

## **Energy Conservation: Science or Ideology?**

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<http://mypages.iit.edu/~johnsonpo/aapt24apr.pdf>

## **Abstract**

Within the domain of classical mechanics, conservation of mechanical energy does not follow directly from **Newton's Laws**, but involves rather artificial assumptions as to the nature of the forces between objects.

Furthermore, there are additional difficulties in this matter. Namely, does energy conservation follow for a system consisting of an infinite number of elastically colliding **point masses** – provided that the total mass of the system is finite?

Or, do we have a problem, **Houston**?

## **Reference:**

“Nonconservation of Energy and Loss of Determinism:

I. Infinitely Many Balls ; II. Colliding with an Open Set”,

Foundations of Physics 39, 937 (2009); 40, 179 (2010),

P. W. Johnson (with D. Atkinson).

Energy conservation appears to be a direct and inevitable consequence of Newtonian mechanics, in that, for systems which interact through conservative potential fields or which undergo elastic collisions, it leads directly to conservation of energy. The work-energy theorem accounts for the energy balance, the net work being either stored as potential energy, or else presumably converted into other forms of energy.

## 1 Newton's Balls

Actually, there has been a substantial amount of recent exploration of this question – by philosophers rather than by physicists. These analyses have pointed out certain difficulties, which can be encapsulated in the classic elementary demonstration apparatus known as **Newton's balls**.

- Pérez Laraudogoitia, J.: A Beautiful Supertask. *Mind* **105**, 81-83 (1996).
- Earman, J. and Norton, J.D.: Comments on Laraudogoitia's 'Classical Particle Dynamics, Indeterminism and a Supertask'. *Brit. Jour. Phil. Sci.* **49**, 123-33 (1998).
- Pérez Laraudogoitia, J.: Earman and Norton on Supertasks that Generate Indeterminism. *Brit. Jour. Phil. Sci.* **50**, 137-41 (1999).

Here one places a series of identical masses precisely in a row, and gives the first mass a velocity  $v_0$  toward its nearest neighbor, the others being initially at rest. After a sequence of collisions, the last mass in the row proceeds off with velocity  $v_0$  in the direction of the initial motion, with all other masses being at rest.

This demonstration is a classic and concise demonstration of energy and momentum conservation in classical mechanics. However, what happens if there is no "last mass"? That is, what occurs if there is an infinite sequence of masses in the row. Do the energy and the momentum simply disappear into the continuum of mass?

## 2 Zeno Balls

Within a few heartbeats, a physicist might point out that such a system is “unphysical”, inasmuch as it would require an infinite amount of mass, and that it would be impossible to align the masses so as to maintain collinearity of the collisional process, without introducing friction or other dissipative effects. We will set aside the second objection for the present, since within the context of classical mechanics there is no intrinsic level of uncertainty or misalignment implicit in the formalism, and that it should be only a matter of sufficient care and cleverness to achieve a given level of precision. Does the requirement of a finite total mass in the system fix the problem? Remarkably, the answer is **NO**!

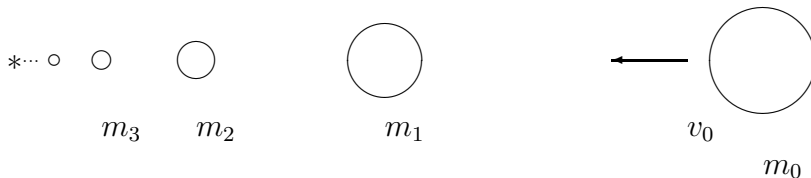


Figure 1. Collision of an infinite number of progressively smaller balls

Let us consider an infinite sequence of masses  $(m_0, m_1, m_2, \dots)$  aligned in order along a line, as shown. Suppose further that the mass  $m_0$  is given a velocity  $v_0$  toward the first mass  $m_1$ , which is at rest, along with all the other masses. After the collision the mass  $m_0$  leaves with velocity  $V_0$ , and the mass  $m_1$  goes forward with velocity  $v_1$ . The mass  $m_1$  strikes the mass  $m_2$ , leaving with velocity  $V_1$ , the mass  $m_2$  going forward with velocity  $v_2$ . This collisional round continues until all subsequent balls have been undergone two collisions. The requirements of energy and momentum conservation in this round of collisions are

$$\begin{aligned} m_k v_k &= m_k V_k + m_{k+1} v_{k+1} \\ \frac{1}{2} m_k v_k^2 &= \frac{1}{2} m_k V_k^2 + \frac{1}{2} m_{k+1} v_{k+1}^2 \end{aligned}$$

where  $k = 0, 1, \dots$ . Equivalently, we have

$$\begin{aligned} v_k &= \frac{m_{k+1} + m_k}{2m_k} v_{k+1} \\ V_k &= v_{k+1} - v_k = \frac{m_k - m_{k+1}}{m_k + m_{k+1}} v_k \end{aligned}$$

Let us write these relations as

$$\begin{aligned} v_{n+1} &= \frac{2}{1 + \mu_n} v_n \\ V_n &= \frac{1 - \mu_n}{1 + \mu_n} v_n \\ \text{where } \mu_k &= \frac{m_{k+1}}{m_k} \end{aligned}$$

Rather complete discussions of these collisional sequences are given in these references:

- Atkinson, D.: Losing Energy in Classical, Relativistic and Quantum Mechanics. Stud. Hist. Phil. Mod. Phys. **38**, 170-180 (2007).
- Atkinson, D.: A Relativistic Zeno Process. Synthese **160**, 5-12 (2008).
- Atkinson, D., and Johnson, P. W.: Nonconservation of Energy and Loss of Determinism I. Infinitely Many Colliding Balls, Found. Physics **39**, 937-957 (2009).

### 3 Constant Recoil Velocities

Here we shall draw special attention to the case in which all particles recoil with a common speed; i. e.,  $V_n = V$  for all  $n$ . When this occurs, the particles move in **lock step** after the first round of collisions, and there are no subsequent collisions. In terms of the parameters

$$\alpha_n = \frac{1 + \mu_n}{1 - \mu_n}$$

the requirement of a constant recoil speed  $V$  may be written as

$$\begin{aligned} v_{n+1} &= \frac{2}{1 + \mu_n} v_n \\ \alpha_{n+1} V &= \frac{2}{1 + \mu_n} \frac{1 + \mu_n}{1 - \mu_n} V \\ \alpha_{n+1} &= \frac{2}{1 - \mu_n} = 1 + \frac{1 + \mu_n}{1 - \mu_n} \\ \alpha_{n+1} &= \alpha_n + 1 \end{aligned}$$

There is a one-parameter family of solutions to this latter recursive formula for  $\alpha_n$ :

$$\alpha_n = \lambda + n$$

where the parameter  $\lambda > 1$  is otherwise arbitrary. The intermediate velocities are

$$v_n = \alpha_n V = (\lambda + n) V$$

and the parameter  $\lambda$  may be determined from the original velocity of the incident ball:

$$\lambda V = v_0$$

The corresponding mass ratios are

$$\mu_n = \frac{\alpha_n - 1}{\alpha_n + 1} = \frac{\lambda + n - 1}{\lambda + n + 1}$$

We may determine the masses themselves:

$$\frac{m_n}{m_0} = \prod_{k=0}^{n-1} \mu_k = \frac{\lambda(\lambda - 1)}{(\lambda + n)(\lambda + n - 1)}$$

Therefore, the total mass of all the balls is

$$M = \sum_{n=0}^{\infty} m_n = \lambda m_0$$

The initial momentum and kinetic energy are given by

$$\begin{aligned} P_i &= m_0 v_0 \\ 2T_i &= m_0 v_0^2 \end{aligned}$$

and the final values are

$$\begin{aligned} P_f &= M V = m_0 v_0 \\ 2T_f &= M u^2 = \frac{m v_0^2}{\lambda} \end{aligned}$$

For this case, as well as for a wide variety of other cases, momentum is conserved whereas energy has been lost. The intermediate velocities  $v_n$  increase with  $n$ , and in the limit the intermediate kinetic energy approaches a non-zero limit:

$$2 T_n^{\text{lost}} = m_n V_n^2 = m_0 u^2 \frac{\lambda(\lambda-1)}{(\lambda+n)(\lambda+n-1)} (\lambda+n)^2 \rightarrow m_0 v_0^2 \left(1 - \frac{1}{\lambda}\right)$$

This amount of energy disappears into the continuum in the process.

## 4 Completely Inelastic Collision

The case of a common recoil speed can be compared with the completely inelastic collision of a particle of mass  $m_0$  and initial speed  $v_0$  with a solid body of mass  $M - m$  that is initially at rest. The two masses coalesce during the collisions, and afterward the entire system of mass  $M$  moves with speed  $V$ . For the collision the momentum is conserved:

$$m_0 v_0 = M u$$

whereas this amount of energy is lost – presumably converted into “heat”.

$$2 T^{\text{lost}} = m_0 v_0^2 - M u^2 = m_0 v_0^2 \left(1 - \frac{1}{\lambda}\right)$$

This energy is converted into “heat” in this inelastic process.

The original collision sequence could be interpreted as a microscopic rendition of this corresponding inelastic collision, without the requirement of “binding forces” to keep the final mass intact. However, it is a mystery as to how an elastic collisional sequence could possibly mimic an inelastic process.

Note also that the intermediate speeds  $(\lambda + n) v$  become arbitrarily large at large  $n$ .

## 5 AB Balls

Suppose that an infinite set of masses  $\{m_n\}$ , with finite total mass  $M$  is confined by walls to a one-dimensional region of finite length. Suppose, as before, that the heaviest particle, with mass  $m_0$ , initially has speed  $v_0$ , the rest of the particles being at rest. Let the particles then collide elastically with one another, and subsequently bounce elastically off the walls.

This seems like a simple extension of what we have done above, but it is not. To see this, given the case of constant recoil, let us consider the state of the system after all the balls have collided with one another, but before any of them have reached a wall. They are all moving with the same speed towards the left, and will continue to do so until they hit the leftmost wall. Or will they? Which ball will strike the wall first? No ball can do so, for if ball number  $n$  were to hit the wall, its neighbor to its left should have struck the wall first, and this applies to any ball at all.

The inconsistency is identical to the one discussed by Alper and Bridger, in which an additional ball approaches the point of accumulation of an infinite set of balls that are all initially at rest. Indeed, by looking at the constant-recoil scenario from a co-moving frame of reference, we see a wall moving to the right, approaching the point of accumulation of the positions of an infinite set of stationary balls: exactly the Alper-Bridger case.

We have advocated a radical way around the impasse, namely that of embracing Aristotle's "potential infinity", in contrast to "actual infinity". The distinction is that between the limit of a finite system, as the size grows without bound, and an infinite system *ab initio*. A merely potentially infinite set of balls can consistently undergo collisions with a wall.

- Alper, J.S., Bridger, M.: Newtonian Supertasks: A Critical Analysis. *Synthese* **114**, 335-369 (1998).
- Atkinson, D. and Johnson, P. W.: Nonconservation of Energy and Loss of Determinism II. Colliding with an Open Set, *Found. Physics* **40**, 179 (2010).