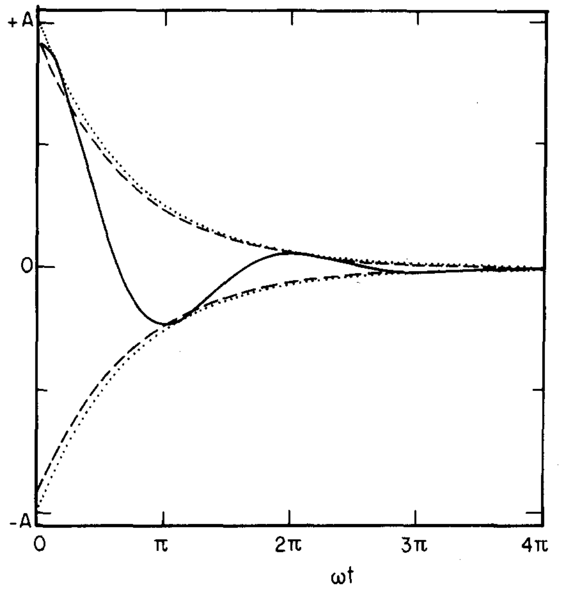

Fig. 1. Motion of damped harmonic oscillator for $\gamma/\omega_0 = 0.4$ with the initial condition $v_0 = 0$. Solid curve: $x(t) = A \exp(-\gamma t) \cos(\omega t + \phi)$. Dashed curve: $A'(t) = \pm A \exp(-\gamma t) \cos[\tan^{-1}(\gamma/\omega)]$. Dotted curve: $A(t) = \pm A \exp(-\gamma t)$.

envelope intercept the oscillation curve at different points. The time difference between the contact points of the envelope with the oscillation curve and the corresponding points of amplitude maximum is given by

$$t_n - t'_n = \tan^{-1}(\gamma/\omega)/\omega\,. \qquad (7)$$

Figure 1 presents these results when $\gamma/\omega_0 = 0.40$. The relations above clearly show the mistake made by these authors. However, we must say that in the limiting case where $\gamma \ll \omega_0$, we have $\gamma/\omega \simeq \gamma/\omega_0 \simeq 0$ and consequently the erroneous results become an excellent approximation.

ACKNOWLEDGMENTS

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Entropy change when charging a capacitor: A demonstration experiment

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INTRODUCTION

In recent issues of this Journal, two articles[1,2] appeared which contribute to the treatment of the second law of thermodynamics in an undergraduate student course in physics. Both papers illustrate the statement that an irreversible process can be transformed into a reversible one, if the transition from a certain initial state to a given final state is divided into N quasistatic steps and if N tends to infinity.

The paper by Gupta *et al.*[2] describes an experiment for the undergraduate laboratory which illustrates this fact by means of the extension H of a spring under the action of a gravitational force Mg. The authors show that the increase in entropy is $\Delta S = \frac{1}{2}MgH/N$ if the extension is produced by adding N masses, each M/N, successively. Thus ΔS tends to zero, if the number of steps N tends to infinity. As a classroom exercise, the authors also mention the stepwise charging of a capacitor.

In this paper we would like to describe a quantitative, but very simple lecture-room experiment which can be demonstrated within a few minutes. From our experience, this experiment is very helpful in making students more familiar with the subtle concepts of entropy and reversibility in thermodynamics. If a capacitor C is charged to a final voltage U_0 via a resistor R, the following energy considerations have to be taken into account. In the final state,

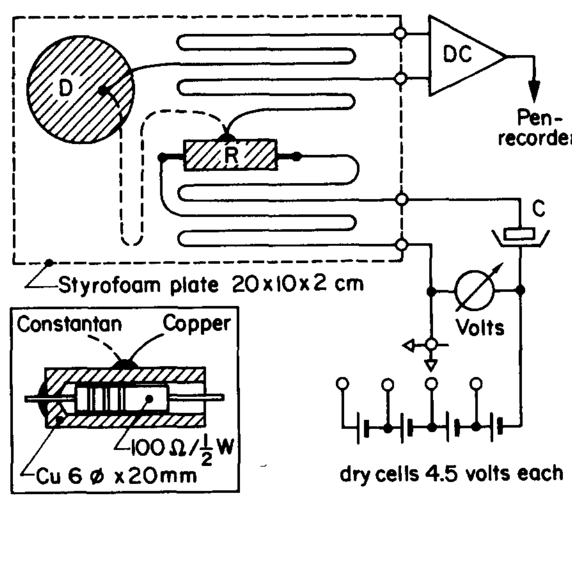

Fig. 1. R: 100-Ω resistor mounted into a copper tube (see inset). D: Copper disk 30⌀×5 mm as a temperature reference for the copper–constantan thermocouple. R and D are embedded in a styrofoam plate. Wiring: 0.2-mm copper wire meander shaped to reduce heat exchange with the environments. C: Electrolytic capacitor 10 000 μF. DC: Direct current amplifier, voltage again 100–1000. (⌀ = diameter.)

the energy $\frac{1}{2}CU_0^2$ is stored in the capacitor. During the charging process the current $I(t) = U_0 \exp(-t/RC)/R$ develops the heat

$$Q = R\int_0^\infty I^2(t)dt = \frac{1}{2}CU_0^2 \qquad (1)$$

in the resistor. It should be noted that Q does not depend on R.[3] Thus the energy delivered by the voltage source is $2\frac{1}{2}CU_0^2$. This is true for a one-step charging process. If, however, the charging process is divided into N steps, each of them with a voltage increase U_0/N, the total heat produced at the resistor will be

$$Q_N = \frac{1}{2}C\left(\frac{U_0}{N}\right)^2 \cdot N = \frac{1}{2}\frac{CU_0^2}{N} \qquad (2)$$

and the energy delivered by the source is $\frac{1}{2}CU_0^2(1 + 1/N)$. Assuming a constant temperature of the environments, the increase of entropy is given by

$$\Delta S_N = \frac{Q_N}{T} = \frac{CU_0^2}{2T}\cdot\frac{1}{N}. \qquad (3)$$

This relation shows that ΔS_N decreases with increasing number of steps N. In the limiting case of a quasistatic process where N tends to infinity, ΔS_N tends to zero.

II. EXPERIMENT

The experimental setup is shown in Fig. 1. The heat produced while charging the capacitor is measured with a very simple calorimetric device. It consists of a $\frac{1}{2}$-W radioresistor built into a small copper tube (see inset). The thermal contact is improved by soldering one of the resistor leads to the tube and by adding some transistor grease. The heat capacity mc of the whole arrangement is about 1.3 J/K. Although the resistance R does not appear in the foregoing equations, some arguments should be taken into account. First, the time constant RC should be small compared to that of the heat transfer from the resistor to the copper tube. Since the latter is found to be of the order of 5 s, a favorable value is $RC \leqslant 1$ s. On the other hand, R should be large in comparison with the internal resistance of the voltage source. For a capacitance $C = 10\,000\,\mu$F and dry cells as a voltage source, a value $R = 100 - 150\Omega$ turns out to be a good compromise. For thermal insulation, the calorimetric device is embedded in a styrofoam plate. The temperature increase ΔT is measured with a copper–constantan thermocouple directly soldered to the copper tube. The

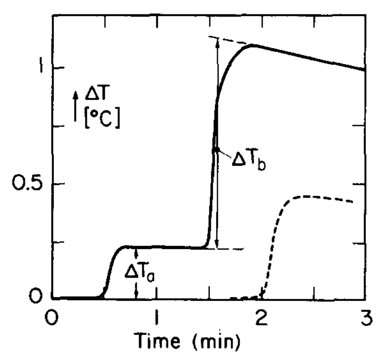

Fig. 2. Pen recorder plot of ΔT versus time for discharging a 10 000-μF capacitor loaded to $U_a = 9$V (ΔT_a) and $U_b = 18$V (ΔT_b). The dotted curve is obtained with $U_a = 9$V and $C = 2 \cdot 10^4\,\mu$F.

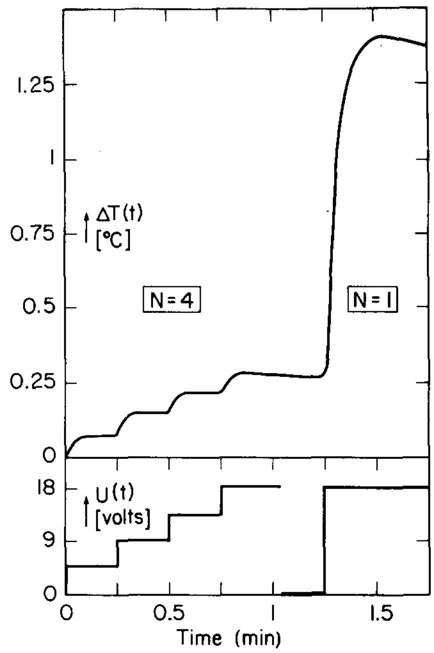

Fig. 3. Pen recorder plot of ΔT versus time for a four-step process ($N = 4$) and a one-step process ($N = 1$). The lower part shows the corresponding voltmeter reading.

copper disk D embedded in the same styrofoam plate acts as a temperature reference. Since the temperature increase

$$\Delta T_N = \frac{Q_N}{mc} = \frac{CU_0^2}{2mc}\cdot\frac{1}{N} = \frac{T}{mc}\cdot\Delta S_N \qquad (4)$$

is fairly small (1 °C or less), a much better insensitivity with respect to temperature fluctuations in the surroundings is achieved than by using an external thermocouple reference. During the experiment, the whole arrangement is covered with another styrofoam plate of the same size. The rather small thermoelectric voltage (40 μV/°C) is amplified by a factor of 100–1000 before being fed into the pen recorder. A chain of dry cells acts as a voltage source, but a chain of Zener diodes can also be used.

III. MEASUREMENTS

As an introductory experiment, the relation $W = \frac{1}{2}CU^2$ for the energy stored in a capacitor can be demonstrated. For this purpose, the capacitor is charged to a voltage $U_a \simeq 9$V and discharged via the resistor R. As can be seen from Fig. 2 the corresponding temperature increase is ΔT_a. A few seconds later, the capacitor is disconnected from R, charged to the voltage $U_b = 2U_a$, and once more discharged via R. According to the equation $mc\Delta T = \frac{1}{2}CU^2$, the temperature increase now yields $\Delta T_b = \Delta T_a(U_b/U_a)^2 = 4\Delta T_a$. The proportionality $W \sim C$ is demonstrated by connecting two capacitors in parallel (see dotted curve in Fig. 2). Operating at much higher voltages is not recommended, because then the heat loss to the environments (being proportional to ΔT) increases, causing a faster decay of the temperature after the experiment. Keeping the capacitor at a medium voltage for some time before the experiment is another precaution, because the capacitance of electrolytic capacitors is known to be slightly dependent on preformation. For more accurate results the well-known rules for the evaluation of ΔT in calorimetric experiments should be obeyed as indicated in Fig. 2 in the case of ΔT_b.

The results obtained for a four-step and a one-step charging process of 10 000-μF capacitor up to a final voltage $U_0 = 18$ V are shown in Fig. 3. The lower part of the figure indicates the increase of the voltage in steps of 4.5 V every 15 s. The corresponding response of the temperature $\Delta T(t)$ shows clearly that in the final state, i.e., the fully charged capacitor, ΔT differs by a factor of 4 for the charging process, with $N=4$ and $N=1$, respectively, as expected from (4). If the experiment is carried out with a chain of six dry cells, 3 V each, the ratio of the temperature increase yields $\Delta T_1/\Delta T_6 = 6$.

ACKNOWLEDGMENT

I am very grateful to my students for discussing the difficulties in understanding certain concepts in thermodynamics.

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[3]If the circuit is assumed to be free of ohmic and dielectric losses, LC oscillations will occur and the energy Q will be emitted in the form of electromagnetic radiation.

Electrostatic levitation of a dipole

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In a recent paper[1] F. H. J. Cornish has shown that an electric dipole can undergo self-sustaining accelerated motion in a direction perpendicular to its axis, with

$$a = (2c^2/d)\sqrt{(e^2/2mc^2d)^{2/3} - 1}\,, \tag{1}$$

where a is the acceleration, $\pm e$ are the charges, d is the separation distance, $2m$ is the dipole mass, and c is the speed of light.[2] According to the equivalence principle, it should therefore be possible for a dipole to "float" in a uniform gravitational field of strength

$$g = a\,. \tag{2}$$

The purpose of this note is to confirm that such levitation does indeed occur, and to elucidate the mechanism responsible.

Viewed from a freely falling reference system, there is no gravity, and the dipole accerates upward in accordance with Cornish's formula (1). Viewed from a stationary reference frame, the dipole is subject to a gravitational force downward

$$F_{\text{grav}} = 2\,mg \tag{3}$$

which must be balanced—if the dipole is to be at rest in this system—by an electrical force *upward*. But why should there be an upward electrical force on the dipole? The answer goes back to an observation of Boyer[3] and others that the electric field lines "droop" in the presence of gravity (see Fig. 1). At the location of $-e$ the field of $+e$ has a downward component, and hence there is an upward force on the charge. Boyer calculated the electric field to first order in g, and showed that the vertical force precisely accounts for the electrostatic contribution to the gravitational mass of the object. For present purposes, however, we require the *exact* electric field to all orders in g. This is to be found, for instance, in a classic paper of Rohrlich[4]: At a horizontal distance d from a point charge e, the vertical component of the electric field is

$$E = -eg/\{2c^2d\,[1 + (dg/2c^2)^2]^{3/2}\}\,. \tag{4}$$

Thus the net upward force on the dipole, due to drooping of the field lines, is

$$F_{\text{elec}} = e^2g/\{c^2d\,[1 + (dg/2c^2)^2]^{3/2}\}\,. \tag{5}$$

For perfect levitation, the electrical force upward (5) must balance the gravitational force downward (3):

$$2\,mg = e^2g/\{c^2d\,[1 + (dg/2c^2)^2]^{3/2}\}\,. \tag{6}$$

Solving for g, we recover Cornish's formula (1)—with $a = g(2)$.

Unfortunately, one is unlikely to witness this levitation in the laboratory. For the electron, $e^2/mc^2 = 2.8\times 10^{-15}$ m (the classical electron radius), and the dipole separation d would have to be almost exactly half of this: 1.4×10^{-15} m.

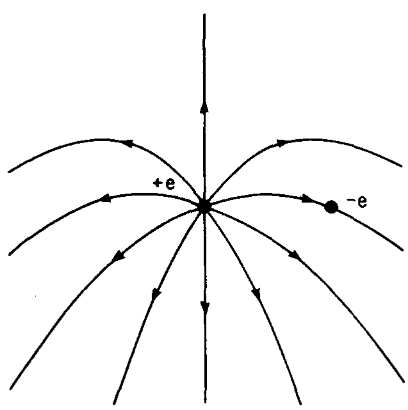

Fig. 1. "Drooping" electric field lines in a uniform gravitational field.

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[3]T. H. Boyer, Am. J. Phys. **47**, 129 (1979).

[4]F. Rohrlich, Ann. Phys. **22**, 169 (1963), Eq. (7.10).