ESTABLISHMENT OF MAGNETIC FIELDS IN COILS

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Magnetic fields are set up around current carrying conductors or coils in accordance with Ampere's Law. These magnetic fields are initially established by displacement currents, in the propagating electromagnetic waves that radiate from the source when the current changes. Changes in conductor current are accompanied by radiating EH fields that establish a new level of magnetic field in the surrounding space, and this field is then maintained by the steady conductor current. This paper considers how the magnetic fields are set up around conductors, air cored coils and coils with magnetic cores, via electromagnetic waves when the current changes.

1. INTRODUCTION

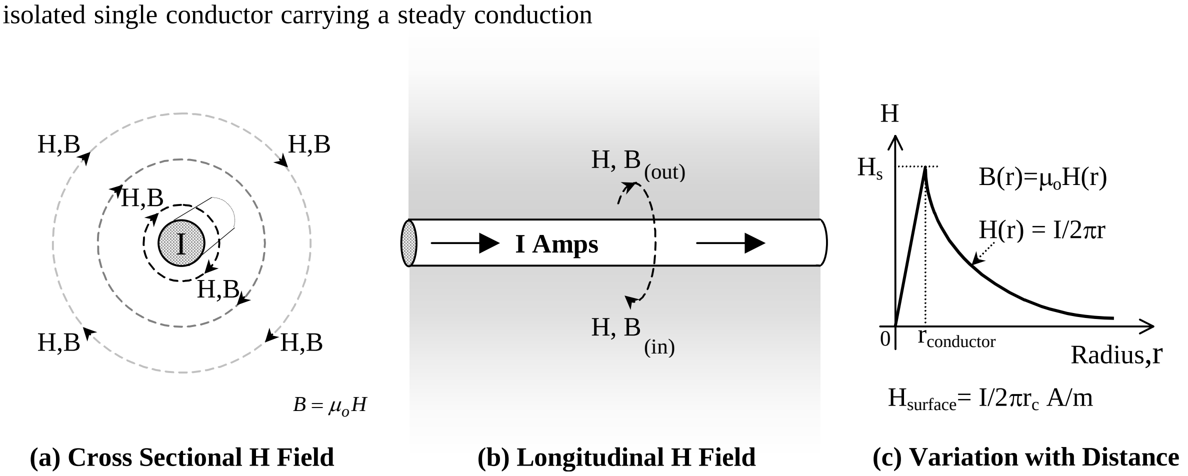
Electromagnetic wave propagation and reflection are fundamental to the process whereby magnetic flux is established in and around coils [1][2][3]. The initial electro-magnetic process that sets up the current in the coil itself will be neglected. It will be assumed that currents can be instantly established throughout the whole cross section of the conductors, or at least uniformly distributed on all surfaces instantly, and that the H fields around the conductors are established by conduction current in the conductors themselves, instead of through source generated displacement currents in the surrounding medium. Although these assumptions may be somewhat unreal, they will allow us to gain an insight into to processes involved in establishing magnetic fields around various configurations of conductors, ranging from single isolated conductors to coils with magnetic cores.

2. H-FIELD OF ISOLATED CONDUCTOR

The familiar circular magnetic field around an isolated single conductor carrying a steady conduction current I is shown in Fig.1. For steady values of current the H field only has *Curl* inside the conductor where the current exists, but the H field itself exists both inside and outside of the conductor as shown in Fig. 1(c). Associated with this H field is a magnetic flux $B=\mu_o H$. The magnitude of H (amps/meter) decreases reciprocally with distance from the conductor, as shown in Fig 1(c), since its integral around the ever-increasing path length is equal to the constant conductor current I.

Propagation From Conductor

Any changes in conductor current changes the H field in the outside space at the surface of the conductor and hence the magnetic flux. This change in magnetic flux induces an E field in the air since $Curl\ E = -\frac{dB}{dt} = -\mu_o \frac{dH}{dt}$. This E field produces a displacement current, $J_{disp} = \partial E/377$, together with an associated H field, since $Curl\ H = J_{disp}$.



(a) Cross Sectional H Field **(b) Longitudinal H Field** **(c) Variation with Distance**

Fig. 1 Steady-state Magnetic Field H around Isolated Conductor

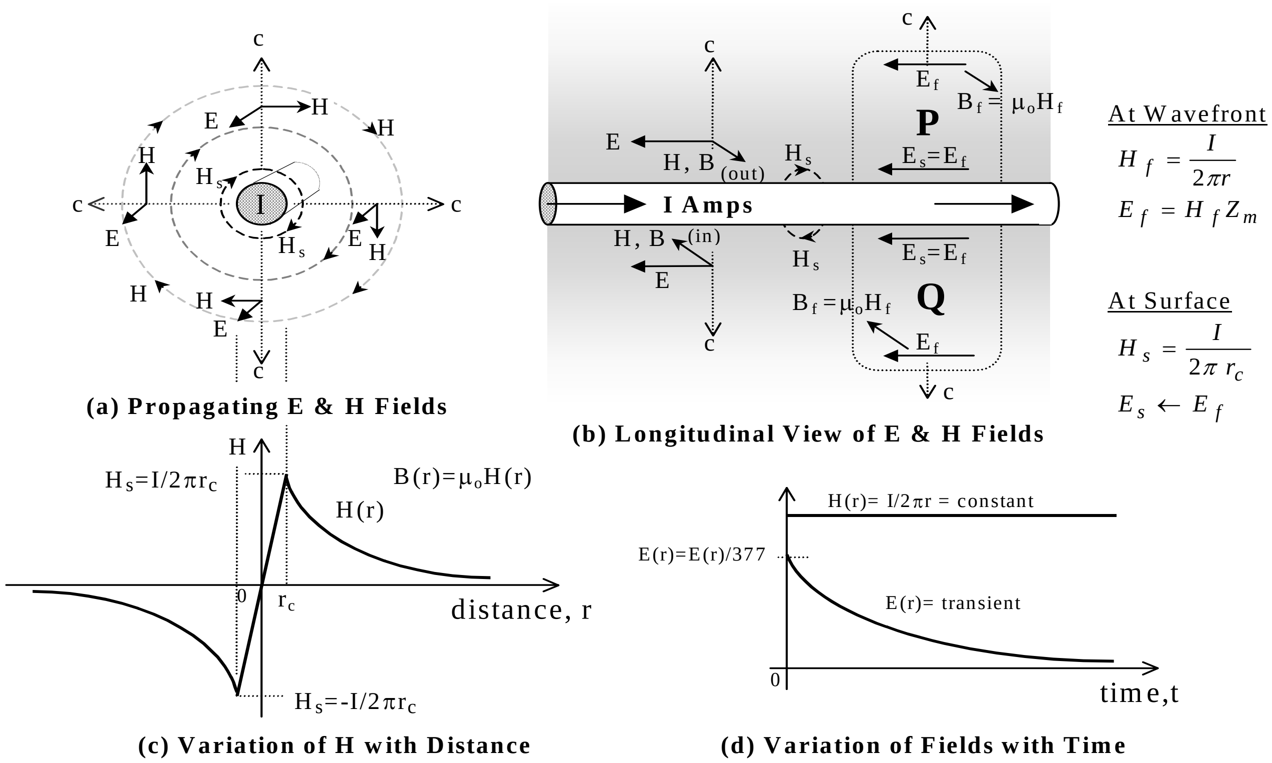


Fig 2. Establishment of Magnetic Field around Isolated Conductor ($\sigma \to \infty$)

These *E&H* fields radiate out from the conductor in all directions at the speed of light as shown in Fig.2. Thus the *H* field has an associated *E* field that produces a displacement current, with its own associated *H* field, as it propagates through the external medium. The velocity of propagation is such that the integral of this propagating *H* field around the conductor, is equal to the change in conductor current that created it, and is $v = c = 1/\sqrt{\mu_o \varepsilon_o}$, giving free space an impedance, $Z_m = \sqrt{\mu_o/\varepsilon_0} = 377$ ohms.

Even the steady *H* field around the conductor, due to constant conductor current *I*, has got there by previous changes (ie. wavefronts), H_f, traveling out from the conductor at velocity *c*. The forward traveling H_f field decreases as the wavefront moves further and further from the conductor, since $H_f = I/2\pi r$. The forward traveling E_f field is generated at the wavefront itself, which is the only place where the flux *B* is actually changing, since $Curl\, E_f = -\dfrac{dB_f}{dt} = -\mu_o \dfrac{dH_f}{dt}$.

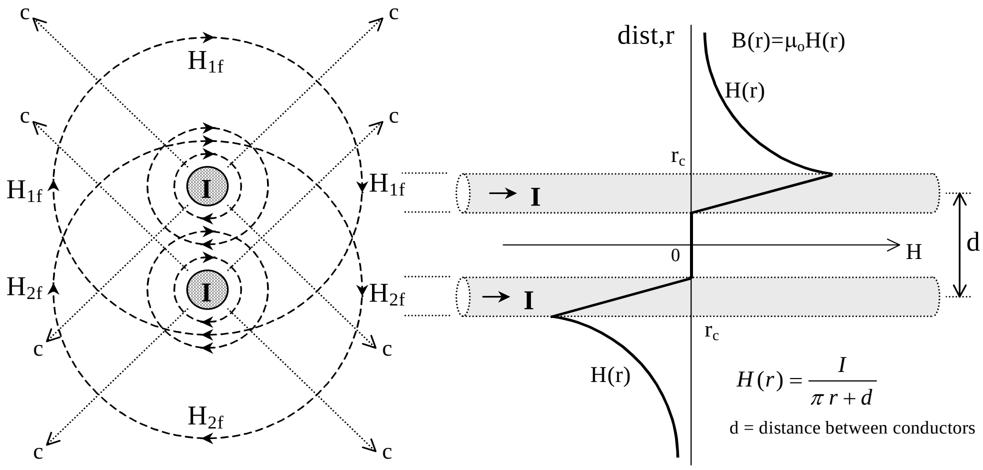
Since H_f decreases reciprocally with distance from the conductor so also does E_f, since $\dfrac{\partial E}{\partial H} = \dfrac{E_f}{H_f} = 377 ohms$. The situation is that of a tapered transmission line in which the characteristic impedance $Z_o = \dfrac{Length\ of\ E}{Length\ of\ H} Z_m$, reduces reciprocally with radial distance from the conductor.

Considering the areas marked *P* and *Q* in Fig. 2(b), it can be seen that the only magnetic flux change in these areas occurs at the wavefronts themselves. Now the longitudinal electric field in area *P* is given by, $\oint E_P.dl = \int_P \dfrac{dB_P}{dt}.ds = \dfrac{d\phi_P}{dt}$ and neglecting propagation delays is the same throughout the region, since the flux density *B* only changes at the wavefront itself. The negative going changes in E_f are continually propagated back to the conductor, reducing the *E* field in the region as it moves back. Therefore the *E* field at the surface of the conductor continually falls since, $E_{front} \rightarrow E_{regoin} \rightarrow E_{surface}$. Thus the longitudinal electric field is transient and only exists when changes are moving outwards and energy is required from the source to establish the surrounding *H* field. However, the *H* component of the propagating *E&H* wave remains unchanged at all points behind the wavefront, since it is maintained by the conductor current *I*. Any further changes in conductor current produce corresponding changes H_f, that are added to the existing steady-state values of *H* as they propagate outwards from the conductor.

This isolated conductor is somewhat unrealistic since there must be a return path for the current, and fields will generally concentrate into the space between the forward and return conduction current paths. As well as these transient surface E_s field due to magnetic flux changes in the space surrounding the conductor there will in practice also be a longitudinal ohmic voltage drop along a conductor to support ohmic losses. This surface electric field, due to the ohmic voltage $V=IR$, does not radiate any fields and in fact absorbs power from external fields to support the I^2R losses inside the conductor.

3. H FIELD OF PARALLEL CONDUCTORS

The *H* fields associated with two parallel conductors each carrying a current *I* is shown in Fig 3. If the currents in each of the conductors are suddenly switched on the *H* fields propagate away from the conductors as indicated. It can be seen that the *H* fields tend in cancel each other out in the space between the conductors, and reinforce one another everywhere else. The integral of the *H* field along a path enclosing both conductors is of course 2*I* in the steady state. This is not true during the initial transient since there are also displacement currents in the propagating wavefronts that also have to be taken into account when applying Ampere's law. It is these displacement currents that allow Amperes Law to be satisfied at points beyond the approaching wavefront.

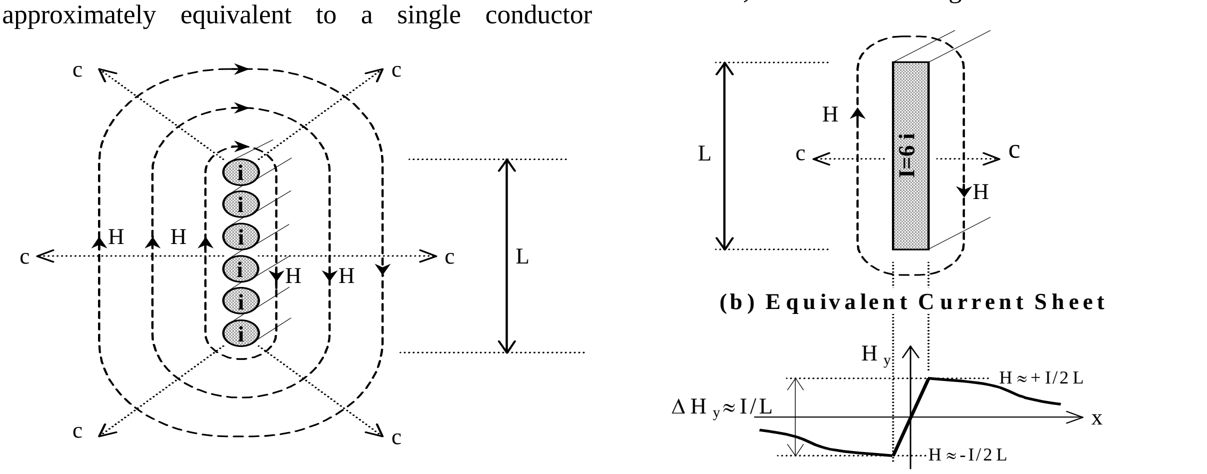


(a) Propagating H fields **(b) Steady State H Around Conductors**

Fig. 3 Magnetic Field Around 2 Parallel Conductors Each Carrying Current I

4. MAGNETIC FIELD OF CURRENT SHEET

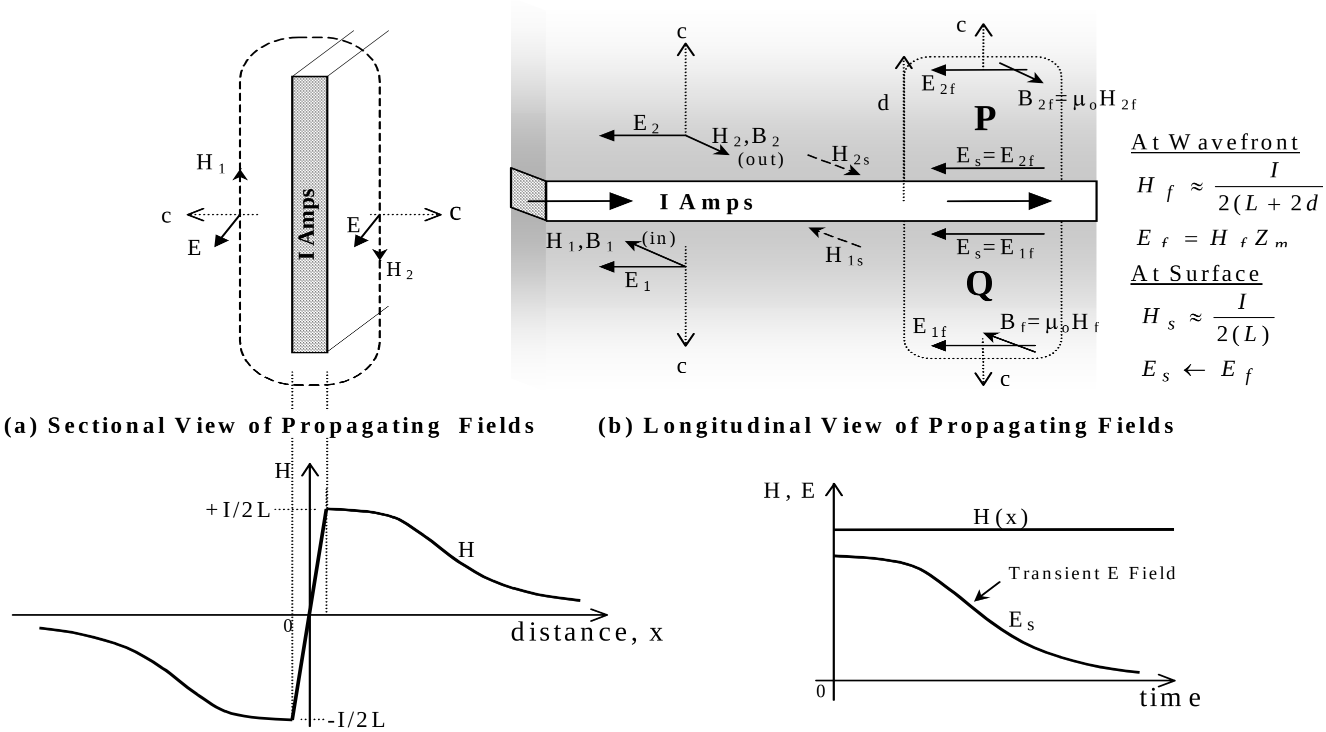
A linear array of *N* parallel conductors, each carrying a steady conduction current *i*, is shown in Fig 4(a). The interaction between the *N* propagating fields is such there is negligible *H* field between the parallel conductors. Thus the array of *N* conductors is approximately equivalent to a single conductor (current sheet) carrying a current *I* = *Ni* as shown in Fig 4(b). The steady state *H* field on each side of the current sheet is shown in Fig 4(c). It should be noted that the change in *H* on passing through a current sheet is $\Delta H \approx I/L$, where *L* is the length of the current sheet.



(b) Equivalent Current Sheet

(a) Linear Array of Parallel Conductors **(c) Steady-state H Field of Current Sheet**

Fig. 4 H Field Around Linear Current Sheet



(a) Sectional View of Propagating Fields

(b) Longitudinal View of Propagating Fields

(c) Variation of H Field with Distance

(d) Variation of Fields with Time

Fig. 5 Propagating H and E Fields around Current Sheet when Current is Changed.

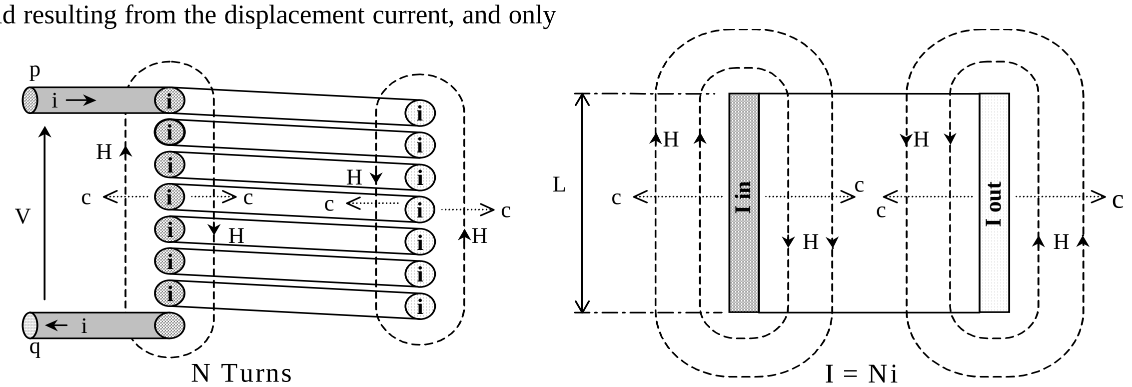
Propagation From Linear Current Sheet

When a current *I* is suddenly switched into a linear current sheet fields H_1 and H_2 propagate outwards from the sides of the conductor, accompanied by associated *E* fields, as shown below in Fig 5(a).

The surface values of magnetic field are $H_{1s} = H_{2s} \approx I/2L$. The magnitude of the wavefront fields E_f and H_f decrease, as the waves get further and further from the current sheet, and flow into ever increasing space as indicated in Fig 5(c). Although the propagating energy gets spread over greater and greater volumes as the distance from the conductor increases, there is an every decreasing surplus that gets reflected back to the conductor due to the ever diminishing E_f fields. The E_f field is generated at the wavefront by the magnetic field resulting from the displacement current, and only exists behind the wavefront while energy is required to magnetize the surrounding space, as illustrated inside the areas *P* and *Q* in Fig 5(b). Thus the *E* fields due to the propagating H_1 and H_2 are transient, but the conduction current *I* maintains the *H* fields. Thus after the initial transient, the *E* fields disappears but the H fields remain, as indicated in Fig 5(d). In practice as well as this induced transient E_s field there will also be a ohmic voltage drop along the conductor due to its resistance.

5. MULTI-TURN COILS

Multi-turn coils are used to intensify the magnetic fields produced by a current. Consider a coil is made up of *N* turns wound around as indicated in Fig 6(a).



(a) Multi-turn Coil (Rectangular or Circular)

(b) Equivalent Single Turn Current Sheet

Fig. 6 Multi-turn Coil and its Single Turn Equivalent

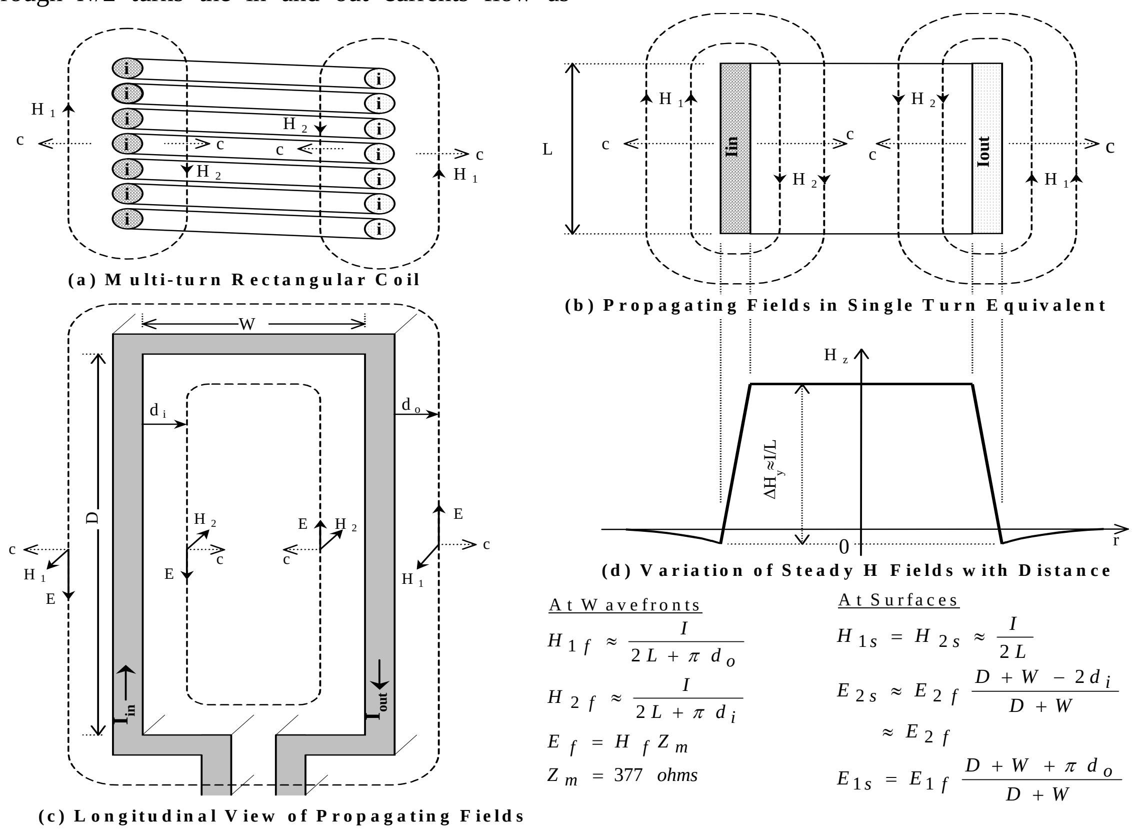
Very often these multi-turn coil are considered to act as a single turn current sheet carrying a current $I = Ni$ as shown in Fig 6(b). The main difference is in transient conditions, when the current i is suddenly turned on. In the multi-turn coil, current enters the top turn at point p and leaves at bottom turn point q. The coil acts like a transmission line, and initially current is returned through the coil as displacement current, flowing in the gaps between the turns. These displacement currents initially have to flow through N inter-turn gaps, and the characteristic impedance is NZ_{ot}, where Z_{ot} is the characteristic impedance of the inter-turn space between 2 adjacent turns.

The supply and return currents move into the multi-turn coil at velocity c, and after a one turn traverse the characteristic impedance as been reduced to $(N\text{-}2)Z_{ot}$. At this point the top turn carries conduction current into the coil, the bottom turn carries conduction current out of the coil, while the conduction current in the inner (N-2) turns is still zero. The multi-turn coil therefore acts like a tapered transmission line, with an ever-reducing field impedance, during the establishment of steady state conditions. After moving through N/2 turns the in and out currents flow as conduction currents in all the turns, and meet in what is effectively a short circuit, at the midpoint of the coil.

The single turn equivalent current sheet shown in Fig 5(b) however has a transition time that is N times faster than the multi-turn coil, and a different characteristic impedance.

6. AIR CORED COILS

An air cored rectangular coil with N turns carrying current i can be considered as a single turn current sheet carrying a current $I = Ni$ as shown in Fig 7. In order to simplify the situation further, the transmission-line process that actually establishes the coil current will be overlooked. It will be assumed that conduction currents can be instantly established in the coil, and only the fields produced by these currents will be considered. In spite of these assumptions, it is a useful concept to gain a better understanding of the magnetization processes. With these assumption the multi-turn coil and its single turn equivalent are shown in Fig 7(a) and (b). When the current I is switched into the coil fields H_1 and H_2 are set up at the surface of the conductors as indicated in Fig 7(a) & (b).



(a) Multi-turn Rectangular Coil

(b) Propagating Fields in Single Turn Equivalent

(c) Longitudinal View of Propagating Fields

(d) Variation of Steady H Fields with Distance

At Wavefronts

$$H_{1f} \approx \frac{I}{2L + \pi d_o}$$

$$H_{2f} \approx \frac{I}{2L + \pi d_i}$$

$$E_f = H_f Z_m$$

$$Z_m = 377 \ ohms$$

At Surfaces

$$H_{1s} = H_{2s} \approx \frac{I}{2L}$$

$$E_{2s} \approx E_{2f} \frac{D + W - 2d_i}{D + W} \approx E_{2f}$$

$$E_{1s} = E_{1f} \frac{D + W + \pi d_o}{D + W}$$

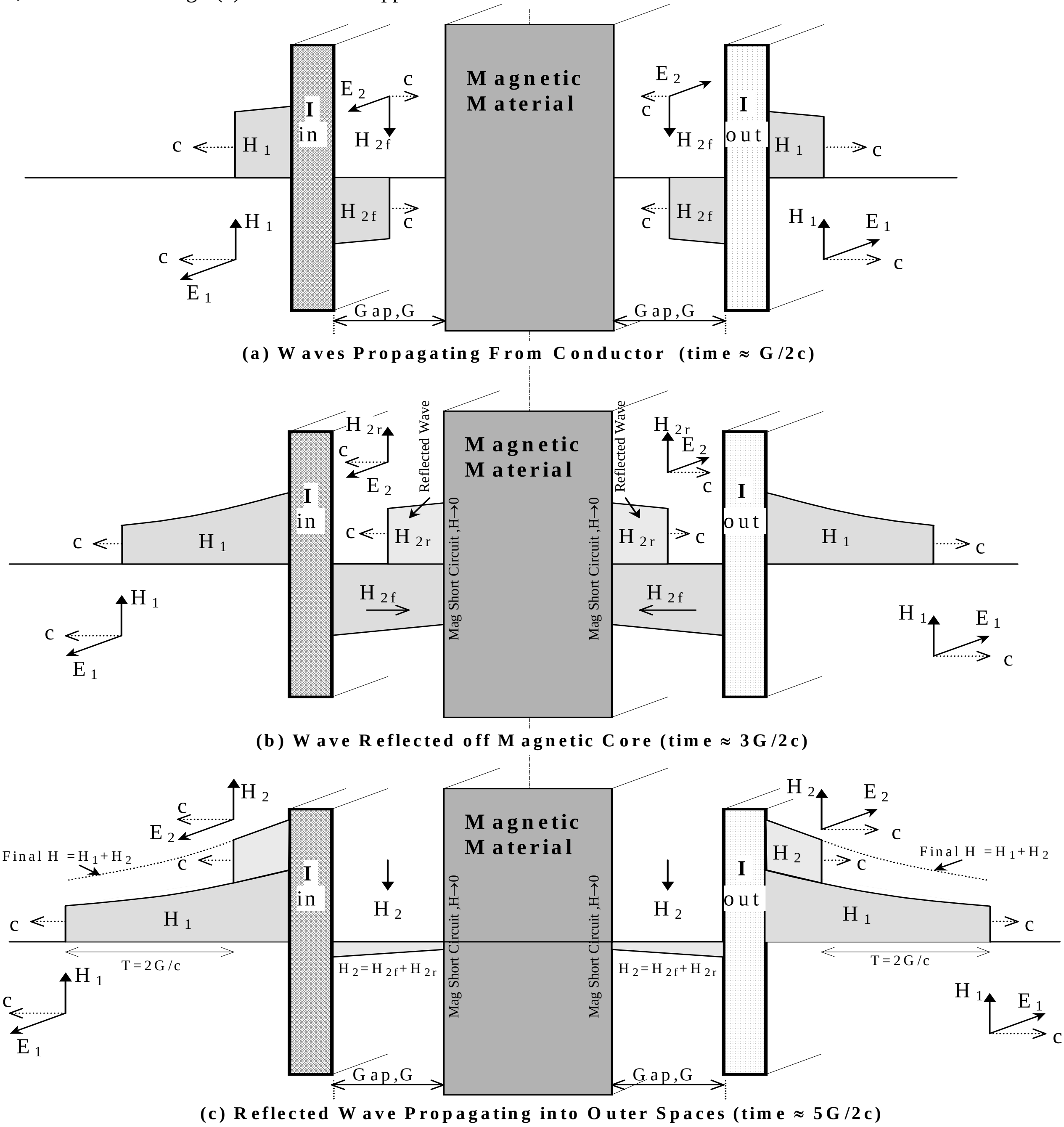
Fig. 7 Propagating Fields in Rectangular Coil

Although these fields are shown half way up the coil in Fig 7(c) they could be considered to start at the source end of the coil and be working themselves up towards to top, at the speed of light, in accordance with the actual transmission line reality. The change in H, and corresponding B fields, around the conductors creates E fields, and these E&H fields propagate away from both sides of the conductors as shown in Fig 7(b) & (c). The E fields are generated at the leading edge of the wavefront and the H fields are maintained behind the front by the coil conduction currents in the usual manner. As the H_2 fields move further into the coil space they meet at the centre and reinforce each other from there on. The oppositely directed *E* fields in the two wavefronts however, cancel each other at the vertical centre line plane of the coil, that could be considered as an electrical short circuit. By the time the waves have traveled the full width W of the coil space the magnetic field inside the coil is approximately $2H_2$, maintained by the coil current. The H_2 fields then cross over to the outer sides of the conductor and propagate outwards from there, as indicated in Fig 7(d). These fields oppose the original H_1 fields so the outside field reduces to H_1-H_2 ≈ 0.

7. COIL WITH MAGNETIC INSERT

When a magnetic core coil is inserted into a coil the situation is reversed with a very small magnetic field inside the coil, and the field concentrated mainly on the outside as indicated in Fig 8.

Normally the coil would be very close to the core, but in this case the gap G between the coil and the core as been increased, in order to show the electromagnetic fields that propagate through the gap when the coil current changes.



(a) Waves Propagating From Conductor (time ≈ G/2c)

(b) Wave Reflected off Magnetic Core (time ≈ 3G/2c)

(c) Reflected Wave Propagating into Outer Spaces (time ≈ 5G/2c)

Fig 8. Establishment of Magnetic Fields Around Coil with Magnetic Core.

The single-turn current sheet equivalent is shown in Fig 8. When the current in the coil changes the magnetic fields H_1 and H_2 propagate out from the outer and inner surfaces conductor respectively, accompanied of course by associated *E* fields, due to *dB/dt*, as shown in Fig 8(a). The *H* field is maintained by the coil current but the *E* field is transient

When the inwardly propagating wavefront H_2 reach the magnetic material the *H* field falls to almost zero, and the wavefront is reflected off the surface of the core back towards the coil as shown in Fig 8(b). The surface of the core is a magnetic short circuit and behaves like an electrical open circuit. Thus the direction of the *H* field is reversed by the reflection while the *E* field is unchanged. As the reflected wavefront moves back towards the coil it cancels out most of the original H_2 field, so that the resultant field inside the coil is very small, as indicated in Fig 8(c).

When the wavefront reaches the inner coil surface the reflected *H* field is transferred to the outer surface of the coil where it is adds to the original H_1., as indicated in Fig 8(c) The *H* field at the outer surface now rises to approx $H_{1s} = I/L$ and propagates into the outside spaces in the usual manner. Thus the steady state *H* fields are compatible with the rule that $\Delta H = I/L$ on passing through a current sheet.

The magnitude of the resultant *H* field inside the coil depends upon the ratio of the length of the coil to its width, the gap *G* between the windings and the permeability of the magnetic insert. The important thing to note is that there is very little *H* field within the coil itself and $H \approx I/L$ at the outside edges of the coil.

8. COIL WOUND ON MAGNETIC CORE

Consider a magnetic core suddenly excited by a current flowing through a multi-turn coil as in Fig 9

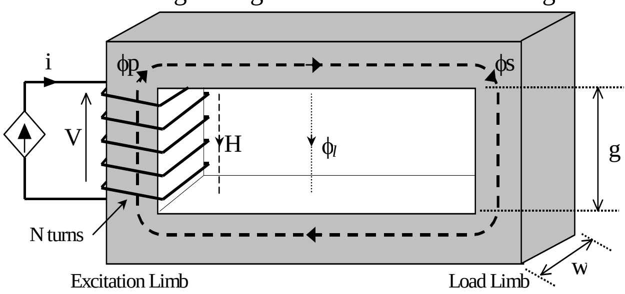


Fig 9. Coil with Continuous Magnetic Core.

. When the current is turned on the magnetising force *NI* produces a magnetic field $H = H_1+H_2$ that radiates from the coil conductors as *E* & *H* fields. H_1 moves away from the coil while H_2 moves into the interior of the coil towards the core. The H_2 field is reflected off the inner surface of the core in an inverted form as indicated in Fig 10(a). This reflected H_2 reduces the *H* field inside the coil itself, but reinforces the initial H_1 field moving into the inner core window as indicated in Fig10(b).

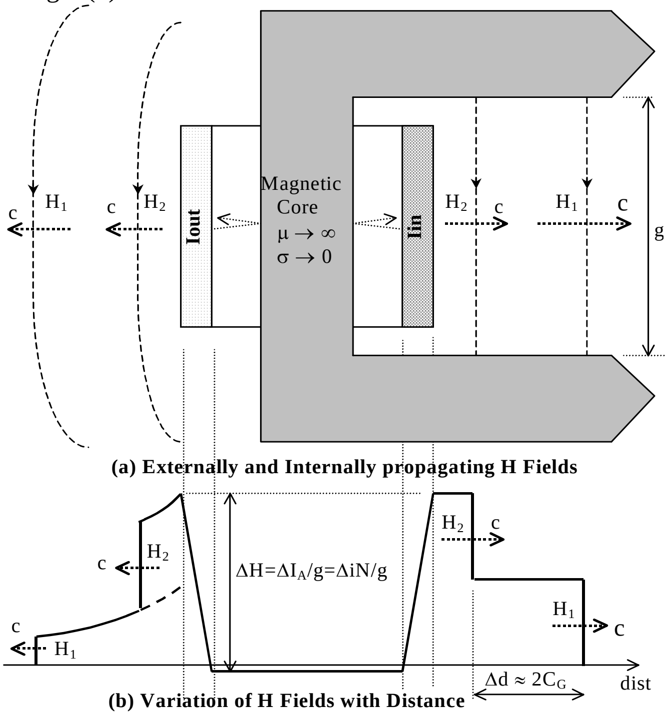


Fig 10. Propagation of H fields into Core Window

The resultant Field $H = H_1+H_2$ = Ni propagates into the core window with the horizontal limbs acting as guides. The characteristic impedance of the core ≈ 377w/g ohms, and the core is magnetised via the leakage flux and multiple reflection of the *EH* wave off the load limb [4]. Wave propagation continues in the window until the whole of the H field is eventually transferred to the surface of the core and diffuses into the interior [5]. The final flux, ϕ_{core} = mmf/Reluctance.

9. CONCLUSIONS

The propagation and reflection aspects associated with the establishment of magnetic fields in and around coils have been considered, at least at a conceptual level. It should be realized that these concepts are very fundamental to basic electrical engineering.

10. REFERENCES

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