Obtaining Force on a Moving Charge using Magnetic Vector Potential

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1. Introduction

The magnetic vector potential **A** (bold characters represent a vector) is claimed to be more fundamental than the magnetic field **B** since **B** can be derived from **A** via the vector curl function. But historically magnetic field effects were discovered, and formula derived, before the magnetic vector potential was conceived; since then the **A** field has played a minor role in physics. Although it is known that **A** fields can exist where there is no **B** field (i.e. when curl **A** = 0) the behaviour of electric charge moving within such non-curl fields is still the subject of controversy. To resolve this some authors have used the so-called convective derivative from fluid dynamics to deduce the force on a charge moving within *any* **A** field, deriving an effective **E** field given by $\mathbf{E} = -(\mathbf{v}\ .\ \nabla)\mathbf{A}$. Then via a series of vector identities they arrive at two force terms

$$\mathbf{E} = (\mathbf{v} \times \mathbf{B}) - \nabla_A(\mathbf{v}\ .\ \mathbf{A})$$

Where the subscript $_A$ means the ∇ (gradient) term only applies to A, velocity derivatives are suppressed. The first term is the well-known electric force on charge moving through a magnetic field while the second term is not yet accepted physics. When **B** = 0 only the second term remains and this is taken to apply to *any* non-curl **A** field. If we are to accept that the **A** field is fundamental, what is needed is a method for calculating forces in any **A** field independent of their curl. We show such a method using finite element math that yields exactly half the correct answer for fields that have curl. This discrepancy is discussed revealing some inconsistency in how we view the interaction between the field and the charge. Accepting the force doubling and then applying it to non-curl fields reveals some striking results. Accurate finite element analysis is a relatively new concept only made possible by the use of computers. Certainly the early pioneers in magnetics did not have this facility and could not analyse fields that have complex patterns. Even today finite element programs are limited to analysis of magnetic **B** fields, very few people have used them for **A** fields where **B** is not present (non-curl fields) since it is believed that this is of no interest.

2. What is wrong with $\mathbf{E} = -\nabla(\mathbf{v}\ .\ \mathbf{A})$?

Figure 1 shows a uniform **A** field where the arrows depict the vector direction and their lengths depict the vector magnitude. Also shown is a circular trajectory for a point charge q moving at velocity **v**. There is no magnetic field since curl **A** = 0. Taking the full $-\nabla(\mathbf{v}\ .\ \mathbf{A})$ term the scalar product (**v . A**) is by definition equal to the tangential component of **A** along the trajectory multiplied by the speed v of the charge.

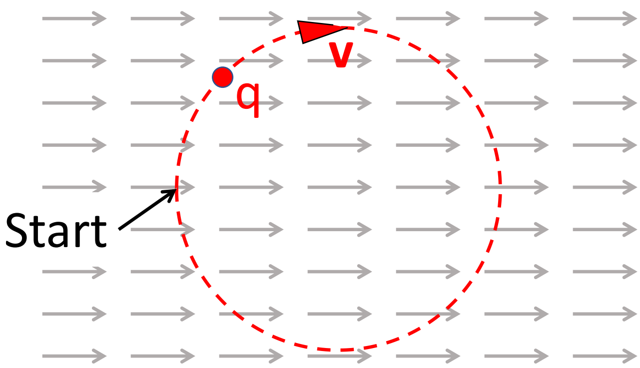

Figure 1. Circular trajectory in uniform field

The (**v . A**) product plotted against time from the starting position is of course simply a sine wave. Its gradient (the **∇** symbol) along the trajectory becomes a cosine function that yields zero voltage around the closed loop *but does not yield zero voltage between a point 90 degrees from start and one diametrically opposite that point.* That suggests that a slipring placed at that circle with brushes at the diametrically opposite positions would generate a voltage across the brushes, Figure 2.

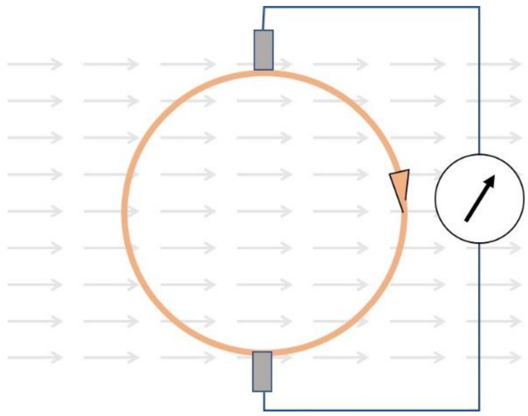

Figure 2. Slip ring in uniform field

No such voltage has ever been detected, even in the earth's **A** field that has the huge value of 200 webers/m at the equator and runs east-west. The reason there is no induced voltage is simply **A** being spatially uniform there is no change of **A** direction or of **A** magnitude around the loop so dA/dt is zero whatever the direction or magnitude of **v,** Figure 3.

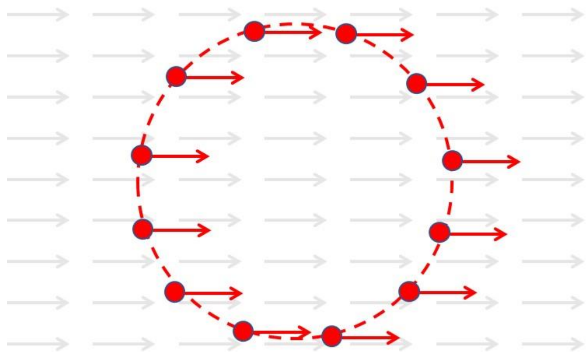

Figure 3. Showing constant A field around the ring

We need some method for computing voltage induction in any **A** field.

The product of charge q with the field vector **A** (q**A**) is like a hidden momentum and q endures a force if that momentum changes. This is not new and is mentioned in many texts but is not taught directly to students. That force can come about by (a) a change of momentum magnitude where the direction of the force is along **A**, or (b) change of momentum direction where a rotating **A** vector will create a force that is at right angles to **A** (this is why we get centrifugal force, the momentum **p** of a mass on a centrifuge gets a force at right angles to momentum **p**). If at a fixed point in space **A** varies with time then (a) explains transformer induction where a coil surrounds a transformer core that carries time-changing magnetic flux but there is no magnetic field at the conductor. Charge movement through a spatially non-uniform **A** field also imposes a time-changing A onto the charge. When we do the above (a) and (b) derivations for fields that have curl (a **B** field) we find each of the (a) and (b) force vectors have a longitudinal component (along the velocity direction) and a transverse component (at right angles to the velocity direction), so we end up with four components. The two longitudinal components are equal and opposite so when they are summed we get zero. The two transverse components are in the same direction so they sum to a new value. We obtain a result that is exactly half the classical **E** = **v** X **B** vector multiplication formula for magnetic fields. Ignoring that half factor (this appears in many other formula) this gives us a clearer understanding of what lies behind the formula that applies to all motors and generators. For those not familiar with vector math this vector multiplication is often taught as Fleming's LH and RH rules.

The fact that we can deduce force from the **A** vector pattern for magnetic forces, this procedure should also apply to a non-curl **A** field pattern where there is now no magnetic field present. Using this procedure we find the longitudinal components do not cancel, there is longitudinal induction present. This is something that has been missed in classical physics, and if it is true it has profound consequences in the search for new sources of energy.

I first reached this conclusion by carrying out a direct analytical approach, where the force components could be resolved by trigonometry. I have since found a method for deducing the force for any trajectory across any field pattern using a finite element approach to get the pattern. I am not aware that any one has used this finite element method correctly before.

3. Using a Finite Element Program

The program I have used is FEMM that is predominantly a 2D x-y solution that applies to systems that are very long in the z direction. Figure 4 is a standard 2D plot for an infinitely long conductor carrying a current.

The outer blue circle is the open boundary of the simulation, while the inner circle is the cross section of the conductor such as copper. Current flowing out of the page is uniformly distributed across that inner area. The arrows indicate the magnitude and direction of the magnetic induction **H** field that forms circular closed lines both within and without the conductor. The **H** field is of interest because of its direct relation to the current where the closed line integral around any circle equates to the current flowing through that circle. This gives rise to the **H** magnitude increasing with radius inside the conductor and decreasing with radius outside the conductor.

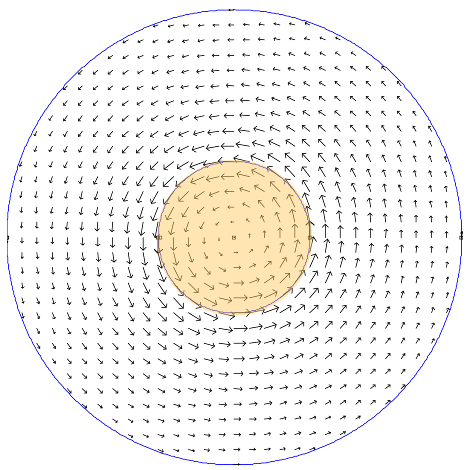

Figure 4. H field around an infinitely long conductor

It so happens that for an infinitely long magnetic core carrying a magnetic flux that is uniformly distributed over its cross section the **A** field follows those same rules. Thus we can use FEMM to compute **A** field lines simply by using a current value in Amps that is the wanted flux value in Webers, then the computed **H** field amps/meter become **A** field webers/meter, Figure 5.

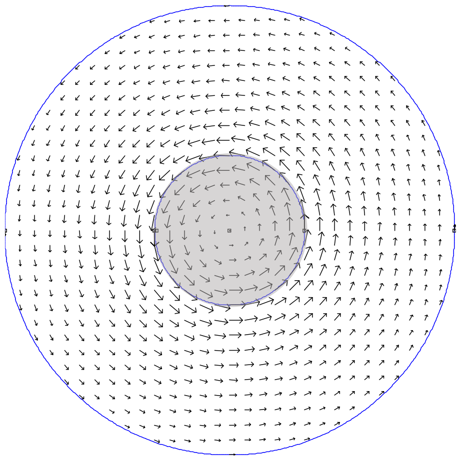

Figure 5. A field around an infinitely long core

The **A** field pattern inside the core has vector curl which computes to the **B** value in webers/m^2 that is emplaced there (as amps/m^2). The **A** field outside the core does not have curl, there is zero magnetic field there.

With computed **A** field at our fingertips, we can now turn attention to computing the forces on a charge moving through that field. Fortunately FEMM has the facility to export data for any line drawn across the output shown in Figure 5, this data can then be imported into a spreadsheet for further computation. Charts of the H field tangential component Ht, normal component Hn and magnitude |H| are available, and these become At, An and |A|. If we take a charge moving in a circular trajectory about the axis in Figure 5, one that follows an **A** field line inside the material where **B** is present, we find that At is constant and An = 0. The prediction could be that there is no dA/dt therefore no force on the charge, but we know that there will be a radial force due to the magnetic field **B**. How can we resolve this dilemma? The answer lies in our knowledge of inertial forces. A mass moving in a circular motion will endure a centrifugal radial force related to its mass m, and that comes about because its momentum m**v** is continually changing direction; that force is at right angles to **v** and has a magnitude ωmv where ω is the angular rotation rate of momentum. The charge also endures a radial force related to q because its electromagnetic momentum q**A** is continually changing direction; that force is at right angles to **A** and has a magnitude ωqA where ω is the angular rotation rate of EM momentum due to movement to a position where **A** has changed direction.

If we have a concentric circular trajectory outside the material where there is no **B** field we get the same result. This suggests a simple experiment where a loop carrying current is placed around a long magnet and this radial force is observed, either tending to expand or contract the loop. Perhaps this could be a long narrow helix bent around into a loop thus allowing it to be seen to expand or contract. The magnet could have a U-shaped keeper to act as a flux return so the loop would not see any magnetic **B** field, only the **A** field.

If we now turn to establishing forces for any trajectory we need to take account of rate of change of **A** magnitude *and* direction. To establish how the **A** vector has actually changed magnitude dA between successive points along a trajectory we simply use the change in |A|. FEMM charts the data against distance so we have the distance between successive points that with the known velocity along the trajectory gives the time interval dt, hence we now have d|A|/dt. This gives us an **E** vector parallel to **A** that can be used to compute components along the trajectory or normal to it. We already have At and An for the two successive points that can be used via the arctangent function to obtain the angle between each **A** and the trajectory line. Because this angle might change slightly over the small distance increment we take the average of the two angles, then using the cosine and sine of that angle gives the longitudinal and transverse components of this induction respectively.

To establish how the **A** vector has actually changed angle (rotation) at successive points along the line it is necessary to determine the angle of the line with reference to a fixed axis (this is to remove the effect of line curvature that would otherwise introduce an imaginary rotation. We do this by taking the arctangent of At and An, *then subtracting the angle of the line with reference to the fixed axis* (note that for straight line trajectories we don't need this subtraction). The change in this angle divided by the time increment dt yields angular velocity ω. Then the average of the two |A| values multiplied by ω yields the **E** vector that is at right angles to **A**. Multiplying this E value by the sine or cosine of the average arctangent values gives the longitudinal and transverse components of this induction respectively. Care must be taken to get the correct sign of each component.

Of interest is the voltage induction along a conductor. The two values of longitudinal component can be summed to get the total E value between successive points, then this can be integrated along the trajectory to yield induced voltage. Note that the voltage is proportional to the electron velocity, thus a current must flow for the voltage to be produced. For a conductor with constant area (as are most wires) electron drift velocity is directly proportional to current, thus the induced voltage has the effect of inducing a resistance value to be added to the actual resistance of that trajectory length of wire. The aspect of electronically inducing a resistance is not unknown, it is used extensively in systems with feedback and especially in oscillators where the positive feedback is analysed as inducing a negative resistance at the input. What is new is the possibility of inducing a negative resistance as described above thus producing oscillations that would otherwise be considered as anomalous.

To test this new method I had the advantage of someone prepared to place a circular slipring close to the surface of a large 100mm diameter NdFeB disc magnet. Here the **B** field (hence also the **A** field pattern) is the same both inside and outside the magnet close to the pole surface. Figure 6 shows the **A** field with the slipring superimposed on it, the diameter of the slipring being close to 50mm.

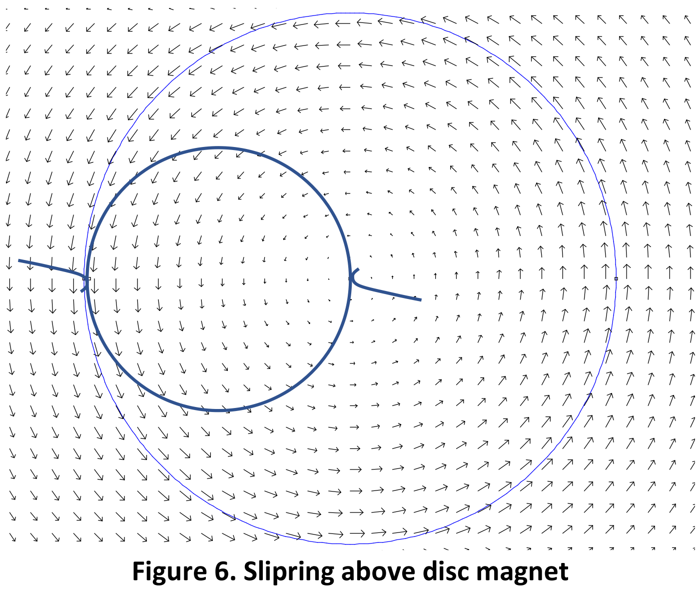

Figure 6. Slipring above disc magnet

Using -$\nabla(\mathbf{v}\,.\,\mathbf{A})$ to obtain the induced E field component an induced voltage would be expected across the slipring. Figure 7 shows that E field for 1000rpm plotted against degrees yielding a voltage of 127mV. No voltage was detected.

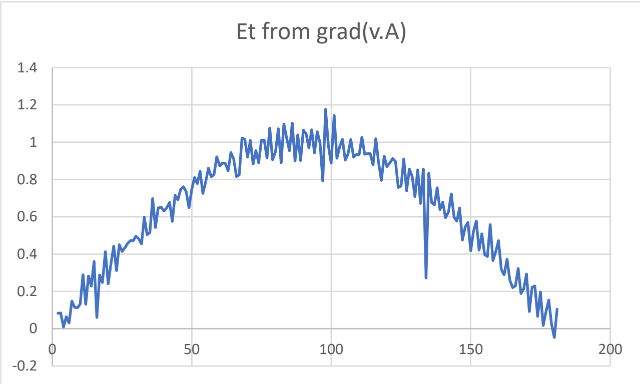

Figure 7. Et from -$\nabla(\mathbf{v}\,.\,\mathbf{A})$

For comparison Figure 8 shows the two values of Et using the amplitude-change plus rotation method outlined above. The induced voltages were 31.3mV and -32.1mV, hence summing to almost zero. The slight difference in voltage values can be put down to computation error due to the noise on those two waveforms, the noise being the result of using only a 1mm grid for the finite element procedure.

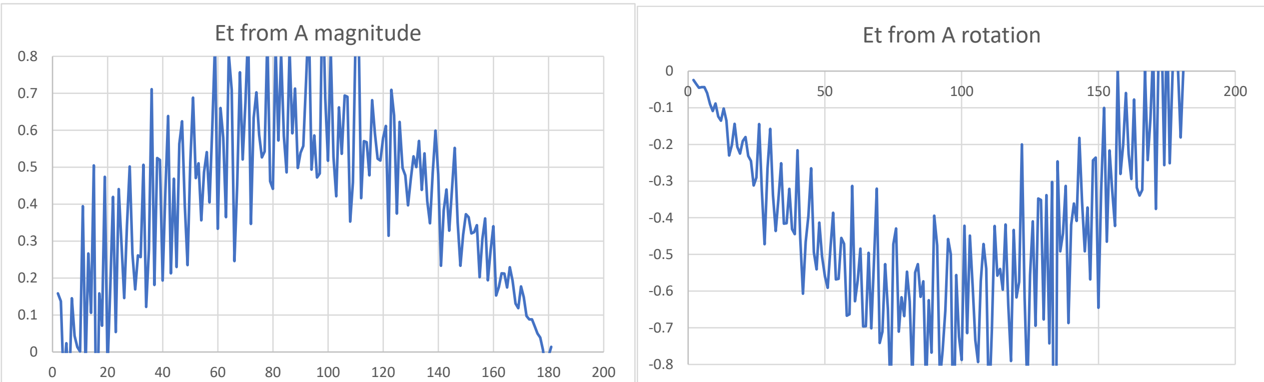

Figure 8. Et from A magnitude and A rotation

Figure 9 shows the same data as that in Figure 8 but this time using a 0.2mm grid showing reduced noise.

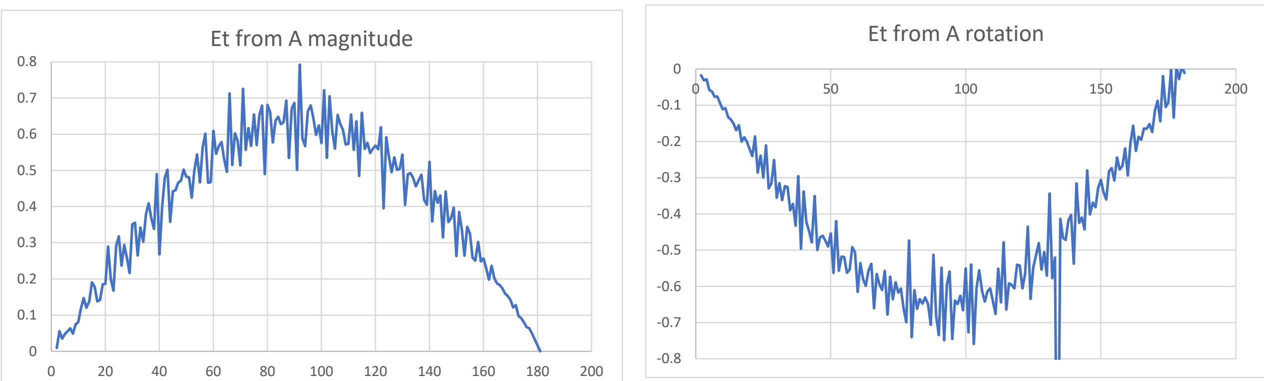

Figure 9. As Figure 8 but with finer grid

Ignoring the computation noise we see that the two Et values cancel everywhere along the trajectory, and this was confirmed in the experiment, it mattered not where the brushes were placed on the slipring, there was no induction. The same cancellation can't be said for the transverse effect. The two transverse component are shown in Figure 10.

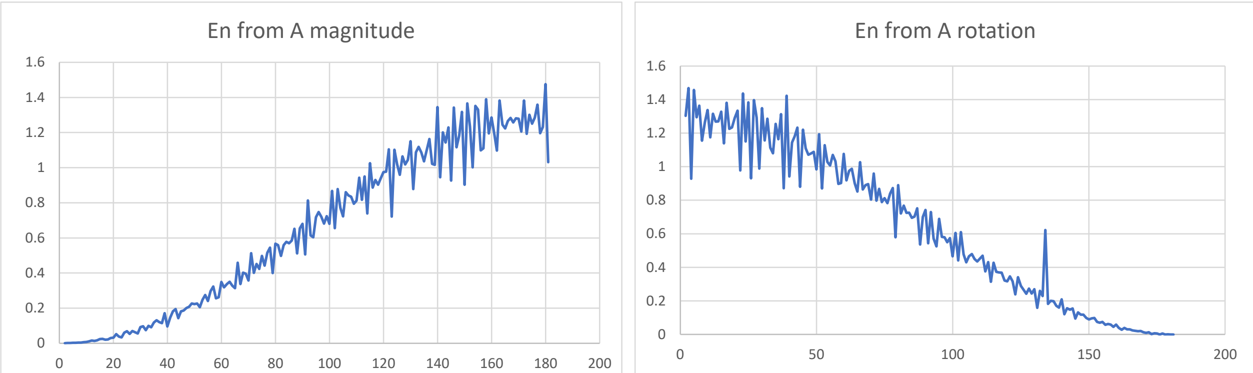

Figure 10. En from A magnitude and A rotation

We see that they sum to a constant value throughout the trajectory. Of course this does not yield anything to be observed in a slipring but were we to have moving charge as current in a circular wire it would endure that radial **E** force that agrees with the value obtained from **E** = **v** X **B** except for a factor of 2.

4. Non-curl A fields.

Having a method of using a finite element program to solve problems using the **A** field we can now turn our attention to fields where there is no **B** field such as fields around transformer cores. A ring core has axial symmetry and FEMM has an axisymmetric solution for systems that are symmetric around one axis, hence using 3D data in a r, θ, z coordinate system with results shown in the r, z plane. An example of such a system is a closed circular loop of conductive material carrying a current where the axis of the loop lies along z. In the displayed r,z plane the current flows into or out of the screen (the θ direction). As before we can use a known value of magnetic flux as a current value then use the computed H field lines as A field lines. Such a solution is shown in Figure 11 where the arrows show the magnitude and direction of the A field lines.

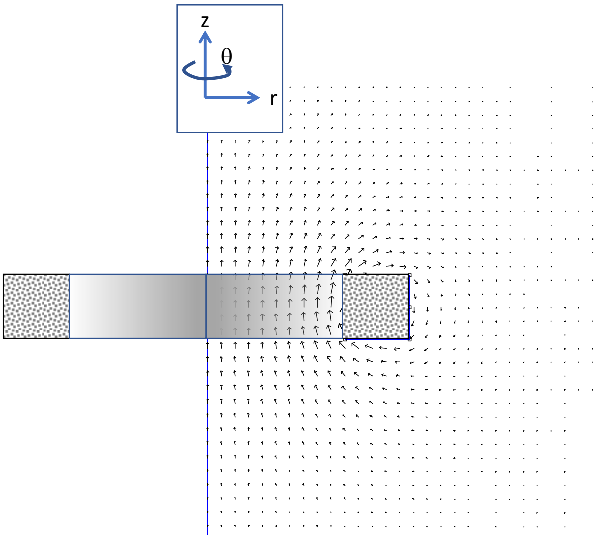

Figure 11. A field around magnetized ring core.

Of interest here are two ring cores magnetized such that the A fields oppose as shown in Figure 12.

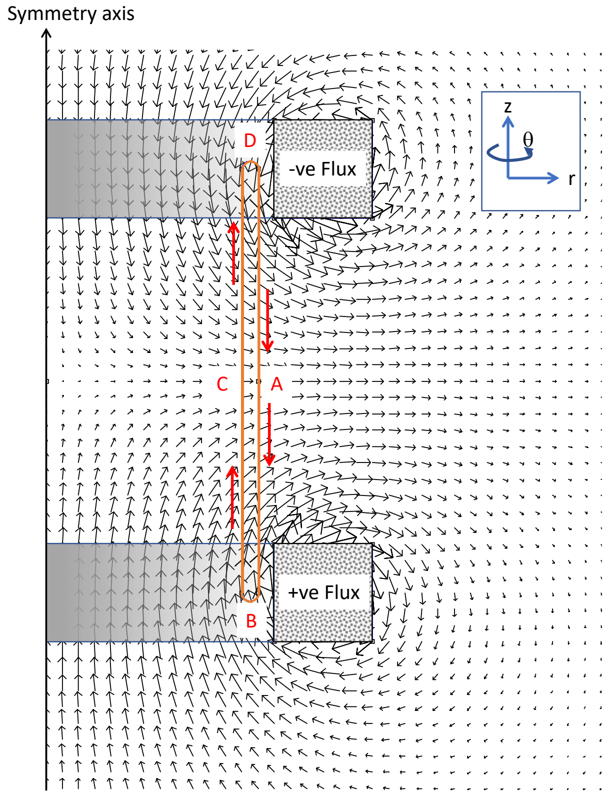

Figure 12. Two magnetized ring cores.

Also shown in Figure 12 is a closed hairpin loop with arrows showing the direction of a current flow around that loop. The reasons for the hairpin loop are (a) the realization that for reciprocal paths the induced voltage has the same polarity and (b) for current flowing around the sharp ends of a hairpin the A field does not change much in magnitude or direction hence there is almost zero induction there, thus there is voltage induction into the closed loop. Figure 13 shows tangential component of A as you move around the hairpin loop starting at position shown by the red A. Successive points around the loop are shown on the waveform.

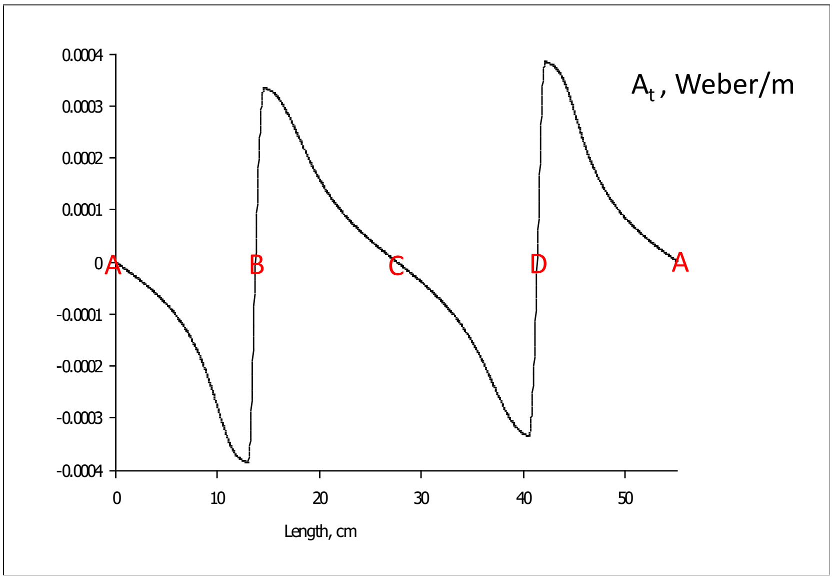

Figure 13. Tangential A component around the hairpin loop

The sudden positive swing at points B and D at the sharp ends of the hairpin do not contribute significantly to the induction because the magnitude of A does not change much there (if the ends were truly sharp the induction there would be zero). Figure 14 is the E field around the loop for a notional 1m/s electron velocity where it is seen to be predominantly positive and integrates to 0.71mV per turn. The mesh size used was 0.2mm while the hairpin was 1mm wide. A finer mesh would minimise the spikey noise at the B and D positions.

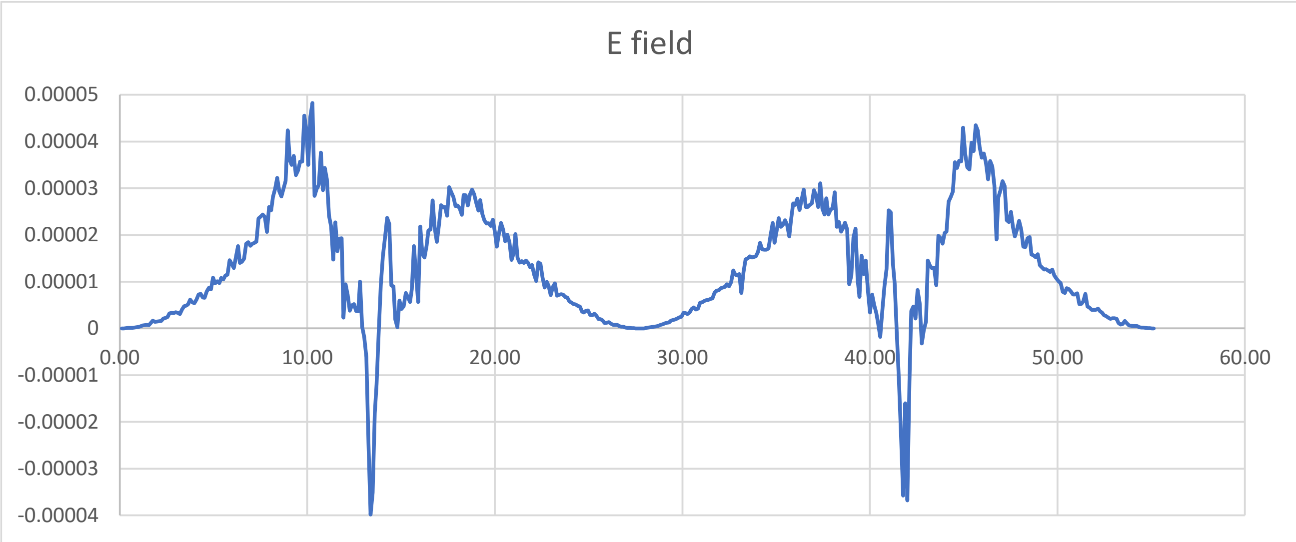

Figure 14. Electric field around the hairpin loop.

5. Practical multiturn system

Figure 15 shows a system where a coil is wound onto a thin-walled cylinder former to create a large number of hairpin loops in series. Note that the voltage is induced only when current flows and is proportional to that current hence effectively is an induced resistance. It should be possible to accurately measure the resistance of the coil and see if it changes slightly in the presence of the two magnetized ring cores.

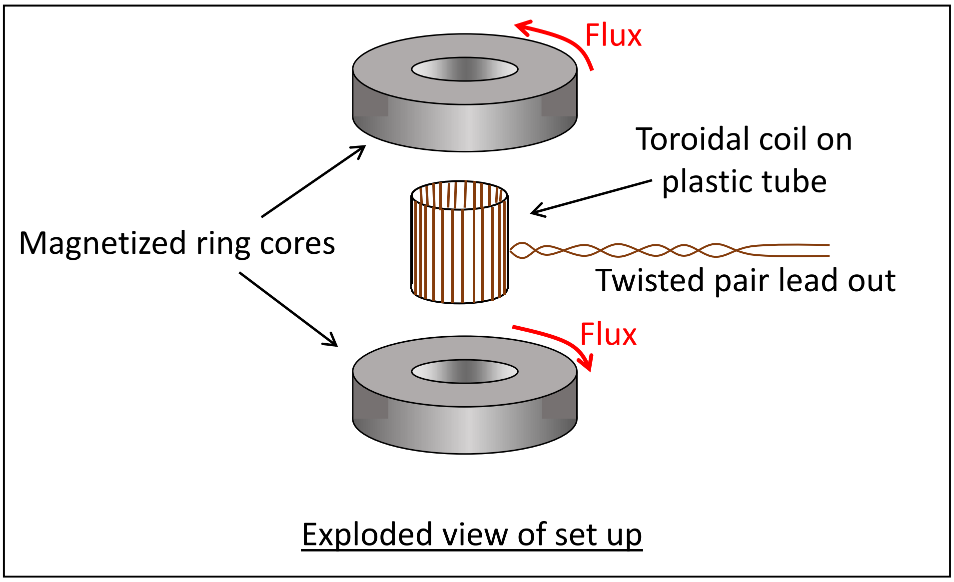

Figure 15. Proposed system

An alternative is to use magnetic soft material in the ring cores and to drive them with AC, whilst at the same time driving the hairpin coil with a slightly different frequency. If the claimed induction does take place the voltage waveform on the hairpin coil should indicate the beat note between the two frequencies.

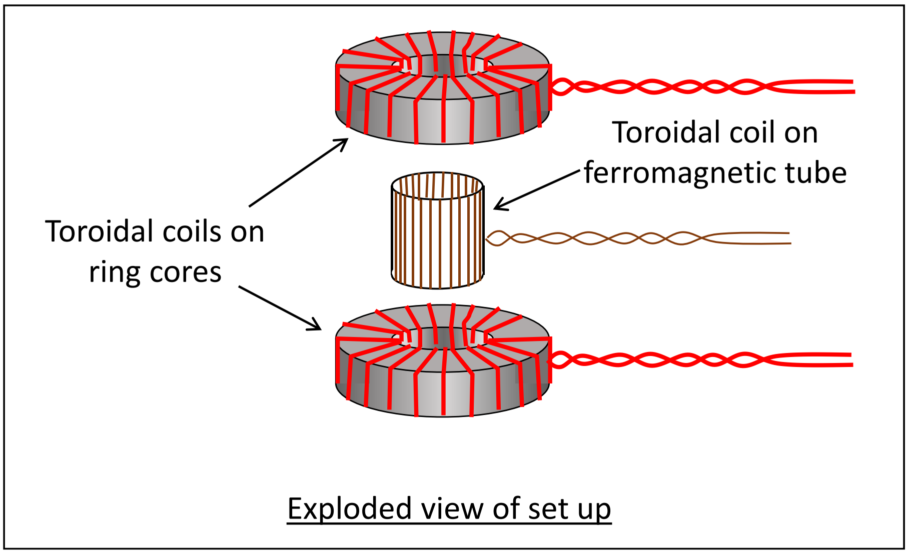

Figure 16. AC driven experiment

6. Using a superconducting coil

For electron drift velocity in copper wires the induced resistance value is orders of magnitude below the actual resistance. For a wire diameter d carrying a current i the electron velocity is given by $v = 9.35\times10^{-11} i/d^2$ m/s. Thus a 1mm diameter wire carrying 1 amp has drift velocity of only about 0.1mm/s. The wire length for the E field shown in Figure 15 is 55.1mm per turn and 1mm diameter copper wire will have a resistance of 1.21 milliohms/turn. The induced voltage for 1 amp current is 6.55×10^{-8}, hence the induced resistance is 6.55×10^{-8} ohms/turn, some 5 orders of magnitude down on the actual resistance. Such a small change in resistance would not be noticeable under normal circumstances and could explain why this effect has never been seen in experiments.

If we turn our attention to a superconducting hairpin coil then the induced resistance becomes the working resistance, and this can be negative. The superconducting coil will have inductance L that with a series negative resistance R will build up current exponentially from some starting value, this positive exponential having a time constant of L/R. In the absence of any applied starting value of current the build-up will start from thermal noise. This feature is well known in super-regenerative receivers where the negative resistance is created by positive feedback. That such negative resistance can be induced via a static magnetic vector potential field is a new discovery that needs exploring.

In super-regenerative receivers the final value reached in the build-up is many decibels greater than the noise or signal level and represents a single sample of the signal level. The current is then allowed to fall back to signal level and the process is repeated at this quenching rate. In our case the built-up current represents energy stored in the inductor, that energy can be extracted and the process repeated at the quenching rate. Seemingly the energy has appeared from nowhere, but it may be possible to show that it comes from some electron spins in the magnetized ring cores, more explicitly the spins responsible for the magnetization.

To be continued.