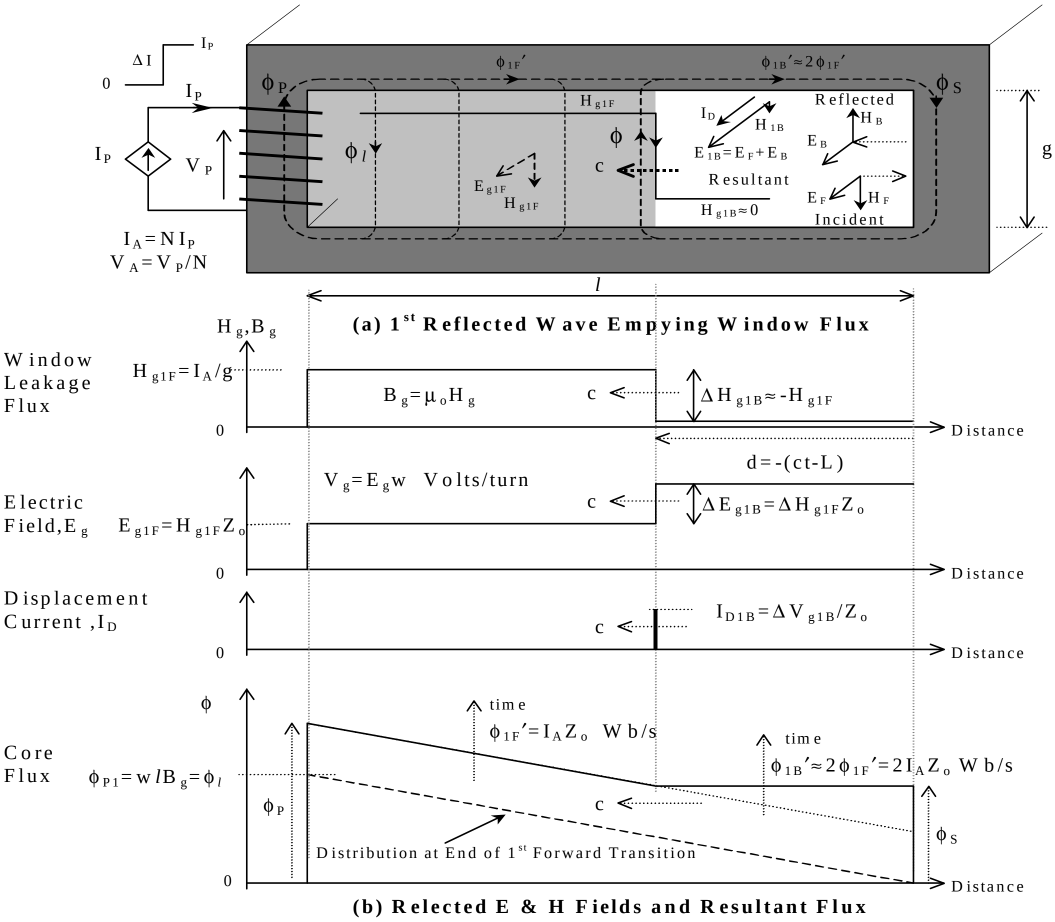

Fig 4. First Reflected Wave Traveling Back up Magnetic Line

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- There is negligible leakage flux in the window.
- Voltage around core ($V=E_g w$) has almost doubled, to approx $2I_A Z_o$ Volts/turn.
- Flux throughout the whole of the core has almost doubled, to $\phi_{P1} \approx \phi_{S1} \approx 2\phi_l$.
- Rate of change of core flux $\phi_{P1}' \approx 2I_A Z_o$ Wb/sec.

On reaching the primary winding the H_g field is increased back to I_A/g, and the change ΔH_g, propagates down the line for the second time, further increasing the E field, and hence the rate of increase in core flux, in its wake.

2.4. Multiple Transitions of Build-Up Core Flux

When the equivalent characteristics of the secondary (limb + any secondary winding + load) are not matched to the characteristic impedance Z_o, of the magnetic line, multiple up and down transitions, of the magnetic line, are required to build up the rate of change of core flux (ie. volts/turn), to that required by the rate of change of primary current. During these transitions, it is the 'leakage' flux that increases the rate of change of core flux. In the case under consideration, the current is a step input, so the final rate of change of core flux is required to be high, and ideally approaches infinity.

In order to establish the basic concepts, it will be assumed that the H field required for the core itself is negligible, and that the core does not saturate.

1st Forward Transition:

$I_{1F} = I_A = NI_P$ ATurns, $V_{1F} = I_A Z_o$ V/turn = ϕ_{P1}' Wb/s

At end of first forward transition*:*

$\phi_{l1F} = B_g wl = \phi_l$, $\phi_{P1F} = \phi_l$, $\phi_{S1F} = 0$

1st Backward Transition:

$I_{1B} \approx 0$ ATurns, $V_{1F} \approx 2I_A Z_o$ V/turn = ϕ_{S1}' Wb/s

At end of first backward transition:

$\phi_{l1B} \approx 0$, $\phi_{P1B} \approx \phi_{S1B} \approx 2\phi_l$

2nd Forward Transition:

$I_{2F} = I_A$ ATurns, $V_{2F} \approx 3I_A Z_o$ V/turn = ϕ_{P1}' Wb/s

At end of second forward transition*:*

$\phi_{l2F} \approx \phi_l$, $\phi_{P1F} \approx 5\phi_l$, $\phi_{S1F} \approx 5\phi_l$

In Summary (for $\mu_{core} \rightarrow \infty$)

At Start 1st Transition

$I_{1F} = I_A = NI_P$ ATurns, $V_{1F} = I_A Z_o$ V/turn = ϕ_{P1}' Wb/s

& $\phi_{core} = 0$ Wb, $d\phi_{core}/dt = V_{1F} = I_A Z_o$ Wb/s (1a)

At Start of n^th Transition

$I_n = I_A$ ATurns, $V_n = (2n\text{-}1)I_AZ_o$ V/turn = f_{P1}' Wb/s

$\phi_{core\ n} \approx f_{core\ (n-1)} + (4n\text{-}2)f_l$ Wb

$(df_{core}/dt)_n \approx f_{core\ n}' = V_n$ Wb/s (1b)

The forward and backward transitions due to $\pm\Delta H_g$» I_A/g can be considered to pump-up the Voltage/turn, and df_{core}/dt, by an amount ΔV=$\Delta E_g w \approx Z_oI_A$, on each forward and backward transition of the magnetic line. The midline H_g would show the back and fro transitions similar to that existing at the wavefront.

2.5 Flux Build-Up in Non-ideal Core (*m* ¹ ¥, s ® 0)

In practice the magnetic core requires a longitudinal H_{Core}= B_{Core}/m, to produce its flux. Thus the changes in the transverse field $\pm\Delta H_{gn}$, that propagates up and down the magnetic line, get smaller and smaller (ΔH_{gn}= I_A /g – $H_{Core\ n}$) as the core flux increases, during each transition. However while ΔH_{gn} reduces, H_{gn} only falls to H_{Core} during return transitions, so the average value of H_{gn} remains reasonably constant. Hence ΔH_{gn}, and the associated ΔE_{gn} in the E field, reduce with each transition of the magnetic line, and the amount by which the voltage/turn gets pumped up by the propagating $\pm H_g$, continually decreases. The rate of increase in core flux is proportional to the Voltage/turn (ie.E_g). Although the core flux actually increases, it does so at ever reducing rates, during each transition of the line.

Eventually a transition is reached where the fall in ΔH_{gn}, due to the increases in the longitudinal $\Delta H_{Core\ n}$ (since $H_{Core\ n} \propto f_{Core\ n}$), is equal to the increase in ΔH_{gn} due to the wave front H_g being returned to I_P/g at the start of the n^{th} transition. At this point the whole of the H field, due to the driving current I_A, has been used up longitudinally by the core flux, ($H_{Core} \approx H_{Applied}$) and very little is available for further propagation. At this point the longitudinal H_{Core} field has increased to the level where most of the Poynting vector (ie. source power) is directed into the core itself, and not into the gap fields, as indicated below in Fig 6.

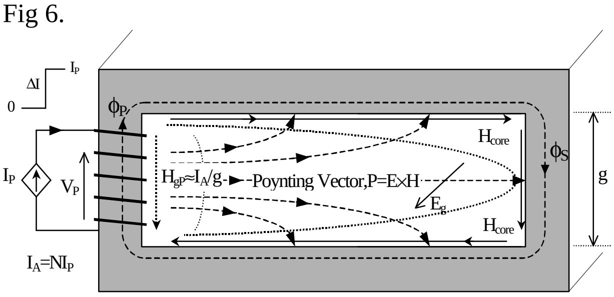

Fig 6. Power Flow into Magnetic Core ($\mu_{core} \neq \infty$)

The E field, and hence the rate of increase of core flux (df_{core}/dt), has reached its maximum level. After this point the rate of increase in core flux continually reduces, together with the associated voltage/turn.

The leakage flux in the window is now steady, and varies linearly along the length of the core. It is a maximum at the primary end of the window (B_g=m_oI_A/g), and falls to almost zero at the secondary end of the window.

3. SURFACE FLUX DIFFUSION INTO CORE

The core flux takes the path of least energy, and initially distributes itself on the surface of the core/laminations. Once propagation has almost ceased $H_{Surface}$ = H_{Core} = $H_{Applied}$, = I_A = NI_P, and the surface flux density has reached its maximum. The core flux increases as the constant surface H field, and its associated surface flux, diffuse into the interior of the core. However the rate of increase continually decreases and hence the voltage/turn falls.

Eventually the H field is uniformly distributed over the whole cross sectional area of the core, and the core flux has reached its maximum level. The core flux is now at its steady state value (f_{core}=*mmf/reluctance*), and the voltage/turn has fallen to zero.

4. EXPERIMENTAL RESULTS WITH CORES

4.1 Experimental SetUp

A step current is applied to the coil wound around the primary limb of a laminated steel magnetic core, as indicated in Fig 6.

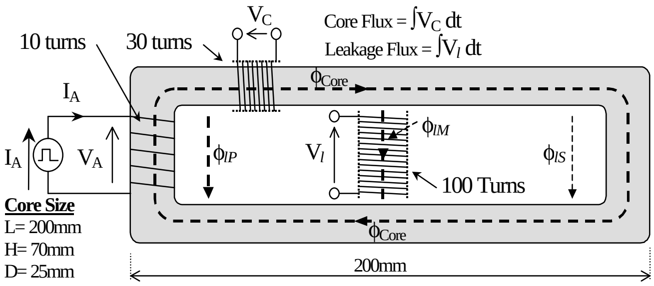

Fig 7. Set-Up for Fluxes in Laminated Steel Core

The results of applying 180mA current pulse to the 10 turn winding is shown in Figs 8,9,10,11. The current is measured via the drop across a 1Ω sensing resistor.

4.2 Mid Line Leakage and Core Fluxes

Fig 8 shows that the leakage flux (ie.H_g field) rises rapidly towards its final value, compared to the core flux that gets established on the surface and then slowly diffuses into the core interior as shown in Fig9.

For closer examination of the mid–line flux, the pulse rate is increased as shown in Fig 10. There are no direct signs of the back and fro transitions, because the line is so short and transition times of the order of ½ nsec. However the average value of leakage flux (ie.H_g) is still expected to rise very quickly (almost instantaneously) with the applied current.

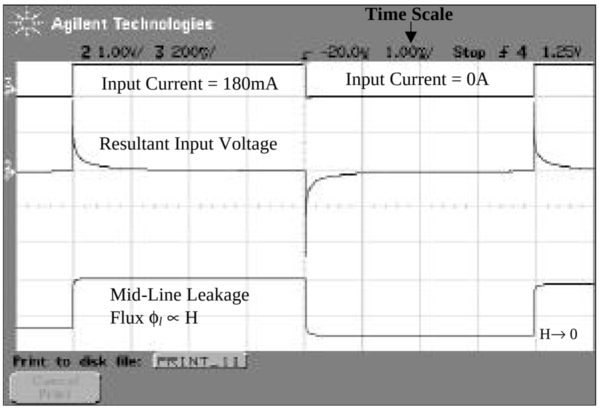

Fig 9 Mid-Line Leakage Flux (ie.H field): (1ms/div)

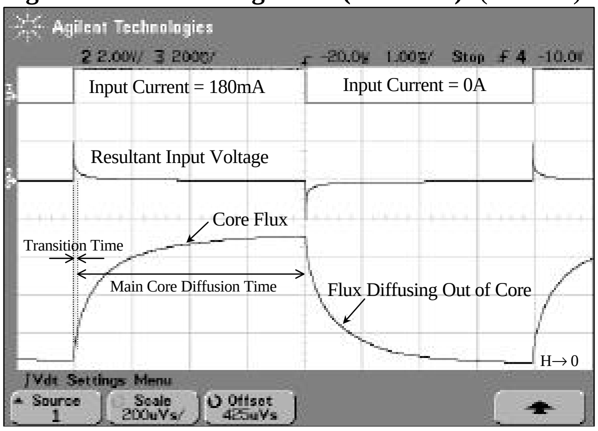

Fig9. Flux Diffusing into Laminated Core: (1ms/div)

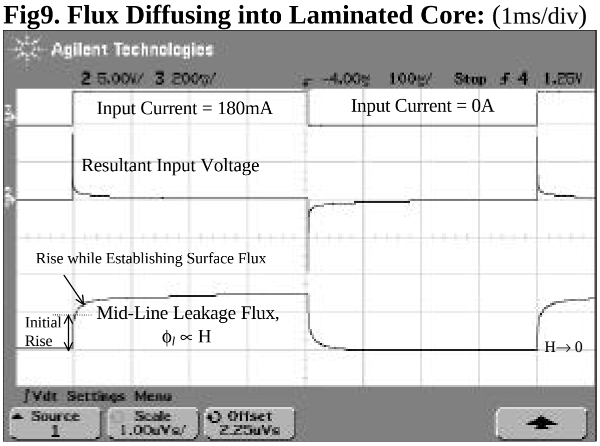

Fig 10. Mid-Line Leakage Flux (ie.H): (100μs/div)

It can be seen from Fig 10 that the mid-line leakage flux initially rises quickly to approx 0.6 of its maximum value followed a much slower rise. The initial fast rise in the mid-line leakage flux (ie propagating H_g field) is due to the outward propagating H_1 field, as described in section 2.1. The final rise is due to the gradual reflection of the H_2 field off the surface of the core, surrounding the coil, as the surface magnetic domains are aligned. This is gradual because energy is required to reorient the surface magnetic domains to form fewer larger domains with a reduced $H_{Surface}$. This energy requires time to move into the core surface from the incident propagating wave.

4.3. Primary End Leakage Flux

The 100 turn search coil used for measuring the leakage flux is moved from the center to the primary end of the window, and results are shown in Fig 11.

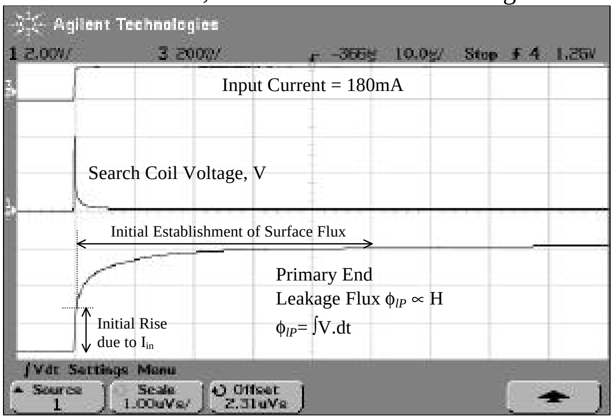

Fig 11. Primary End Leakage Flux (10msecs/div)

It should be realized that we are only able to see averaged values of leakage flux, due to the fast transitions, and as expected, these are approx twice as large at the primary end of the core than seen previously at the mid line. This would be the case even with an idealized core, since the mark to space ratio of the H_{gap} transitions is 100% at the primary end of the window, decreasing to 0% at the secondary.

The initial rise in leakage flux (ie. H_{gap}) due to the increasing input current can be clearly seen, and this is mainly due to the outward propagating wave (H_1) from the coil, as described previously. Since the core is non-ideal the inward propagating (H_2) has to establish the flux (ie. lower the H field) on the core surface inside the coil before being reflected back to reinforce the forward wave, and thus increase the leakage flux in the core window.

5. CONCLUSIONS

It has been shown that displacement currents, leakage flux, electromagnetic propagation and diffusion are essential for establishing flux in magnetic cores. Although the actual transitions cannot be seen, experimental results support the theory. They show the fast establishment of the direct H_1, and reflected H_2 fields, by means of the associated leakage flux at the primary end of the coil. Also shown is the fast establishment of window leakage flux, that establishes the surface core flux, which diffuses relatively slowly into the interior of the core/laminations.

6. REFERENCES

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[2] J.Edwards, T.K.Saha ,"Power Flow in Transformers by Elecromagnetic Fields', pp 86-91 AUPEC'00, Brisbane, 2000.