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(71) Applicant: **UNIVERSITY PATENTS, INC., 537 Newtown Avenue, Norwalk Connecticut 06851 (US)**

(72) Inventor: **Reiss, Howard R., 1255 San Lucas Drive, Tucson Arizona 85704 (US)**

(74) Representative: **Rackham, Stephen Neil et al, GILL JENNINGS & EVERY 53-64 Chancery Lane, London WC2A 1HN (GB)**

(54) **Method and apparatus for induced nuclear beta decay.**

(57) Certain nuclear beta decay transitions, normally inhibited by angular momentum or parity considerations can be induced to occur by the application of an electromagnetic field. The energy released by these induced nuclear transitions is useful for the controlled production of power. These induced beta decay transitions are also useful to reduce the halflives of long-lived fission product wastes from nuclear fission power plants. Theoretical results are given for induced beta decay halflives as a function of the intensity of the applied field. The nuclides that can be treated in this way are all those found in Nature which are potentially useful energy sources, as well as ^{90}Sr and ^{137}Cs— the most radioactive of fission wastes. It is shown that electromagnetic fields of the type and intensity required to achieve useful power production and/or fission waste disposal can be produced in a practical way.

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METHOD AND APPARATUS FOR INDUCED NUCLEAR BETA DECAY

This invention relates to a method and apparatus for inducing nuclear beta decay transitions that are normally inhibited by angular momentum or parity considerations.

According to one aspect of this invention a method of inducing nuclear beta decay transitions comprises providing a medium which includes atomic nuclei that have forbidden beta decay transitions in which the initial and final nuclear states do not have the same intrinsic pairty or have total angular momenta which differ by more than one quantum unit of angular momentum, and applying to the medium an electromagnetic field which has an intensity sufficient to provide the angular momentum or intrinsic parity necessary to overcome the forbiddenness of the beta decay transitions of the atomic nuclei, thereby to induce the beta decay transitions.

According to another aspect of this invention an apparatus for inducing beta decay transitions comprises a medium which includes atomic nuclei that have forbidden beta decay transitions in which the initial and final nuclear states do not have the same intrinsic parity or have total angular momenta which differ by more than one quantum unit of angular momentum, field producing means for producing an electromagnetic field in the medium and means for energising the field producing means to establish the field at an intensity sufficient to provide the angular momentum or intrinsic parity necessary to overcome the forbiddenness of the beta decay transitions of the atomic nuclei.

The energy released in these induced nuclear transitions is useful for the controlled production of power. The induced beta decay transitions are also useful to reduce the halflives of long-lived fission product wastes from conventional nuclear fission power plants.

The background leading to this invention, theoretical predictions and practical examples will now be described and explained.

I. BACKGROUND AND FOUNDATIONS OF THE INVENTION

A. Introduction and Prior Art.

There is little history of work on causing changes in the rates of beta radioactivity. The common understanding is that it is an immutable natural process. There are two theoretical treatments of the influence on beta decay of extremely intense constant magnetic fields.[1/] These studies conclude that there would be essentially no effects for fields up to about 10^{12}G, but above about 10^{13}G beta decay rates would be increased noticeably. The problem is that the largest field that can be produced in the laboratory at present is about 10^{6}G.

The work just cited is of interest in an astrophysical context. Another astrophysical treatment of beta decay modification treats photon effects on beta decay in a stellar interior. The mechanism is one in which the photon produces a virtual electron-positron pair, with the positron being absorbed by the nucleus in lieu of beta-particle emission.[2/] The process can become of importance at temperatures of the order of 10^{8} K.

The present invention involves induced emission from a certain type of metastable nuclear state. There is precedent for this in atomic physics. The 2s state of the hydrogen atom is metastable; but it can be induced to decay to the 1s ground state by a nonresonant electromagnetic field. The emission occurs with at least one photon of inducing field type, plus another photon carrying the remaining energy of the 2s-1s energy level difference. The theory for this process was given by Zernik[3/]

for a first order process in the inducing field. The theory of arbitrarily high order processes involving a low frequency inducing field has also been developed.[4,5] Experimental verification of the lowest order induced process in hydrogen has been accomplished.[6] This invention is conceptually closely akin to this atomic work in that an externally applied electromagnetic field permits a relaxation of the conservation conditions that cause the metastability of the system with no field present. It differs from the atomic analogue in that the metastable state is nuclear, rather than atomic; the metastability is against emission of beta particles and neutrinos, rather than photons; and the emitted radiation therefore consists of a mixture of beta particles, neutrinos, and photons, rather than photons only.

B. Qualitative Effects of the Applied Field.

The present invention relates to the production of nuclear energy by the process of induced beta radioactivity. (One could use the words "stimulated" or "accelerated" rather than "induced." However, the word "stimulated" is suggestive of laser physics, where the stimulating radiation is resonant with an atomic or molecular transition, so that the stimulated radiation and stimulating radiation are of the same type. The word "accelerated" might be more acceptable, although it seems inappropriate in those cases where the nuclear species in question exhibits no radioactivity at all when not subjected to inducing radiation.) A number of nuclear species exist having real or potential beta decay transitions classed as "forbidden." The term "forbidden" is used in beta

decay physics, not as an absolute term, but to indicate that the transition is strongly inhibited. Such species therefore have very long halflives. It is the basic purpose and objective of the present invention to induce the beta decay of such species so as to materially reduce their halflives. With nuclides which normally exhibit beta decay, this would lead to an increased rate of release of energy. In like fashion, those nuclides which only have a potential beta decay can be induced to release that energy. In either case, these species would be useful fuel for the controlled production of power. In addition, since certain radioactive by-products or wastes of nuclear fission power plants have long halflives because of their property of beta decay forbiddenness, the present invention, when applied to these materials, would afford the advantage of rapidly converting such wastes to nonradioactive species. At the same time, useful energy could be extracted therefrom.

It is recognized in nuclear physics that beta decay transitions are unimpeded when the initial and final nuclear states have the same intrinsic parity and have total angular momenta which are either the same or differ by one quantum unit of angular momentum. These beta decays are categorized as "allowed." On the other hand, beta decay transitions are inhibited when the initial and final nuclear states either do not have the same intrinsic parity, or have total angular momenta which differ by more than one quantum unit of angular momentum. These beta

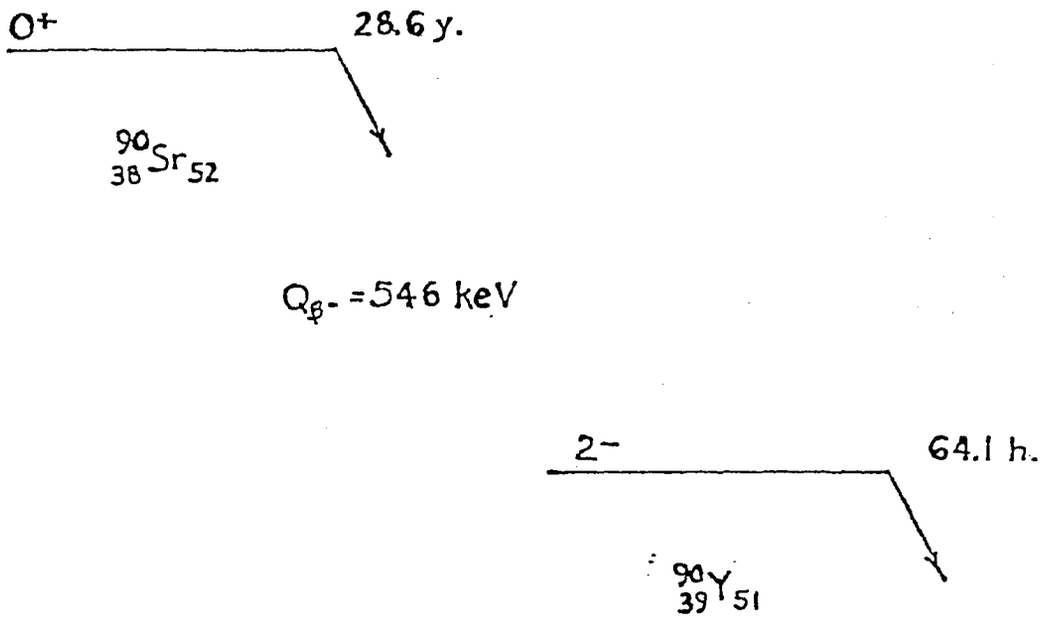
decays are categorized as "forbidden." Forbiddenness has a very strong influence on the observed halflife. For example, strontium-90 (one of the wastes of nuclear fission power plants) has a halflife for beta decay of 28.6 years, because the initial and final nuclear states have an angular momentum difference of two units, and have opposite parity. By contrast, strontium-92 beta decays with a halflife of only 2.7 hours. The two nuclei have very similar nuclear parameters for beta decay, the primary difference being that an allowed decay exists for strontium-92, but not for strontium-90. The degree of forbiddenness varies for different nuclides. Whereas strontium-90 represents a type of "first forbidden" decay, calcium-48 is an example of a "fourth forbidden" decay. In fact, calcium-48 is not observed ever to undergo beta decay, even though it is possible by every conservation rule other than angular momentum. Other nuclei with parameters similar to those for calcium-48, but with an allowed beta decay open to them, have beta decay halflives of the order of forty days.

In accordance with the present invention, forbidden beta decay transitions are rendered allowed. This result is accomplished by employing an externally applied electromagnetic field to serve as a reservoir of angular momentum and parity to remove forbiddenness from the beta decay. The necessity for having an electromagnetic interaction in the beta decay in addition to the usual beta decay interaction invokes a

penalty in the halflife expected. That is, the halflife for a beta decay induced by an electromagnetic field can never be as short as the halflife for an otherwise comparable allowed transition. Nevertheless, the halflife shortening possible through the intercession of an electromagnetic field in a forbidden decay can be very striking.

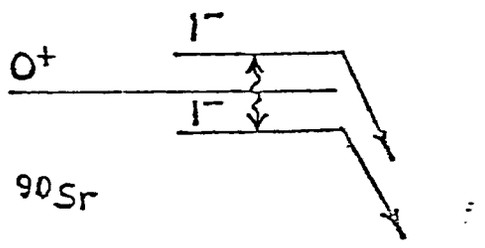
To explain how an applied electromagnetic field can remove forbiddenness from beta decay, it is convenient to introduce the concept of photons. (A photon is the basic elementary particle of the electromagnetic field. The fields considered here are coherent fields involving a superposition of different types of photons, so a photon representation is not suitable for practical calculation. Nevertheless, the photon provides a simple conceptual notion of how forbiddenness is removed.) Each photon of the electromagnetic field carries one quantum unit of angular momentum, and has negative intrinsic parity. (In the language of elementary particle physics, the photon is a pseudovector particle.) The angular momentum and parity of a photon are independent of the energy carried by the photon, and since there are no critical energy or momentum conservation conditions which the photon must satisfy, the choice of the frequency of the applied electromagnetic field is largely determined by practical considerations about the best way to achieve certain values of an interaction strength parameter to be discussed below.

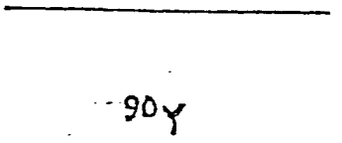
An illustration of the principle involved is provided by the beta decay of ^{90}Sr. The decay scheme for this is[7]



The superscript on ^{90}Sr and on its daughter nucleus ^{90}Y (yttrium-90) indicate the total number of nucleons in the nucleus. The left subscript shows the number of protons, and the right subscript gives the number of neutrons. Thus the beta decay of ^{90}Sr to ^{90}Y involves the conversion of one of the neutrons in ^{90}Sr into a proton, thus causing a transmutation from strontium to yttrium. (The further decay of ^{90}Y into the stable nuclide zirconium -90 is not shown here, since it is not needed for this discussion.) The horizontal lines show the energy levels of the nuclei. The 0^+ at the left of the line means that this ground-state energy level of ^{90}Sr has zero angular momentum and positive parity. The 2^- shown for ^{90}Y signifies two units of angular momentum, and negative parity. The opposite parities of the states, and the need for a change in angular momentum of two units,

accounts for the 28.6-year halflife of ^{90}Sr. In the presence of an applied electromagnetic field, the initial state (^{90}Sr) or final state (^{90}Y) can be thought of as emitting or absorbing a photon, with a resulting change in angular momentum and parity. For example, the ground state of ^{90}Sr in the electromagnetic field would have a 1^- component, so that the beta decay could proceed with a change of only one unit of angular momentum and no parity change, which is an allowed beta transition. An energy level diagram for this is





where the straight diagonal lines represent beta transitions, and the wavy lines represent photon absorption or emission. The amount of energy represented by the photon is greatly exaggerated in this diagram. On the scale of energy set by the difference between the ^{90}Sr and ^{90}Y ground states, a photon of the applied field contributes essentially zero energy.

The result of this interaction with the electromagnetic field is to enhance the transition rate due to removal of forbiddenness from the beta

decay, while accepting some penalty in the total transition rate due to the introduction of an interaction with the electromagnetic field. A significant overall increase in the transition rate achieved by application of the electromagnetic field in accordance with the present invention, has practical importance from at least two points of view. One is achieving useful power production from the beta decay of materials which are long-lived when not induced to decay; and the other is achieving relief from a major aspect of the problem of disposal of radioactive wastes arising from nuclear fission power.

C. Illustrative Nuclear Species to which the Invention Applies.

Some of the nuclear species most useful in the practice of the present invention will now be considered, and these will be discussed under two principal headings: those nuclides, found in Nature, most promising for power production; and the beta-active fission products which present the major burden of radioactive waste disposal, and which could also contribute to power production.

1. Naturally Occurring Nuclides.

The nuclear species relevant to this category are ^{40}K (potassium-40), ^{48}Ca (calcium-48), ^{50}V (vanadium-50), ^{87}Rb (rubidium-87), ^{96}Zr (zirconium-96), ^{113}Cd (cadmium-113), and ^{115}In (indium-115). (Other beta decay species found in Nature--^{123}Te, ^{138}La, ^{176}Lu, ^{180}Ta, ^{197}Re--will not be mentioned further, because of small abundance and/or low decay energy). A striking feature common to all

these nuclides is their very long halflives. The shortest lifetime in the list is possessed by ^{40}K, whose 1.277 x 10^9-year[8] halflife is about 1/4 the age of the Earth. The halflife of ^{87}Rb, 4.80 x 10^{10} years,[9] is more than ten times the age of the Earth. The other nuclei bracket the threshold of detectability. ^{115}In is listed at 4.41 x 10^{14} years.[10] The decay of ^{113}Cd (halflife 9.3 x 10^{15} years[11]) was detected for the first time only recently.[12] ^{48}Ca, ^{50}V and ^{96}Zr have never been observed to decay, even though it is possible in principle, and nuclear data compilations give only a lower limit for their halflives.

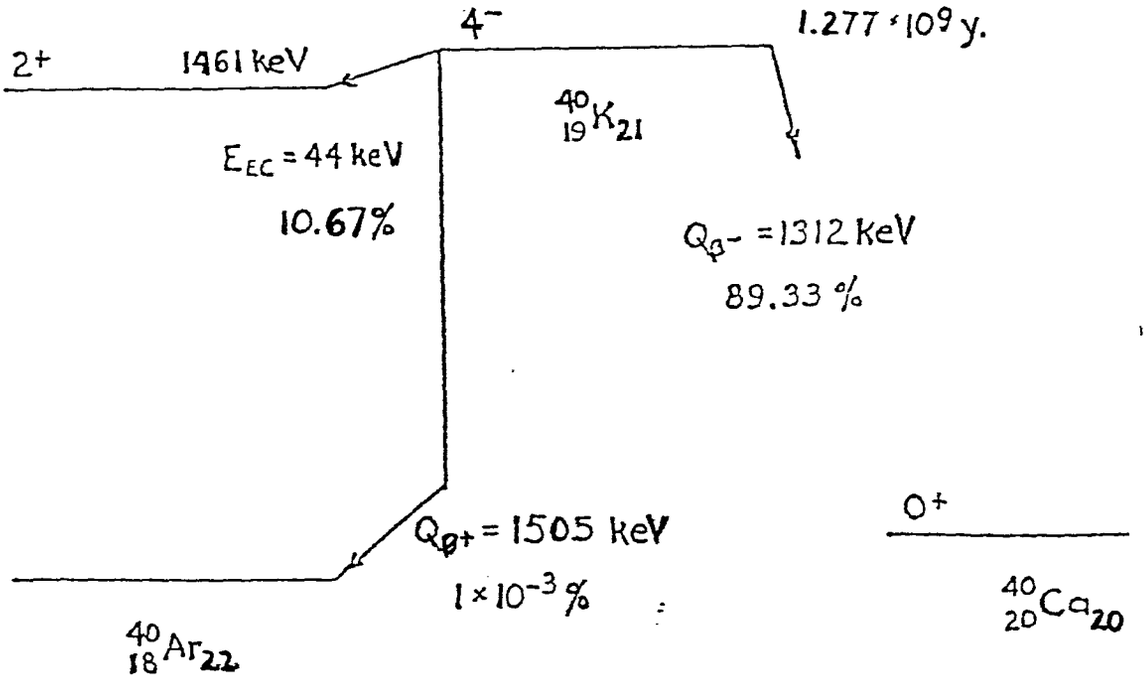
A feature of those materials which decay in a single stage of beta emission is related to the safety of power reactors with such fuels. The enhanced beta activity of the fuel requires the establishment of precisely the correct conditions within the reactor. If the reactor malfunctions, the beta decay enhancement is interrupted, and the fuel immediately reverts to the near-zero radioactivity of its normal state. There is no possibility of a runaway reaction. Furthermore, there is neither induced nor residual radioactivity to deal with upon shutdown. Even if some mechanical accident should breach the integrity of the reactor, any fuel or waste products which might escape are as innocuous as the original charge of fuel. The situation is not quite as straightforward

with ^{48}Ca and ^{96}Zr which experience a spontaneous beta decay following the induced decay. However, since the spontaneous decays have halflives of the order of one or two days, do not induce further activity, and emit nothing gaseous, hazards associated with an accident are minimal. Several weeks delay after an accident would be necessary to permit the activity to disappear.

Some of the nuclides considered here experience only beta decay, with no associated gamma emission. A feature of such a pure beta decay energy source is the prospect of direct generation of electrical energy. Essentially all of the energy in a pure beta decay appears in the charged beta particle, and in a neutral neutrino or antineutrino (with a trivial amount appearing in nuclear recoil). The neutrino energy is irretrievably lost, but if the kinetic energy of the beta particle is used to carry it to a collector separate from the fuel, the consequence is a separation of charge. This separation of charge creates an electric potential difference which can cause electrical current to flow.

The beta decay properties of ^{40}K will now be discussed. The natural decay of ^{40}K exhibits all the types of beta activity. Its beta decay can be

represented by the following energy level diagram, adapted from Endt and Van der Leun.[8]



The horizontal line for ^{40}K is the ground state, with a spin and parity of 4^-. The line slanting down to the right signifies a β^- decay to the 0^+ ground state of ^{40}Ca (calcium-40). This decay arises from the conversion of one of the neutrons in ^{40}K into a proton, which is the reaction

$$n \rightarrow p + e^- + \bar{\nu} \quad .$$

The three emergent particles from the reaction are the proton, electron (or β^- particle) and the antineutrino, $\bar{\nu}$. The antineutrino has such infinitesimally small probability of interaction with anything, that its primary importance in practical application is that it carries away, and thus "wastes," about half of the energy released in the beta decay. The 1.312 MeV of kinetic energy shown in

the diagram for the β^- decay thus over states, by a factor of about two, the average energy retrievable from the process. The 4^- to 0^+ transition is called "unique third forbidden."

The line in the ^{40}K level diagram slanting down to the left represents the capture of an atomic electron by the nucleus, leading to the first excited state of ^{40}Ar (argon-40). This EC (electron capture) is equivalent to the conversion of one of the protons in ^{40}K into a neutron, or

$$" p + e^- \rightarrow n + \nu " \quad .$$

The reaction is placed in quotation marks to emphasize the fact that such a reaction is energetically impossible with free protons and electrons, but can become possible within an appropriate nucleus. The symbol ν on the right hand side is a neutrino, the antiparticle of the antineutrino of β^- decay. The 4^- to 2^+ transition, "unique first forbidden," would be the dominant decay mode of ^{40}K since it is so much less forbidden than 4^- to 0^+, were it not for the very small transition energy involved in the EC decay--only 44 keV as compared to 1312 keV for β^-. These opposite trends give the result that 89.33% of the natural decays occur by β^- and 10.67% by EC. Since the EC process leads to an excited state of ^{40}Ar, it is followed quickly by the emission of a 1.46 MeV gamma ray as the newly-formed argon goes into its ground state.

The last decay mode shown on the diagram is β^+ decay, which is equivalent to

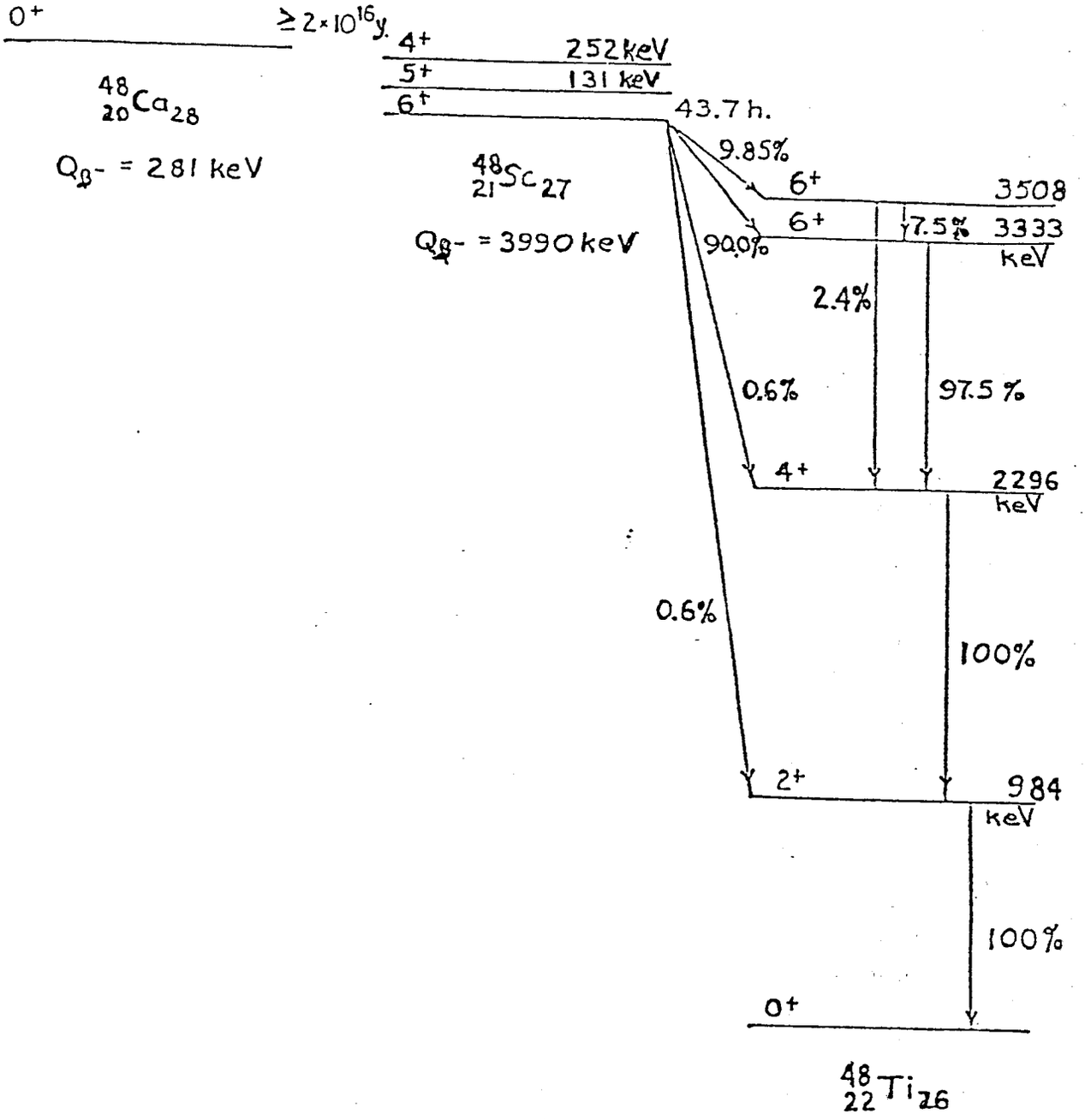
$$" p \rightarrow n + e^+ + \nu " \quad .$$

Again, the quotation marks are a reminder that this reaction is not energetically possible for a free proton, but it can occur in certain nuclei. The line in the diagram showing β^+ decay has a vertical portion followed by a slanted part. The vertical line is an indicator of an energy equal to the combined rest mass energies of an electron and a positron (totaling 1.022 MeV) which enters into the energy balance for β^+ decay. Thus the energy available to the positron and neutrino amounts to 1505 keV less 1022 keV, or only 483 keV. This accounts for the fact that a β^+ transition to the first excited state of ^{40}Ar is not possible. It is also most of the reason why the β^+ decay of ^{40}K is so strongly dominated by the β^- decay, even though both are 4^- to 0^+ transitions. (There are other reasons having to do with details of nuclear structure.)

^{48}Ca presents new features. It appears to be entirely stable, but β^- decay is energetically possible if a large angular momentum forbiddenness is overcome. If decay is induced by an electromagnetic field, the daughter nucleus is radioactive with both β^- (allowed) and gamma emissions.

For further explanation, the energy level diagram

of ^{48}Ca and its (potential) daughter nucleus ^{48}Sc (scandium-48) is useful:[13]



No beta transition is actually observed from ^{48}Ca, but its ground state is 281 keV above the ground state of ^{48}Sc. The two most probable beta decays shown for ^{48}Sc are allowed, so ^{48}Sc has a halflife of only 43.7 hours. Since the only levels in ^{48}Ti available for allowed transitions from ^{48}Sc are well above the ground state, the beta decay of ^{48}Sc is accompanied by gamma ray emissions of 175 keV (7.5%), 1212 keV (2.4%), 1037 keV (97.5%), 1312 keV (100%), and 984 keV (100%). These gamma ray transitions are shown by the vertical lines in the ^{48}Ti diagram. The overall energy difference between the ground states of ^{48}Sc and ^{48}Ti is 3.990 MeV. Thus, although the potential beta decay of ^{48}Ca itself is not particularly energetic, the end result of such a decay, when induced, is the release of a relatively large amount of beta and gamma ray energy.

The other nuclei under this heading will be discussed more succinctly than were ^{40}K and ^{48}Ca. The next heavier candidate, ^{50}V,[14] is interesting because it appears to be totally stable in Nature, and because it is the only case to be listed here in which potential β^+ activity is as significant as β^-.

Rubidium-87[9] is interesting because of its comparatively large isotopic abundance (27.85%), and its relatively great importance in terms of energy resources.

Zirconium-96[15] is very similar in nature to ^{48}Ca. ^{96}Zr is apparently non-radioactive, with the beta-active ^{96}Nb (niobium-96) as its daughter nucleus if decay is induced. ^{96}Nb decays to excited states of ^{96}Mo (molybedenum-96).

The nearly stable nuclide ^{113}Cd[11] has a higher degree of forbiddenness than ^{87}Rb, and slightly more available transition energy. The isotopic abundance

of ^{113}Cd is 12.26%, but it is less widely distributed in Nature than ^{87}Rb.

Finally, ^{115}In[10/] has the same forbiddenness in its beta decay as ^{113}Cd, a more energetic β decay, but nearly as long a lifetime. Natural indium is largely ^{115}In (95.7%).

2. Fission Products.

The second group of nuclides to be examined is the fission products which arise from the breakup of the fissionable fuel in nuclear reactors. A great many different fission products occur, but they all share the property of being neutron-rich when they are created, and so they exhibit β^- decay.

By far the most important beta decay nuclei from the standpoint of fission reactor waste disposal are ^{90}Sr (strontium-90) and ^{137}Cs (cesium-137). For the first 700 years or so of natural decay, ^{90}Sr and ^{137}Cs comprise virtually the entire burden of fission waste radioactivity.[16/] The reason for this arises only in part from the fact that they are among the most likely in occurrence in the probability distribution of fission products. More important is that their beta decays have a moderate degree of forbiddenness. The nuclei with allowed beta transitions decay with sufficient rapidity that their radioactivity is significantly depleted during the first year or so of waiting time after spent fuel rods are removed from the reactor. Nuclei with highly forbidden beta transitions decay so slowly as to moderate the level of radioactivity they present, although their persistence is thereby increased. However, ^{90}Sr and ^{137}Cs both have "unique first forbidden" beta decays (angular momentum change of two, and change of parity)

which give them halflives of the order of thirty years. This makes temporary storage of little use, and yet the levels of activity are high. It is also a particularly obnoxious halflife in terms of health hazards, since thirty years is the order of magnitude of a human lifetime. ^{90}Sr in particular becomes incorporated in bone when ingested, where it continues to damage the host organism. The biological halflife (i.e., the halflife for retention in humans) of ^{90}Sr is 49 years in bone and 36 years on a whole body basis.[17]

The decay of ^{90}Sr is to ^{90}Y (yttrium-90), which, in turn, has a first-forbidden, but more energetic decay to the stable ^{90}Zr nucleus.[7] Application of an appropriate external field would accelerate both ^{90}Sr and ^{90}Y decays, but the ^{90}Sr decay always remains the controlling factor.

In the case of ^{137}Cs, decay is directly to a beta stable nucleus, ^{137}Ba (barium-137).[18] The natural decay is 94.7% to the excited 11/2- state of ^{137}Ba, which is followed by emission of a 662 keV gamma ray. Decay directly to the groundstate of ^{137}Ba occurs in 5.3% of the cases. When induced by an applied field, the relative importance of the two final states in ^{137}Cs decay is dependent on field intensity.

When subjecting beta active fission fragments to decay-inducing fields, the most likely aim would be twofold: to reduce the level of radioactivity of fission wastes, and to produce useful energy thereby. Other long-lived fission products which experience forbidden beta decays include ^{85}Kr (krypton-85) which has a 10.72 year halflife because of the same kind of unique first forbidden decay as ^{90}Sr and ^{137}Cs. Also

included are much longer lived fission products like ^{135}Cs (2.3 x 10^6 years), ^{99}Tc (technetium-99, 2.13 x 10^5 years), and ^{129}I (iodine-129, 1.57 x 10^7 years), all of which have "second-forbidden" transitions. These, with a number of other fission products, could make a contribution to total energy release even though they represent less of a disposal problem than ^{90}Sr and ^{137}Cs. They are listed below, with the probability of occurrence as a fission product (yield)[19], normal halflife, and the maximum beta decay energy available when stimulated.

Nuclide	Yield (%)	Halflife (years)	Maximum Decay Energy (MeV)
^{135}Cs	6.7	2.3 x 10^6	0.209
^{93}Zr	6.4	1.53 x 10^6	0.090
^{137}Cs	6.23	30.17	1.173
^{99}Tc	6.1	2.13 x 10^5	0.291
^{90}Sr	5.9	28.6	0.546 + 2.279
^{144}Ce	5.45	0.7800	0.3182 + 2.996
^{87}Rb	2.56	4.80 x 10^{10}	0.2733
^{147}Pm	2.26	2.6234	0.2247
^{85}Kr	1.33	10.72	0.687
^{129}I	0.9	1.57 x 10^7	0.189

D. Potential Energy Resources.

1. Resources from Naturally-Occurring Nuclides.

The potential resources of energy contained in naturally occurring nuclides with inducible beta decays are difficult to state in absolute terms. Nevertheless, a relative statement of resources as compared to some more familiar material can serve to set the scale. The table below gives the energy potentially available from induced beta decay in a variety of fuels as compared to the energy in ^{235}U (uranium-235) and in ^{6}Li (lithium-6). ^{235}U is the only nuclide found in Nature which is subject to fission by slow neutrons. It is the present basis of the nuclear power industry. ^{6}Li is the ultimate practical source of the tritium that is necessary for the DT (deuterium-tritium) nuclear fusion reaction. The DT reaction represents the most likely prospect for success for practical energy from nuclear fusion. In the table, 200 MeV of energy is presumed to be available from each ^{235}U nucleus, and each ^{6}Li nucleus is taken to give rise to the 17.6 MeV of the DT reaction. The beta decay nuclides are evaluated in terms of the sum of half the beta decay energy plus all the gamma decay energy emitted in the progress of the decay to the final state. The abundance data used[20/] are atom abundances (atoms per 100 silicon atoms) of the elements as they occur in the igneous rocks of the Earth's crust.

Element	Atoms per 100 Si	Isotope	Isotopic Abundance %	Energy Per Isotopic Atom MeV	^{6}Li Energy / Energy	^{235}U Energy / Energy
potassium	4.42	^{40}K	0.01178	0.66	2.9×10^{-3}	1.5
calcium	9.17	^{48}Ca	0.185	3.87	0.55	280
vanadium	0.030	^{50}V	0.25	0.75	4.7×10^{-4}	0.24
rubidium	0.036	^{87}Rb	27.85	0.14	1.2×10^{-2}	6.1
zirconium	0.026	^{96}Zr	2.8	2.92	1.8×10^{-2}	9.2
cadmium	1.3×10^{-5}	^{113}Cd	12.26	0.15	2.0×10^{-6}	1.0×10^{-3}
indium	7×10^{-6}	^{115}In	95.7	0.24	1.3×10^{-5}	7.0×10^{-3}
lithium	0.091	^{6}Li	7.52	17.6	1.0	520
uranium	1.6×10^{-4}	^{235}U	0.7205	200	1.9×10^{-3}	1.0

Energy potentially available from induced beta decay of materials in the Earth's crust is seen to compare favorably with other types of nuclear energy. Beta energy resources are about one half of DT fusion energy resources, and they are about three hundred times greater than ^{235}U fission energy resources. This second comparison signifies that beta energy resources exceed the resources available in total from uranium, with breeding included. Furthermore, assessments based on igneous rock understate beta energy resources since calcium, for example, is much more abundant in sedimentary rock. An energy resource comparison of ^{235}U with ^{48}Ca in limestone favors ^{48}Ca by a factor of the order of 10^4.

Certain of the beta energy resources occur extensively in seawater, so the resources in the Earth's hydrosphere should be considered in addition to the resources of the lithosphere listed above. Seawater is not a significant source of either lithium or uranium, so a direct comparison as just done for igneous rock is not available. Instead, an index of resource assessment introduced by Hubbert[21] can be employed. He compared DD fusion resources with fossil fuels, based on the extraction of 1% of the deuterium from the oceans. With the same 1% extraction assumed for the beta energy fuels, and with the composition of seawater as given by Rankama and Sahama[22], the resource figures in the following table are arrived at.

Element	Weight Percent	Atoms Per H Atom	Isotope	Isotopic Abundance %	Energy Per Isotopic Atom MeV	Energy Total Fossil Fuel Energy	Energy Petroleum Energy
potassium	3.8×10^{-2}	8.8×10^{-5}	^{40}K	0.01178	0.66	13	240
calcium	4×10^{-2}	9.0×10^{-5}	^{48}Ca	0.185	3.87	1.2×10^{3}	2.4×10^{4}
vanadium	3×10^{-8}	5.3×10^{-11}	^{50}V	0.25	0.75	1.9×10^{-4}	3.5×10^{-3}
rubidium	2×10^{-5}	2.1×10^{-8}	^{87}Rb	27.85	0.14	1.5	29

The resources of potential beta decay energy are seen to be very large indeed. As compared to the total initial world supply of petroleum, the energy of ^{48}Ca is twenty thousand times as large, and ^{40}K and ^{87}Rb are also impressively larger in magnitude than the energy resources of petroleum. Since Hubbert has estimated that DT fusion energy resources are of the same order of magnitude as total fossil fuel energy, a comparison between beta energy resources in the hydrosphere and in the lithosphere can be made. For ^{40}K, ^{87}Rb, and ^{48}Ca, resources from the oceans are much greater than from the rocks; ^{50}V is similar in importance from either source; while ^{113}Cd and ^{115}In are available only from the lithosphere.

2. Resources from Fission Products.

Whether the primary intent of stimulating forbidden beta transitions in fission products is to reduce the burden of radioactive wastes, or to achieve useful energy therefrom, an assessment of the size of the power source thus available is appropriate. If nuclear fission power capacity reaches a level of 900,000 megawatts, then the long lived beta active fission products generated per year by this nuclear industry would have an energy content of the order of 2000 megawatt years. That is, if it should be possible to consume these fission products on a steady-state basis as they are produced, the total power available from the fission products is about 2000 MW, or about 800 MW of electricity if thermal losses are considered. Of this total, ^{90}Sr and ^{137}Cs taken together represent about 80%, and ^{135}Cs and ^{99}Tc together represent another 10% or so.

II. EXPLANATION OF THE INVENTION

A. Theory.

The basic idea of this invention is that forbidden beta decays can have their forbiddenness removed by the intervention of the angular momentum and parity contained in an applied electromagnetic field. There are some nuclear species whose beta decays are so highly forbidden by angular momentum and parity selection rules that their halflives are of the order of, or greater than, the age of the solar system. Other nuclides have such long halflives that no beta decay activity has ever been observed in them, even though it is possible in principle. Such quasi-stable nuclear species are thus still to be found among the mineral resources of the Earth. Other nuclides with forbidden beta decays are generated as byproducts of nuclear fission reactions. Both natural and manmade forbidden beta species contain potential energy resources which can be released for practical use when their beta decays are induced to occur by an applied field. Independently of (or conjointly with) any utilization of energy therefrom, the induced beta decay of fission products serves to reduce a major radioactive waste disposal problem.

The theory of induced beta decay is developed by first deducing the quantum mechanical dynamical equations for the relevant internal nuclear coordinates in the presence of an external field. This both specifies the equation of motion which must be solved, and serves to exhibit the effective charge with which the beta active portion of the nucleus is coupled to the external field. A formalism is developed which is the extension of the usual beta

decay theory to the case where the nuclear states and beta particle experience interaction with the applied field. Specific calculational examples are given to demonstrate the formalism in computing a final result.

The nature of the applied field is examined in its context as input to the nuclear calculation. The electromagnetic field experienced by a nucleus is a superposition of the externally applied field and the internal fields in the medium contributed by the atom or solid in which the nucleus is embedded. As expressed in terms of electromagnetic field potentials (in Coulomb gauge), it is shown that it is the vector potential which is unaffected by fields internal to the medium, and it is the vector potential which is effective in causing induced beta decay. The scalar potential is strongly modified by the internal fields, but the scalar potential is of no significance to induced beta decay.

B. Embodiments.

One example of an electromagnetic field source to induce beta decay is a coaxial transmission line operating in TEM (transverse electromagnetic) mode. The fuel is incorporated as the dielectric medium between the inner and outer conductors of the coaxial line. The power transmitted along the line is dumped into an absorptive load which is cooled by the same coolant employed to remove energy from the fuel. The electromagnetic field in the simplest TEM mode in a coaxial transmission line has just the form presumed in the theoretical treatment developed here. An example of the application of this system is given.

Another electromagnetic field source is a resonant coaxial cavity. This is similar to the coaxial transmission line, except that the line is terminated by reflectors at a cavity length equal to an integer number of half wavelengths of the cavity field (in simplest TEM mode). Other cavity lengths are possible, depending on the design of the input circuit, and how the termination is loaded. An example is given.

Many other field sources are possible. For example, transmission lines other than coaxial can be used, such as two-wire, four-wire, coaxial cage, strip line, etc. Furthermore, even some very ordinary circuit elements carrying alternating current will possess in their vicinity electromagnetic fields, a fractional amplitude of which corresponds to the TEM mode of a propagating plane wave as considered in the theoretical development. For example, one can use the fields in close proximity to a long conducting cylinder or to a solenoid. An in-between case with certain advantages is a hollow conducting torus. In these cases, the fuel is placed in those regions near the conductors where the field configuration and intensity are most advantageous. This would be, for example, in a cylinder coaxial with the long conducting cylinder, or a torus enveloping the hollow conducting torus.

Particular embodiments of a method and apparatus in accordance with this invention will now be discussed and described with reference to the accompanying drawings; in which:-

Figure 1 is a diagrammatic representation of a laboratory experiment carried out to verify the theory of the present invention;

Figure 2 is a diagram of a first embodiment of an apparatus including a coaxial transmission line; and,

Figure 3 is a diagram of a second embodiment of and apparatus including a coaxial resonant cavity.

III. DETAILED DESCRIPTION OF THE INVENTION

A. Theory.

1. Introduction.

To ascertain the effect of an externally applied electromagnetic field on the internal coordinates of a nucleus, the nucleus is considered to consist of two parts: a "core" and a "fragment." The core is a stable sub-nucleus of zero total angular momentum; and the fragment contains the nucleon (or nucleons) which is a candidate for beta decay, plus any other nucleons which are angular momentum coupled to it in initial or final states. The equation of motion is then separated into center-of-mass (CM) and relative coordinate equations giving, respectively, the dynamical equations for the motion of the center of mass of the entire nucleus and the relative motion of the fragment with respect to the core. It is this latter equation which must be solved.

The theory of induced beta decay involves a coupling of the nuclear fragment both to the external electromagnetic field and to the weak (beta decay) interaction. The coupling constant of the weak interaction is very small. On the other hand, the coupling constant to the electromagnetic field is very much larger, particularly in view of the relatively large intensity of the applied field. Furthermore, the field can be regarded as being on for a time approaching infinity before and after the beta decay occurs. Therefore, the weak interaction is treated as a perturbation which causes a transition of the nucleus-plus-field system from one state to another.

The combined nuclear-electromagnetic field system is explicitly time dependent, so the standard derivation of the perturbation formalism of beta decay (based on stationary nuclear states) is not appropriate. However, a derivation which is applicable in the presence of explicit time dependence gives a result which has the standard form.

The perturbation theory just described requires a knowledge of the state vector for the nuclear fragment in the presence of the field. The interacting nuclear wave function employed is the momentum translation approximation.

The electron emitted in the beta decay does not appear until the decay has occurred, and so its interaction with the field might be thought to be of little consequence. However, the field intensity parameter associated with induced beta decay is so large (and the mass of the electron sufficiently small) that the onset of effective interaction of the electron with the field occurs on a shorter time scale than the Heisenberg uncertainty time of the beta decay interaction. The onset of field-electron interaction is also much faster than the transit time of the newly created beta particle across the nucleus. The electron is therefore represented by a Volkov wave function, which is an exact solution for a free charged particle in the presence of an electromagnetic field.

A general expression for the transition probability for induced beta decay contains matrix elements for Fermi and Gamow-Teller transitions which are generalizations of those which occur in ordinary beta decay. Coupling of the electromagnetic field to the beta particle causes the transition probability to split into three parts corresponding to: direct

interaction of the field with the electron charge, interaction of the field with the spin of the electron, and an interference between the direct and spin terms. For the field intensities of interest here, the direct term and the spin terms are of approximately equal importance for the more energetic beta decays, although the direct term dominates for low energy decays.

The final form for the transition probability per unit time, or equivalently, for the halflife for induced beta decay, is written for any order of forbiddenness which is to be overcome by the inducing field, and for any number of nucleons in the fragment. Results for several fuel nuclei are given.

All of the above theory is predicated on a particular form for the applied electromagnetic field, and a relatively large intensity is found to be required. Possible sources of such a field are considered.

2. Separation of Variables.

In the cases of interest here, one can consider the initial nucleus to consist of a stable, relatively tightly-bound "core," plus a "fragment" of one or several nucleons outside the core. This fragment contains the nucleon which is a candidate for beta decay, plus any other nucleons which couple with it to provide the observed total angular momentum and parity of the nucleus. The "core" will always be such as to have spin and parity 0^+. For example, consider ^{90}Sr, which has 38 protons, 52 neutrons, and a total spin of zero and positive intrinsic parity ($J^\pi = 0^+$). The core nucleus can be considered to be ^{88}Sr, which has

50 neutrons, $J^{\pi} = 0^{+}$, and is the principal stable isotope of strontium. ^{88}Sr is particularly stable since the neutron number of 50 is a magic number, and the proton number of 38 corresponds to completed $p_{3/2}$ and $f_{5/2}$ shells beyond the magic number of 28.[23] The "fragment" constituents of two neutrons in ^{90}Sr outside the ^{88}Sr core are both $d_{5/2}$ neutrons, coupled together to give an overall 0^{+} state. One of these two neutrons will decay to a $p_{1/2}$ proton, which will couple with the remaining $d_{5/2}$ neutron to form a 2^{-} state in the daughter ^{90}Y nucleus.

A table of such separations into core and fragment will be given below for all of the nuclides of interest here.

When the Schrödinger equation is separated in terms of the CM coordinate $\vec{R}$ and the relative coordinate $\vec{r}$ of the fragment with respect to the CM, the result is [24]

$$i\partial_t \Psi_R = \left[e_t \phi(\vec{R},t) + \frac{1}{2m_t}\left(-i\vec{\nabla}_R - e_t \vec{A}(t)\right)^2 \right] \Psi_R \qquad (1)$$

$$i\partial_t \Psi_r = \left[\tilde{e} \phi(\vec{r},t) + \frac{1}{2m_r}\left(-i\vec{\nabla}_r - \tilde{e} \vec{A}(t)\right)^2 + V(r) \right] \Psi_r \ , \qquad (2)$$

where m_t and e_t are the total mass and total charge

$$m_t \equiv m_1 + m_2 \ , \qquad e_t \equiv e_1 + e_2 \ , \qquad (3)$$

and m_r and $\tilde{e}$ are the reduced mass and reduced charge

$$m_r \equiv \frac{m_1 m_2}{m_1 + m_2} \ , \qquad \tilde{e} \equiv \frac{e_1 m_2 - e_2 m_1}{m_1 + m_2} \ . \qquad (4)$$

The subscripts 1 and 2 refer to fragment and core, respectively. The so-called "natural" units ($\hbar$=c=1) are employed here.

The implication of the reduced charge expression, Eq. (4), is that the fragment behaves as if it has a positive charge when there is a preponderance of protons in the fragment, a negative charge when neutrons predominate, and a near-zero charge when equal numbers of protons and neutrons exist in the nuclear fragment.

3. S-Matrix Formalism.

When the beta decay transition probability is induced by an applied electromagnetic field, it is appropriate to view the asymptotic states as states which contain the full influence of the applied field, and the transition-causing "perturbation" will be the beta decay interaction. This means that the asymptotic states are explicitly time dependent, and not the stationary states normally employed. This is not a "textbook" situation, but a derivation of the appropriate S-matrix element (or transition amplitude) gives the result

$$S_{fi} = -i \frac{G}{2^{1/2}} \int d^4x \left[\overline{\Psi}_f \gamma_\mu (1 - \kappa\gamma^5) \Psi_i\right] \cdot \left[\overline{\Psi}^{(e)} \gamma^\mu (1 - \gamma^5) \Psi^{(\nu)}\right]. \quad (5)$$

This has precisely the appearance of the standard result, except that here it must be remembered that the nuclear and leptonic states are states containing the full effects of the applied electromagnetic field. In Eq. (5), Ψ_i and Ψ_f are the initial and final nuclear states, $\Psi^{(e)}$ and $\Psi^{(\nu)}$ are the electron and neutrino states, all in the presence of the applied

field; G is the weak interaction coupling constant; κ is the ratio of axial vector to vector couplings for nuclear beta decay; and the γ^{μ} , γ^{5} are Dirac matrices.

4. Interacting Nuclear States.

The calculational procedure developed above for induced beta emission is to substitute wave functions including the effects of the applied electromagnetic field. The formalism is otherwise the standard beta decay calculation. The nuclear wave function to be used must represent the effects of the applied field to an order of interaction which is at least as large as the order of forbiddenness of the natural beta decay. It must also be valid in the presence of electromagnetic fields of such intensity that the convergence of conventional perturbation theory is suspect. A technique ideally suited to the present problem is the momentum translation approximation[5/].

The momentum translation expression for the nuclear wave function in interaction with the electromagnetic field is

$$\Psi(\vec{r},t) = \exp(ie\vec{A}\cdot\vec{r})\ \Phi(\vec{r},t) \quad , \tag{6}$$

where $\Phi(\vec{r},t)$ is the nuclear wave function with no electromagnetic field. Validity conditions for the approximation in Eq. (6) are[5/]

$$eaR_0\omega/E << 1 \quad , \tag{7}$$

$$\omega R_0 << 1 \quad , \tag{8}$$

where a is the amplitude of $\vec{A}$, R_o is the nuclear radius, ω is the energy of a photon of the applied field, and E is the total nuclear transition energy. It will become evident later that eaR_o must be of order unity, and ω/E will be many orders of magnitude less than unity. Equation (7) is thus easily satisfied. Equation (8) states essentially that the ratio of the nuclear radius to the wavelength of the applied field is very small, which is amply satisfied for all fields of possible interest. One further condition for applicability of the momentum translation approximation is that no intermediate nuclear states are accessible through interaction with a small number of applied-field photons. This is certainly not possible here. Hence, Eq. (6) is an excellent approximation to employ here.

With the standard product solution for the noninteracting wave function

$$\Phi(\vec{r},t) = \psi(\vec{r})\, e^{-iEt},$$

the initial nuclear wave function in the presence of the field is, from Eq. (6),

$$\Psi_i(\vec{r},t) = e^{i\tilde{e}_i\vec{A}\cdot\vec{r}}\, \psi_i(\vec{r})\, e^{-iE_it} \quad ; \qquad (9)$$

and the final nuclear wave function to be used is

$$\Psi_f(\vec{r},t) = e^{i\tilde{e}_f\vec{A}\cdot\vec{r}}\, \psi_f(\vec{r})\, e^{-iE_ft} \quad . \qquad (10)$$

The reduced charges, $\tilde{e}_i$ and $\tilde{e}_f$ are the appropriate forms of Eq. (4); and $\psi_i(\vec{r})$, $\psi_f(\vec{r})$ are stationary state nuclear wave functions with no field present.

5. Interacting Lepton States.

The leptons emitted in β^- decay are an electron and an antineutrino[25]. The antineutrino is uncharged, and possesses no coupling to the electromagnetic field. The antineutrino is therefore described by an ordinary free-particle wave function. The emitted antineutrino is treated as a neutrino in the initial state with reversed four-momentum, i.e.,

$$\Psi^{(\nu)} = \frac{1}{\sqrt{2E_{(\nu)}V}}\, u^{(\nu)}(k_{(\nu)}, s_{(\nu)})\, e^{ik_{(\nu)}\cdot x} \quad . \qquad (11)$$

In Eq. (11), $k_{(\nu)}$ is the four-momentum with time part $E_{(\nu)}$, $u^{(\nu)}$ is a spinor, $s_{(\nu)}$ is the spin parameter, and V is the normalization volume. The scalar product indicated in the exponential is a four-vector product $k_{(\nu)} \cdot x = k_{(\nu)}{}^{\mu} x_{\mu} = E_{(\nu)} t - \vec{k}_{(\nu)} \cdot \vec{r}$.

The electron emitted in beta decay is a charged particle whose coupling to the electromagnetic field is very significant when the field intensity is high. In ordinary beta decay theory, the electron is treated as a free particle, although Coulomb corrections are sometimes introduced. In the present situation, the free particle electron solution is replaced by the Volkov solution[26], which is an exact wave function for a free, charged particle in the presence of a plane wave electromagnetic field. The circumstances which the Volkov solution are to describe are that the electron suddenly appears at some time (say $t = 0$) in an electromagnetic field which has been on for a long time prior to the creation of the electron. It is thus appropriate to consider the field to be monochromatic. The electromagnetic field is specified as

$$A^{\mu} = \alpha \varepsilon^{\mu} \cos(k \cdot x + \rho) \quad , \qquad (12)$$

where ρ is a phase shift reflecting the fact that the beta decay cannot be expected to occur in phase with the field. The polarization four vector ε^{μ} in Eq. (12) has the scalar invariant $\varepsilon^2 = -1$. The required solution is

$$\Psi^{(e)} = \sqrt{\frac{m}{E_e V}}\ \exp\left\{-i\left[p_e\cdot x + \eta k\cdot x + \zeta \sin(k\cdot x+\rho) + \tfrac{1}{2}\eta \sin 2(k\cdot x+\rho) - \zeta\sin\rho - \tfrac{1}{2}\eta\sin 2\rho\right]\right\}\left(1 - \frac{e}{2p_e\cdot k}\not{k}\not{A}\right) u^{(e)}(p_e, s_e)\ , \tag{13}$$

with the definitions

$$\zeta \equiv -\frac{ea\, p_e\cdot\epsilon}{p_e\cdot k} \quad ; \quad \eta \equiv \frac{e^2a^2}{4p_e\cdot k}\ . \tag{14}$$

(The minus sign is introduced in the definition of ζ to account for the fact that a gauge with $\varepsilon^0 = 0$ will be used, in which case $p_e\cdot\varepsilon = -\vec{p}_e\cdot\vec{\varepsilon}$).

6. Transition Probability per Unit Time.

The wave functions given in Eqs. (9), (10), (11), and (13) give the input necessary to apply the S-matrix of Eq. (5). Standard procedures of quantum mechanics are then employed to arrive at a transition probability per unit time for induced beta decay. The final result divides naturally into three separate parts which can be identified as a direct term, a spin term, and an interference term. This comes about because the Volkov solution, Eq. (13), contains the factor $1 - e\not{k}\not{A}/(2p_e\cdot k)$. The first term in this factor occurs in the corresponding solution for a spinless particle, whereas the second term is associated directly with the half-integral spin of the electron. When the square of the S-matrix is formed, these two

terms lead to three terms in the transition probability, one of which arises as an interference term between the direct and spin terms.

The results are quite complex in form, in general, but an important simplification can be introduced, based on the relative magnitudes of two intensity parameters which occur. One intensity parameter can be associated with the interaction of the nuclear particles with the electromagnetic field. It is given by

$$z \equiv (eaR_o)^2 \quad , \tag{15}$$

where a is the amplitude of the vector potential of the field as given in Eq. (12), and R_o is the nuclear radius. This quantity is typical of intensity parameters which arise in bound-state intense-field problems.[27,28] The other intensity parameter is associated with the interaction of free electrons with the electromagnetic field.[27,28] It is

$$z_f \equiv \frac{e^2a^2}{2m^2} \quad . \tag{16}$$

The two parameters are related by

$$z_f = \frac{1}{2(mR_o)^2} z \approx (3 \times 10^3)z \quad . \tag{17}$$

Of these parameters, it is the z of Eq. (15) which plays the more important role in the interchange of angular momentum and parity between the beta-decay system and the field. Although the specific value of z for which optimum conditions for induced beta decay to occur depends upon the particular beta decay, it is generally true that this optimum z is roughly of order

unity. Then, however, Eq. (17) shows that z_f will be large. This justifies the use of asymptotic approximations for certain generalized Bessel functions[27] which appear in the present theory, as they do in all relativistic intense-field theories involving charged particles with spin. These asymptotic approximations make possible a relatively simple final form for the transition probability per unit time.

When $z_f >> 1$, as discussed above, the transition probability per unit time, W, is of the form

$$W = \frac{G^2 m^5}{2\pi^3} f_{ind} |M_{ind}|^2 , \tag{18}$$

where

$$f_{ind} = f_1 + f_2 + f_3 \tag{19}$$

is a spectral integral consisting of three parts arising from the direct, spin, and interaction terms; and where the squared nuclear matrix element is

$$|M_{ind}|^2 = \frac{1}{4\pi (2z_f)^{1/2}} \left[\left| \cos\left(z^{1/2} \frac{r}{R_0} \cos\theta\right) \right|_{fi}^2 + \left| \sin\left(z^{1/2} \frac{r}{R_0} \cos\theta\right) \right|_{fi}^2 \right.$$
$$\left. + \kappa^2 \left| \cos\left(z^{1/2} \frac{r}{R_0} \cos\theta\right) \vec{\sigma} \right|_{fi}^2 + \kappa^2 \left| \sin\left(z^{1/2} \frac{r}{R_0} \cos\theta\right) \vec{\sigma} \right|_{fi}^2 \right] . \tag{20}$$

The form (18) corresponds to the standard form for allowed beta decay, where

$$W_0 = \frac{G^2 m^5}{2\pi^3} f_0 |M_0|^2 ,$$

with

$$|M_0|^2 = |1|_{fi}^2 + \kappa^2 |\bar{\sigma}|_{fi}^2 \quad ;$$

and, when Coulomb corrections are neglected, as they are in the present work, the spectral integral is

$$f_0(\mathcal{E}_0) = \int_1^{\mathcal{E}_0} d\mathcal{E}_e \, h_0(\mathcal{E}_0, \mathcal{E}_e) \quad ; \quad h_0(\mathcal{E}_0, \mathcal{E}_e) = \mathcal{E}_e (\mathcal{E}_e^2 - 1)^{1/2} (\mathcal{E}_0 - \mathcal{E}_e)^2 \quad .$$

The separate spectral integrals in Eq. (19) have a fairly complicated form because of an assumption made in their derivation. In the complex pattern of absorptions and emissions of external intense-field energy by the nuclear system, it is presumed that no net energy can be extracted from the external field. This is conservative in that it amounts to neglecting part of the transition probability. The direct, spin, and interference spectral integrals so derived are

$$f_1(\mathcal{E}_0) = \int_1^{\mathcal{E}_0} d\mathcal{E}_e \, h_1(\mathcal{E}_0, \mathcal{E}_e) \quad ,$$

where $h_1(\varepsilon_0, \varepsilon_e)$ is the spectral function

$$h_1(\mathcal{E}_0, \mathcal{E}_e) = \frac{1}{3} \frac{\sigma^3}{\mathcal{E}_e} \ln\left(\frac{(\sigma + \rho_e^2)^{1/2} + |\rho_e|}{\sigma^{1/2}} \right) + \frac{(\sigma + \rho_e^2)^{1/2} |\rho_e|}{45 \mathcal{E}_e} \left[8(\sigma + \rho_e^2)^2 + 10\sigma(\sigma + \rho_e^2) + 15\sigma^2 \right] - \frac{\rho_e^2}{45 \mathcal{E}_e} (8\rho_e^4 + 30\sigma\rho_e^2 + 45\sigma^2) \; ;$$

$$f_2(\mathcal{E}_0) = \int_1^{\mathcal{E}_0} d\mathcal{E}_e \, h_2(\mathcal{E}_0, \mathcal{E}_e) \, ,$$

where the spectral function is

$$h_2(\varepsilon_o,\varepsilon_e)=\frac{4}{21}\frac{\sigma^4}{\varepsilon_e^3}\ln\left(\frac{(\sigma+\rho_e^2)^{1/2}+|\rho_e|}{\sigma^{1/2}}\right)+\frac{8}{105}\frac{1}{\varepsilon_e^3}\left[|\rho_e|(\sigma+\rho_e^2)^{7/2}-\rho_e^8\right]$$
$$-\frac{\sigma\rho_e^4}{3\varepsilon_e^3}\left(\sigma+\frac{4}{5}\rho_e^2\right);$$

and

$$f_3(\varepsilon_o)=\int_1^{\varepsilon_o}d\varepsilon_e\, h_3(\varepsilon_o,\varepsilon_e),$$

involving the spectral function

$$h_3(\varepsilon_o,\varepsilon_e)=\frac{1}{48}\frac{\sigma^4}{\varepsilon_e^3}\ln\left(\frac{(\sigma+\rho_e^2)^{1/2}+|\rho_e|}{\sigma^{1/2}}\right)-\frac{1}{48}\frac{|\rho_e|(\sigma+\rho_e^2)^{1/2}}{\varepsilon_e^3}\left(\sigma^3+\frac{118}{45}\sigma^2\rho_e^2\right.$$
$$\left.+\frac{136}{15}\sigma\rho_e^4+\frac{16}{5}\rho_e^6\right)+\frac{1}{12}\frac{\rho_e^4}{\varepsilon_e^3}\left(3\sigma^2+\frac{8}{3}\sigma\rho_e^2+\frac{4}{5}\rho_e^4\right).$$

In these expressions, ε_e is a dimensionless electron energy, ε_o is a dimensionless nuclear energy change, and ρ_e is a dimensionless electron momentum defined by

$$\varepsilon_e=E_e/m, \qquad \varepsilon_o=E_o/m, \qquad \rho_e=p_e/m,$$

where $E_o = E_i - E_f$. The last definition required is

$$\sigma \simeq 2\varepsilon_e(\varepsilon_o-\varepsilon_e).$$

7. Nuclear Parameters.

Table 1 gives the information required to apply the foregoing formalism to computation of nuclear matrix elements involved in induced beta decay. The first seven nuclides listed are materials found in Nature, and the last two are the principal fission fragment waste products. Nuclear spin and parity

assignments are from "Nuclear Data Sheets" (except for ^{40}K, which is from P. M. Endt and C. Van der Leun, Nucl. Phys. A310, 1 (1978)). Angular momentum assignments for nucleons in the "fragments" are standard shell model assignments.[29] The reduced charge for the fragment comes from Eq. (4).

A few examples of how fragment assignments are made are given here. For example, $^{113}_{48}Cd_{65}$ has a single nucleon fragment. The core nucleus, $^{112}_{48}Cd_{64}$, is a stable nuclide in Nature with spin and parity of 0^+. By the usual single particle model, this means that this "even-even" nuclide has the spins of all of its protons and of all of its neutrons anti-aligned in pairs to give pairwise and overall zero angular momentum. The odd neutron in ^{113}Cd has a shell model assignment of $s_{1/2}$, which should then determine the entire nuclear spin and parity to be $\frac{1+}{2}$, which is the case. Upon beta decay, the unpaired $s_{1/2}$ neutron becomes an unpaired $g_{9/2}$ proton, which then contributes the entire observed $\frac{9+}{2}$ spin and parity of the final $^{113}_{49}In_{64}$ nucleus.

An example of a two nucleon fragment is provided by $^{90}_{38}Sr_{52}$. The core nucleus, $^{88}_{38}Sr_{50}$, is the principal stable isotope of strontium. In particular, N=50 is a "magic number"[23] for the neutron shell in ^{88}Sr, and Z=38 represents the closure of an $f_{5/2}$ shell for the protons, so ^{88}Sr is a clear case of a stable, relatively tightly bound core nucleus. The two neutrons in ^{90}Sr beyond the magic number of N=50 then constitute the fragment, one of whose two neutrons will undergo beta decay. They must be

Table I
NUCLEAR PARAMETERS

Initial Nucleus	Initial J^π	Core Nucleus ($J^\pi=0$)	Type of Decay	Final Nucleus	Final J^π	L	Initial Fragment				Initial Reduced Charge $\tilde{e}_i/e$	Final Fragment				Final Reduced Charge $\tilde{e}_f/e$
							Neutrons		Protons			Neutrons		Protons		
							1	2	1	2		1	2	1	2	
$^{40}_{19}K_{21}$	4^-	$^{38}_{18}Ar_{20}$	β^- β^+	$^{40}_{20}Ca_{20}$ $^{40}_{18}Ar_{22}$	0^+ 0^+	3 3	$0f_{7/2}$ $0f_{7/2}$		$0d_{3/2}$ $0d_{3/2}$		1/20 = 0.05	$0f_{7/2}$	$0f_{7/2}$	$0d_{3/2}$	$0d_{3/2}$	1
$^{48}_{20}Ca_{28}$	0^+	$^{46}_{20}Ca_{26}$	β^- β^-	$^{48}_{21}Sc_{27}$ ^{48}Sc	6^+ 5^+	6 4	$0f_{7/2}$ $0f_{7/2}$	$0f_{7/2}$ $0f_{7/2}$			-5/6 = -0.83	$0f_{7/2}$ $0f_{7/2}$		$0f_{7/2}$ $0f_{7/2}$		1/8 = 0.13
$^{50}_{23}V_{27}$	6^+	$^{48}_{22}Ti_{26}$	β^- β^- β^+	$^{50}_{24}Cr_{26}$ ^{50}Cr $^{50}_{22}Ti_{28}$	0^+ 2^+ 0^+	6 4 6	$0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$		$0f_{7/2}$ $0f_{7/2}$ $0f_{7/2}$		2/25 = 0.08	$0f_{7/2}$	$0f_{7/2}$	$0f_{7/2}$ $0f_{7/2}$	$0f_{7/2}$ $0f_{7/2}$	26/25 = 1.04
$^{87}_{37}Rb_{50}$	$3/2^-$	$^{84}_{36}Kr_{48}$	β^-	$^{87}_{38}Sr_{49}$	$9/2^+$	3	$0g_{9/2}$	$0g_{9/2}$	$1p_{3/2}$		-8/29 = -0.28	$0g_{9/2}$		$1p_{3/2}$	$1p_{3/2}$	20/29 = 0.69
$^{96}_{40}Zr_{56}$	0^+	$^{94}_{40}Zr_{54}$	β^- β^-	$^{96}_{41}Nb_{55}$ ^{96}Nb	6^+ 5^+	6 4	$1d_{5/2}$ $1d_{5/2}$	$1d_{5/2}$ $1d_{5/2}$			-5/6 = -0.83	$1d_{5/2}$ $1d_{5/2}$		$0g_{9/2}$ $0g_{9/2}$		7/48 = 0.15
$^{113}_{48}Cd_{65}$	$1/2^+$	$^{112}_{48}Cd_{64}$	β^-	$^{113}_{49}In_{64}$	$9/2^+$	4	$2s_{1/2}$				-48/113 = -0.42			$0g_{9/2}$		64/113 = 0.57
$^{115}_{49}In_{66}$	$9/2^+$	$^{112}_{48}Cd_{64}$	β^-	$^{115}_{50}Sn_{65}$	$1/2^+$	4	$2s_{1/2}$	$2s_{1/2}$	$0g_{9/2}$		-32/115 = -0.28	$2s_{1/2}$		$0g_{9/2}$	$0g_{9/2}$	80/115 = 0.70
$^{90}_{38}Sr_{52}$	0^+	$^{88}_{38}Sr_{50}$	β^-	$^{90}_{39}Y_{51}$	2^-	1	$1d_{5/2}$	$1d_{5/2}$			-38/45 = -0.84	$1d_{5/2}$		$1p_{1/2}$		2/15 = 0.13
$^{137}_{55}Cs_{82}$	$7/2^+$	$^{134}_{54}Xe_{80}$	β^- β^-	$^{137}_{56}Ba_{81}$ ^{137}Ba	$3/2^+$ $11/2^-$	2 1	$1d_{3/2}$ $0h_{11/2}$	$1d_{3/2}$ $0h_{11/2}$	$0g_{7/2}$ $0g_{7/2}$		-28/137 = -0.20	$1d_{3/2}$ $0h_{11/2}$		$0g_{7/2}$ $0g_{7/2}$	$0g_{7/2}$ $0g_{7/2}$	106/137 = 0.77

considered as a pair because initially they are angular momentum coupled to 0^+, and it is impossible to say which of the two will decay. Finally, the remaining $d_{5/2}$ neutron will couple to the newly formed $p_{1/2}$ proton to give the 2^- state of the $^{90}_{39}Y_{51}$ daughter nucleus.

$^{87}_{37}Rb_{50}$ is an example of a nuclide where the fragment must consist of three nucleons. The odd proton in ^{87}Rb must be part of the fragment because initially this $p_{3/2}$ particle accounts for the entire ^{87}Rb spin and parity of $\frac{3^-}{2}$. The beta decay itself involves a neutron, not the odd proton, and since the beta decay neutron is initially paired with another to give 0^+, then both of these neutrons must also be assigned to the fragment. In the final state, the $g_{9/2}$ neutron which beta decays to a $p_{3/2}$ proton will couple to 0^+ with the initial odd proton, while the remaining $g_{9/2}$ neutron finds itself unpaired in the final state, and so accounts for the $\frac{9^+}{2}$ spin and parity of the ^{87}Sr daughter nucleus.

The state assignments shown in the table for ^{137}Cs require a special comment. ^{137}Cs has a $\frac{7^+}{2}$ ground state, with an L=1 decay to an $\frac{11^-}{2}$ state and an L=2 decay to a $\frac{3^+}{2}$ state of the daughter ^{137}Ba nucleus. The contrast between the spins and parities of these states suggests something unusual. ^{137}Ba has 81 neutrons and ^{137}Cs has 82 neutrons--a magic number. The last two neutron shells to be filled before the magic number is reached are the $d_{3/2}$ and $h_{11/2}$ shells. Between neutron numbers 67 and 79, there is alternation in the filling of these two levels. It can be inferred, therefore, that when ^{137}Cs beta decays to the $\frac{3^+}{2}$ level of ^{137}Ba, it is a $d_{3/2}$ neutron in ^{137}Cs which experiences the beta decay, leaving an unpaired $d_{3/2}$ neutron in ^{137}Ba to contribute the $J=\frac{3}{2}$

angular momentum of the final nucleus; and in decay to the $\frac{11}{2}$ level of ^{137}Ba, it is an $h_{11/2}$ neutron in ^{137}Cs which experiences the decay, leaving an unpaired $h_{11/2}$ neutron to account for the $J=\frac{11}{2}$ final state.

8. Nuclear Matrix Elements.

a. Form of the Nuclear Matrix Element

Total transition probability per unit time is given in Eq. (18). It contains the squared transition matrix element $|M_{ind}|^2$, defined in Eq. (20). This will now be examined in more detail.

Equation (20) is expressed as the sum of four terms. The first pair of terms arises from the vector part of the beta decay interaction, and corresponds to the usual Fermi matrix element of beta decay theory. The second pair of terms (the ones containing the Pauli spin operators $\vec{\sigma}$) comes from the axial vector part of the beta decay interaction, and corresponds to the usual Gamow-Teller matrix element of beta decay theory. However, a simplification can be introduced from isospin considerations, which have not been placed in evidence in the above work. For Fermi matrix elements, the isospin conservation rule is $\Delta T=0$[30,31], where T is the total isospin quantum number. Since this condition is not satisfied for any of the transitions considered here, only the Gamow-Teller matrix elements will be retained. That is, Eq. (20) is now replaced by

$$|M_{ind}|^2 = \frac{K^2}{4\pi (2z_f)^{1/2}} \left[\left| \cos\left(z^{1/2}\frac{r}{R_0}\cos\theta\right)\vec{\sigma} \right|_{fi}^2 + \left| \sin\left(z^{1/2}\frac{r}{R_0}\cos\theta\right)\vec{\sigma} \right|_{fi}^2 \right]. \quad (21)$$

The terms in the square bracket in Eq. (21) are

squared nuclear transition matrix elements, with the f and i subscripts referring to final and initial nuclear states. The coordinate r which occurs in the matrix elements refers to the position vector $\vec{r}$ of the nuclear fragment with respect to the nuclear core. In practical calculation of the nuclear matrix elements, one needs the coordinates of the separate nucleons contained in the fragment. The vector $\vec{r}$ gives the location of the CM of the fragment. Since each nucleon in the fragment can be taken to have the same mass M, then the position vector of the jth nucleon in the fragment ($\vec{r}_j$) is related to $\vec{r}$ by

$$qM\vec{r} = \sum_{j=1}^{q} M\vec{r}_j ,$$

where q is the total number of nucleons in the fragment. Since only one of these q nucleons will undergo beta decay (say the jth one), then whenever $r\cos\theta$ appears in the matrix element, the replacement

$$r\cos\theta \rightarrow \frac{1}{q} r_j \cos\theta_j$$

should be used, where θ_j measures the angle between $\vec{r}_j$ and the polarization vector of the applied field.

Equation (21) can be stated in more detail as

$$|M_{ind}|^2 = \frac{\kappa^2}{4\pi(2Z_f)^{1/2}} \left(|\vec{M}_{fi}^{cos}|^2 + |\vec{M}_{fi}^{sin}|^2 \right) , \qquad (22)$$

where

$$\vec{M}_{fi}^{cos} = \frac{1}{(2j_i+1)} \sum_{m_i} \sum_{m_f} \left(\psi_f , \cos\left(\frac{Z^{1/2}}{q} u_j \cos\theta_j \right) \vec{\sigma}\, \psi_i \right) \qquad (23)$$

$$\vec{M}_{fi}^{sin} = \frac{1}{(2j_i+1)} \sum_{m_i} \sum_{m_f} \left(\psi_f , \sin\left(\frac{Z^{1/2}}{q} u_j \cos\theta_j \right) \vec{\sigma}\, \psi_i \right) . \qquad (24)$$

In Eqs. (23) and (24), u_j is the dimensionless radial coordinate

$$u_j = r_j/R_o \quad ;$$

j_i is the total angular momentum of the initial state, so that $(2j_i+1)^{-1}$ times the sum over m_i is an average over orientations of initial angular momentum; and the sum over m_f is a sum over orientations of the final angular momentum. In practice, only one of the two terms in Eq. (22) will be nonzero. When Ψ_f and Ψ_i have the same parity, only $\vec{M}_{fi}^{\cos}$ will survive; and when they have opposite parity, only $\vec{M}_{fi}^{\sin}$ will survive.

b. Sample Results

The only example in Table 1 which has a single-nucleon fragment is ^{113}Cd. With independent-particle quantum number assignments as given in Table 1, and with harmonic oscillator functions employed for the radial wave functions, Eqs. (23) and (22) lead to

$$|M_{ind}|^2 = \frac{\kappa^2}{4\pi(2z_f)^{1/2}} \frac{2}{3^2 \cdot 5 \cdot 7} e^{-z/2} z^4 \left(1-\frac{z}{8}\right)^2 \left(1-\frac{z}{24}\right)^2 . \quad (25)$$

Further reduction of Eq. (25) follows from the experimentally determined value of κ [32]

$$\kappa = 1.23 \pm 0.01 \quad , \quad (26)$$

and the approximate relation between z_f and z

$$(2z_f)^{1/2} = z^{1/2}/mR_o \simeq 80 z^{1/2} \quad . \quad (27)$$

From Eqs. (25) and (27), the intensity dependence of $|M_{ind}|^2$ for ^{113}Cd is given by $e^{-z/2}z^{7/2}(24-z)^2(8-z)^2$. This function has a broad maximum with a peak value at

$$z_{max} = 3.394 \quad . \tag{28}$$

Low-intensity behavior is proportional to $z^{7/2}$.

An example of a q=2 nucleus, which has the same L=4 forbiddenness as ^{113}Cd is ^{48}Ca. Application of the formalism to this case yields

$$|M_{ind}|^2 = \frac{\kappa^2}{4\pi(2z_f)^{1/2}} \frac{11\cdot 13}{2^{11}\cdot 3\cdot 5\cdot 7^3} e^{-z/8} z^4 \left(1 - \frac{z}{2^2\cdot 11} + \frac{47z^2}{2^6\cdot 3\cdot 11\cdot 13^2}\right). \tag{29}$$

Equations (26) and (27) apply again to this result. The low-intensity behavior of Eq. (29) is proportional to $z^{7/2}$, just as in Eq. (25), since both ^{113}Cd and ^{48}Ca have L = 4. However, the intensity parameter which maximizes Eq. (29) is

$$z_{max} = 22.541 \ , \tag{30}$$

which is a substantially greater intensity than the maximum for ^{113}Cd given in Eq. (28).

Another q = 2 example, but one which has L = 3, is ^{40}K. The squared induced transition matrix element in this case is

$$|M_{ind}|^2 = \frac{\kappa^2}{4\pi(2z_f)^{1/2}} \frac{1}{2^7\cdot 3^2\cdot 5\cdot 7^2} e^{-z/8} z^3 \left(1 - \frac{z}{36} + \frac{z^2}{1728}\right). \tag{31}$$

which has a maximum at

$$z_{max} = 18.647 \ . \tag{32}$$

One further example for which q = 2 is ^{90}Sr, which has L = 1. The calculation in this case leads to a rather more complicated answer than the cases above, and it is not convenient to quote the result entirely in terms of rational numbers, as done above. The ^{90}Sr calculation gives

$$|M_{ind}|^2 = \frac{\kappa^2}{4\pi(2z_f)^{1/2}} \frac{2\cdot 7}{3^2\cdot 5} e^{-z/8}$$
$$z\left[1 - \frac{z}{5} + (1.679\times10^{-2})z^2 - (7.341\times10^{-4})z^3\right. \quad (33)$$
$$\left. +(1.711\times10^{-5})z^4 - (1.920\times10^{-7})z^5 + (8.109\times10^{-10})z^6\right],$$

which reaches a maximum when

$$z_{max} = 1.493 \quad . \quad (34)$$

Finally, a three-nucleon fragment case will be considered. The transition to the $11/2^-$ final state in the beta decay of ^{137}Cs has q = 3 and L = 1. The final result here is

$$|M_{ind}|^2 =$$
$$\frac{\kappa^2}{4\pi(2z_f)^{1/2}}(9.297\times10^{-3})z\,e^{-z/18}\left[1 - (8.709\times10^{-2})z + (2.923\times10^{-3})z^2\right.$$
$$-(4.881\times10^{-5})z^3 + (4.496\times10^{-7})z^4 - (2.370\times10^{-9})z^5 \quad (35)$$
$$\left. +(7.060\times10^{-12})z^6 - (1.098\times10^{-14})z^7 + (6.900\times10^{-18})z^8\right] .$$

which is maximized at

$$z_{max} = 3.355 \quad . \quad (36)$$

9. Induced Halflife.

The halflife for beta decay is related to the transition probability per unit time by

$$t = \ln 2/W \quad .$$

The analogue of Eq. (18) for allowed beta decay gives

$$t = \frac{6.25 \times 10^3}{f|M|^2} \quad , \tag{37}$$

in which the constant

$$\frac{2\pi^3 \ln 2}{G^2 m^5} = 6.25 \times 10^3$$

is arrived at by using the known value for the beta decay coupling constant, G. It is conventional in beta decay work to express results in terms of "log ft" values. Equation (37) would lead to a log ft expression

$$\log ft = 3.80 - \log(|M|^2) \quad .$$

However, in practice, the simple theoretical log ft values for allowed beta decays are always smaller than those found experimentally, except for a special group of beta decays known as "superallowed" decays. Empirically, this can be incorporated into the theoretical expression by modifying the constants shown above, so that, for nuclides of even mass number

$$\log ft = 5.35 - \log(|M|^2) \quad , \tag{38}$$

and for nuclides of odd mass number[33/]

$$\log ft = 5.18 - \log(|M|^2) \quad . \tag{39}$$

Log ft values do not represent the most convenient way to express results for induced beta decays, so the empirical constants expressed in Eqs. (18) and (37) will be converted into revised constants for a halflife expression like Eq. (37) by using

$$t_{ind} = \frac{10^{5.35}}{f_{ind}\,|M_{ind}|^2} \tag{40}$$

for even-A nuclides, and setting

$$t_{ind} = \frac{10^{5.18}}{f_{ind}\,|M_{ind}|^2} \tag{41}$$

for odd-A nuclides.

Consider ^{113}Cd as an example of the application of the above formalism. Equation (25), when evaluated at the intensity stated in Eq. (28), gives $|M_{ind}|^2 = 3.08 \times 10^{-5}$. The spectral integrals can be evaluated from the knowledge that ^{113}Cd has a beta-decay Q of 297 keV, which leads to $\varepsilon_o = 1.581$ from $\varepsilon_o = E_o/m = 1 + (Q/m)$. The spectral integrals give $f_1 = 9.90 \times 10^{-2}$, $f_2 = 3.09 \times 10^{-2}$, and $f_3 = -0.64 \times 10^{-2}$. The negative value for f_3 means that interference between direct and spin terms is a partially destructive interference. The total induced spectral integral is

$$f_{ind} = 0.124 \quad .$$

This value, when substituted in Eq. (41) with the above result for $|M_{ind}|^2$, gives

$$t_{ind} = 1.26 \times 10^3 \text{ years}$$

for ^{113}Cd. This is to be compared with the natural decay halflife of 9.3×10^{15} years.

For ^{48}Ca, Eqs. (29) and (30) give $|M_{ind}|^2 = 3.68 \times 10^{-5}$. The Q value for the 0^+ to 5^+ transition is 150 keV, so $\varepsilon_o = 1.294$. Then the spectral integral results are $f_1 = 5.68 \times 10^{-3}$, $f_2 = 0.98 \times 10^{-3}$, $f_3 = -0.20 \times 10^{-3}$, for a total $f_{ind} = 6.46 \times 10^{-3}$. When Eq. (40) is employed, the induced halflife is

$$t_{ind} = 2.99 \times 10^4 \text{ years.}$$

The ^{40}K case gives $|M_{ind}|^2 = 5.32 \times 10^{-7}$ from Eqs. (31) and (32), and $f_1 = 81.3$, $f_2 - 68.6$, $f_3 = -13.9$, $f_{ind} = 136.2$, leading to

$$t_{ind} = 98.0 \text{ years .}$$

In the case of ^{90}Sr, the calculation leads to $|M_{ind}|^2 = 3.50 \times 10^{-4}$, $f_1 = 1.40$, $f_2 = 0.70$, $f_3 = -0.14$, and $f_{ind} = 1.96$. The induced halflife

$$t_{ind} = 10.4 \text{ years ,}$$

when combined with the natural halflife of 28.6 years, gives an overall halflife in both natural and induced channels of 7.62 years.

Results for ^{137}Cs are: $|M_{ind}|^2 = 1.57 \times 10^{-5}$ from Eqs. (35) and (36); Q = 512 keV, or $\varepsilon_o = 2.001$ for the transition to the $11/2^-$ state; $f_1 = 1.05$, $f_2 = 0.50$, $f_3 = -0.10$, $f_{ind} = 1.45$; and

$$t_{ind} = 210 \text{ years .}$$

In this L = 1 case, the result is actually an induced halflife decay channel that is slower than the natural

decay. However, under optimum induced field conditions, the L = 2 transition to the $3/2^+$ final state would be the dominant transition.

Power density in an induced beta decay fuel can be expressed as

$$\text{Power density} = EW_{ind}\rho, \tag{42}$$

where E is the decay energy involved in the beta decay of a single nucleus, W_{ind} is the total induced transition probability as found from Eq. (18), and ρ is the number of active nuclei per unit volume. If E is expressed in MeV, W_{ind} in sec^{-1}, ρ in nuclei per cubic meter, and power density in watts per cubic meter, then Eq. (42) becomes

$$\text{Power density}(W/m^3) = (1.6\times10^{-13})E(\text{MeV})W_{ind}(sec^{-1})\rho(m^{-3}). \tag{43}$$

This expression can be used inversely to find the density of active nuclei needed to achieve a given power density. For example, assuming the lower limit of power density of practical interest is of the order of 10 watts per m^3, Eq. (43) leads to a minimum density of active nuclei of the order of $10^{14}/EW_{ind}$.

10. Strength of the Electromagnetic Interaction.

From Eq. (18) and the results obtained for the squared matrix element, it is seen that the overall transition probability for induced beta decay at low field intensity is of the form

$$W_{ind} = Cz^{L-\frac{1}{2}}, \tag{44}$$

where C depends on properties of the nucleus, and all field dependence is contained in the $z^{L-\frac{1}{2}}$ factor. One might expect to find a dependence on z^{L} in Eq. (44) rather than $z^{L-\frac{1}{2}}$, since the process being described is an Lth order interaction with the electromagnetic field. The loss of half a power in z comes from the asymptotic forms for the generalized Bessel functions. Physically, the loss in z dependence arises from the emitted electron. The interaction of the relatively light beta particle with the very intense field is so severe that very rapid oscillations in the phase of the electron occur, causing a reduction in the beta decay transition amplitude. Equation (22) shows that contributions arising from an Lth order interaction of the nucleus with the field is diminished by $z_f^{1/2}$, a free electron interaction parameter coming from the beta particle.

The parameter z has been spoken of as a field intensity parameter; but, as is evident from Eq. (44), it is identified also as the coupling strength of the field-nucleus interaction. An alternative way to write z is as

$$z = 4\rho\alpha_o\lambda R_o^2 \quad . \tag{45}$$

In Eq. (45), α_o is the fine structure constant, which is conventionally taken in quantum electrodynamics to be the measure of the strength of coupling between the electromagnetic field and an elementary particle of charge e. However, the electromagnetic field is a Bose field, and the more photons there are in a given mode of the field, the more the interaction probability involving that mode is increased. This enhancement is measured by the photon density ρ. The factors λR_o^2 in Eq. (45) define an effective

interaction volume, so that $\rho\lambda R_o^2$ is a measure of the number of photons which are in interaction with the charged nuclear system. The interaction volume can be viewed as a box whose cross-sectional area is defined by the area of the nucleus, and whose length is the wavelength of the field.

The intensity parameter z as defined in Eq. (15) appears superficially to be dependent on the choice of gauge for the electromagnetic field. It is, in fact, gauge-invariant in any relativistically-stated "simple" gauge.[24/]

A way to avoid possible confusion about gauge invariance of z is to express it directly in physical quantities. One way is to write the intensity parameter for the plane wave in terms of the electric field as

$$z = e^2 E_o^2 R_o^2 / \omega^2 \quad , \tag{46}$$

or in terms of the magnetic induction as

$$z = e^2 B_o^2 R_o^2 / \omega^2 \varepsilon_r \quad , \tag{47}$$

where E_o and B_o are the amplitudes of the $\vec{E}$ and $\vec{B}$ fields and ε_r is the dielectric constant of the material in which the wave propagates. When $\hbar$ and c factors are inserted where appropriate, $\vec{E}$ and $\vec{B}$ are given in Mks units (as volts per meter and teslas, respectively), the frequency $\nu(\omega = 2\pi\nu)$ is given in Hertz, and R_o is taken to be 5×10^{-15} meters, then we can express the intensity parameter z as

$$z = 1.362\ E_o^2/\nu^2 = (1.224\times10^{17}) B_o^2/\nu^2\varepsilon_r \quad . \tag{48}$$

Equation (48) can be used inversely to find field parameters necessary to achieve a given intensity parameter. For example, assuming the lowest z of practical interest is of the order of 10^{-3}, then the magnetic induction needed to achieve this at the frequency ν is of the order of

$$B_o/\nu\varepsilon_r^{\frac{1}{2}} \gtrsim 10^{-10}$$

where B_o is in teslas and ν is in Hz.

The intensity parameter z must be roughly of the order of unity for induced beta decay to be important. A value for the intensity parameter of the order of unity is difficult to achieve. Some possibilities will be reviewed here. First, the energy flux of the applied field will be expressed in terms of z. If the energy flux is stated in units of watts per square centimeter (W/cm^2), and all other quantities are in Gaussian units, the connection is

$$\text{energy flux } (W/cm^2) \equiv P = 10^{-7} \frac{1}{4} \frac{z}{\alpha o} \frac{\hbar\omega c}{\lambda R_o^2} \quad .$$

The factor 10^{-7} is for conversion from ergs to joules. The factor $\hbar\omega\, c/\lambda R_o^2$ is the energy flux associated with the passage of a single photon through the interaction volume, and the factor z/α_o converts this into the overall energy flux. If z is set to unity, and R_o is replaced approximately by $\lambda_c/80$, then the applied field must supply

$$P \simeq (8.8 \times 10^{13})/\lambda^2 \ ,$$

where λ is in centimeters and P in W/cm^2. A central fact is the inverse square dependence on wavelength, strongly favoring long wavelength sources, other

things being equal. "Other things," however, are not equal, since the technological capability for producing large radiated power is very non-uniform across the electromagnetic spectrum. Some representative values of P are given here for certain well developed sources.

Wavelength(Cm)	Type of Radiation	P(for z = 1), W/Cm^2
1.06×10^{-4}	Nd - glass laser	8×10^{21}
1.06×10^{-3}	CO_2 laser	8×10^{19}
3	Microwave	10^{13}
3×10^{2}	VHF	10^{9}
3×10^{5}	RF	10^{3}

The energy fluxes listed above are very large. The figure given for the Nd-glass laser is beyond present capabilities. The CO_2 laser might reach the required intensity, but only in a very small volume with a short pulse. The energy input would greatly exceed output. The microwave requirement is also unreasonably large, even in a high-Q cavity. At long wavelengths, however, practical systems become possible.

11. Field Potentials at the Nucleus.

When a nucleus is incorporated in a solid, the applied electric field at the position of a nucleus will largely be cancelled by counter-fields generated within the solid of which the nucleus is a part. The applied magnetic field will be essentially unaffected. It is very important to note that the internal fields which accomplish cancellation of the applied electric field are entirely quasistatic (i.e., oscillating

Coulombic fields with negligible radiation component), so that, in Coulomb gauge, they are describable by an oscillating scalar potential. The internal fields do not contribute to the vector potential. (Again, this statement applies to Coulomb gauge.) These statements are explicated by

$$\vec{E}_{int} = -\vec{E}_{ext} = \vec{\nabla}\phi_{ext} + \partial_t \vec{A}_{ext}$$

$$\vec{E}_{int} = -\vec{\nabla}\phi_{int} \quad .$$

When these two expressions for $\vec{E}_{int}$ are equated, and an integration performed, then

$$\phi_{int} = -\phi_{ext} - \int \partial_t \vec{A} \cdot d\vec{r} \quad .$$

where $\vec{A}$ without subscript is $\vec{A}_{ext}$ since $\vec{A}_{int}$ can be ignored. The total potential experienced by the nucleus is the sum of internal and external fields, so

$$\begin{aligned} \Phi_{tot} &= \phi_{int} + \phi_{ext} = -\int \partial_t \vec{A} \cdot d\vec{r} \\ \vec{A}_{tot} &= \vec{A}_{ext} = \vec{A} \quad . \end{aligned} \tag{49}$$

The next question concerns the algebraic form of $\vec{A}$. Consider the specific example of a coaxial transmission line given below. The $\vec{E}$ and $\vec{B}$ fields of Eqs. (55) and (56) are associated with the vector potential in Coulomb gauge given by

$$\vec{A} = -\hat{\rho}\frac{C}{\omega\rho}\sin(\omega t - kz) \quad . \tag{50}$$

The ρ,z coordinates which appear in Eq. (50) are macroscopic coordinates. The integral which appears in Eq. (49) is over nuclear, or microscopic coordinates. Hence, the ρ, z coordinates in Eq. (50) can certainly be taken to be constant numbers as far as any nuclear motions are concerned. Therefore, in an equation of motion like Eq. (2), Eq. (50) can be expressed simply as $\vec{A}(\vec{r},t) = \vec{a}\cos\omega t$, where $\vec{a}$ is a constant amplitude vector, the trigonometric phase has been shifted, and the long-wavelength approximation is employed.

A scalar potential in the form of Eq. (49) leads to a scalar interaction term in the relative coordinate equation for the nucleus (Eq. (2)) of the form $V = -\tilde{e}\partial_t \vec{A}\cdot\vec{r}$. Such a scalar interaction term is of no consequence for induced beta decay. To show this in simple fashion, consider a time-dependent perturbation theory matrix element between an initial state and some intermediate state. After the time integration is done, a scalar potential of this type gives the result

$$(\psi, \frac{e\vec{a}\cdot\vec{r}}{2}\psi_i)(\frac{\omega}{E-E_i-\omega})$$

where $\vec{a}$ is the vector amplitude of the trigonometric term in $\vec{A}$. By contrast, the vector potential $\vec{A}$, through the perturbing term $-A\cdot(i\vec{\nabla})$ gives the result

$$(\psi, \frac{e\vec{a}\cdot\vec{r}}{2}\psi_i)(\frac{E-E_i}{E-E_i-\omega})$$

Since $|E-E_i|$ is a nuclear energy difference, and ω is the energy of a single rf photon, the scalar potential contribution is totally negligible as compared to the vector potential.

B. Laboratory Test of the Theory.

1. Experimental Apparatus

To subject the foregoing to experimental test, a field source based on a low-frequency standing wave in a resonant coaxial cavity was employed. The field in the cavity can be regarded as the superposition of two plane waves of equal amplitude traveling in opposite directions. Because the transverse dimensions of the coaxial line are very small as compared to a wavelength, only the TEM (transverse electromagnetic) or plane-wave-like mode can exist. The coaxial cavity had an air dielectric, with physically very small radioactive sources attached to the central conductor at a location where the fields are such that $|\vec{E}| = c|\vec{B}|$. The cavity was operated as a three-quarter-wavelength stub 11, off a coaxial transmission line 20 as shown schematically in Fig. 1a. The power supply 4, was a 4.1 MHz radio transmitter sending an unmodulated 40 kW signal down the transmission line into a water-cooled non-reflecting resistive load 3.

Two sources 10 were emplaced at the $|\vec{E}| = c|\vec{B}|$ point in the cavity, shown by the arrow in Fig. 1a. One source was approximately 15 μCi of ^{137}Cs, and the other was about 100 μCi of ^{7}Be. The ^{137}Cs is the "active" source whose first forbidden beta decays to the first excited state of ^{137}Ba give rise to 661.64 keV gamma rays. It is this source which should show the effects of the electromagnetic field. ^{7}Be is a "normalizing" source whose electron capture transition to ^{7}Li is superallowed, and thus is expected to show little or no effects from the applied field. A 477.57 keV gamma ray is emitted following electron capture. All measurements of Cs counts were

normalized with respect to Be counts as a way of eliminating spurious non-field-induced influences on the Cs count rate. Both sources consisted of 1 mm diameter pellets of ion exchange resin containing salts of the radioactive materials. They were attached to the central conductor with teflon tape, over which was shrunk a teflon tube. The entire arrangement of sources and attaching materials was electrically nonconducting.

Radioactive decay of the sources was monitored by detection of the gamma rays emitted following the decay. These gamma rays easily penetrate the outer conductor of the coaxial cavity, and were detected by a Ge(Li) (lithium-drifted germanium) crystal outside the cavity. As a way of increasing field intensity at the location of the sources, they were emplaced in a specially constructed test section of very small diameter. In the test section, the inner conductor diameter was 6 mm, and the outer conductor diameter was 14 mm. The test section and detection crystal were both encased in a special low-radiation background shield. Output from the detector was processed by a 8192 channel analyzer, which provided background subtraction routines to permit determination of the net gamma-ray count from each of the two sources. A schematic diagram of the nuclear detection apparatus is given in Fig. 1b, which shows the sources 11, attached to the inner conductor 2, of the coaxial line. The gamma ray detection crystal 13, is located outside the outer conductor 14 of the coaxial line 11.

2. Form of the Data

The experiment was conducted by alternating equal periods of time with the rf power on and with the power off. Each power-on and power-off part of the cycle was divided into four equal periods of length determined by presetting "live time" on the multichannel analyzer to 135 seconds. This corresponded to about 2.5 minutes of clock time. The reason for this choice is that the ^{137}Cs decay leads to an isomeric state in ^{137}Ba which has a 2.55 minute halflife for decay to the ground state. There is no corresponding delay in emission of the gamma ray following ^{7}Be decay. The isomerism in ^{137}Ba gives a characteristic buildup and decay pattern to the Cs/Be count ratios through the successive power-on and power-off cycles.

The desired result to be obtained from the experiments is a knowledge of the change in beta decay transition probability in ^{137}Cs caused by the field. The experiment measures the gamma rays emitted from ^{137}Ba as a consequence of beta decay from ^{137}Cs. With the terminology that state a is the initial ^{137}Cs state, state b is the first excited state in ^{137}Ba, and state c is the ground state of ^{137}Ba, then a solution of the differential equation for the population of state b as a function of time is

$$N_b(t) = N_a^o \left(\frac{W_a}{W_b - W_a}\right) e^{-W_a t} + \beta e^{-W_b t} ,$$

where N_a^o is the initial population of state a, W_a is the transition probability for the beta decay from state a to state b, W_b is the transition probability for the gamma transition from state b to state c, and

β is an integration constant determined by initial conditions. When the field is turned on, the transition probability for $a \rightarrow b$ is modified from W_a to Ω ,

$$W_a \rightarrow \Omega = W_a + \Delta \ ,$$

where Δ is the incremental transition probability caused by the field. The experiment is conducted by alternating power-off and power-on cycles of duration T. For each of these cycles, the integration constant β is evaluated anew by taking the final condition from each cycle as the initial condition for the following cycle. After a number of such cycles, the results are expressible as

$$\downarrow N_b(t) = N_a^0 \frac{1}{W_b} \left[W_a e^{-W_a t} + \Delta \frac{e^{-W_b t}}{(1+e^{-W_b T})} \right]$$

$$\uparrow N_b(t) = N_a^0 \frac{1}{W_b} \left[\Omega e^{-\Omega t} - \Delta \frac{e^{-W_b t}}{(1+e^{-W_b T})} \right] \ ,$$

where $\downarrow$ refers to power-off cycles and $\uparrow$ refers to power-on cycles. In these results, the inequality $W_b >> W_a$ is used, and the origin of time t starts anew at every switch between on and off cycles.

The rate of gamma-ray emission is

$$\Gamma(t) = N_b(t) W_b \ ,$$

so the number of gamma-ray emissions in time T is $\int_0^T dt\, \Gamma(t)$. This is measured experimentally. When emission rates are identified as coming from power-on or power-off cycles by up or down arrows as above,

then the quantity

$$\alpha = \frac{\int_0^T dt\uparrow\Gamma(t) - \int_0^T dt\downarrow\Gamma(t)}{\int_0^T dt\uparrow\Gamma(t) + \int_0^T dt\downarrow\Gamma(t)} \tag{51}$$

can be determined directly from the experiment. In terms of α (measured in the experiment), and the quantity

$$c = \frac{1}{W_b T}\left(\frac{1-e^{-W_b T}}{1+e^{-W_b T}}\right) \tag{52}$$

which is fixed by experimental conditions, then the relative change in beta decay transition probability is given by

$$\frac{\Delta}{W_a} = \frac{2\alpha}{1-2c-\alpha} \tag{53}$$

In actual application of these results to analysis of the experiment, each of the quantities

$$\int dt\uparrow\Gamma(t) \quad , \quad \int dt\downarrow\Gamma(t)$$

in Eq. (51) is divided by a decay-corrected count of gamma emissions from ^{7}Be decay within the same on and off cycles as the ^{137}Cs counts. When corrected for the 53.29-day halflife of ^{7}Be, these ^{7}Be counts are time-independent, and so do not affect the analysis leading up to Eq. (53).

3. Experimental Results

The excited-state decay in ^{137}Ba occurs with known transition probability. When this is combined

with the measured value

$$T = 0.1748 \text{ hours}$$

for each complete on-cycle or off-cycle, then

$$W_b T = 2.8487 ,$$

which yields, from Eq. (52)

$$c = 0.3126 .$$

The outcome of 200 separate, successive, complete cycles with power off followed by power on, as analyzed in accordance with Eq. (53), gave the final result

$$\frac{\Delta}{W_a} = (9.17 \pm 4.12) \times 10^{-4} . \qquad (54)$$

The number given after the ± sign in Eq. (54) is the "standard error", which is the standard deviation divided by the square root of the number of separate determinations of $\Delta / W_a - 200$ in this case. The first number in Eq. (54) is the measured mean value for Δ / W_a.

One way to assess the meaning of the result in Eq. (54) is through the concept of confidence intervals[34]. With the use of Student's t-distribution[34], one can state that the experiments establish that the value of the true mean (as opposed to the measured mean) is greater than zero at a confidence level of 98.6%. This is a one-sided confidence interval, rather than the more usual two-sided interval which states that the true mean lies between a pair of numbers symmetrically disposed

about the measured mean. The result of the experiment can then be restated as establishing at a 98.6% confidence level that the applied field has caused an increase in beta-decay transition probability.

C. Examples of Embodiments.

1. Coaxial Transmission Line System

One embodiment of this invention employs the electromagnetic field propagated in lowest TEM mode along a coaxial transmission line of circular cylinder configuration. The fuel constitutes the dielectric medium that lies in the cylindrical annulus between the inner and outer conductors of the transmission line. The nuclear radiations emitted by the fuel are converted to thermal energy by being stopped within the fuel and/or surrounding materials. This thermal energy is then converted in the conventional manner to drive rotating machinery, or it can be further converted to electrical energy in conventional fashion.

The coaxial transmission line operating in the simplest TEM mode represents a straightforward application of the theory of induced beta decay, since apart from a radial decrease of the amplitude of the fields, the fields are of pure plane wave type.

The fuel should be in the form of a non-conducting material. One approach is to use a solid material of high melting point (e.g., $K_2Si_2O_5$, $CaCO_3$, CdF_2, $SrSiO_3$) to minimize problems in case of a loss-of-coolant accident. Coolant can be passed through channels within the fuel annulus, and/or at the outer periphery of the outer conductor, and/or within the inner conductor. Another strategy is to

have the fuel in the form of an aggregate of geometrical shapes over which a gaseous or liquid coolant flows. Another approach is to use a fuel with low melting point (but preferably a high boiling point), so that the fuel is a dielectric liquid at normal operating temperatures. The fuel itself can then be used as the heat transfer medium, circulating between the region of the fuel annulus and an external heat exchanger. An advantage of this technique is that the circulating fuel can be continuously purged of decay products and replenished with fresh fuel to maintain a steady fuel concentration.

Figure 2 shows a system based on a coaxial transmission line, consisting of an outer conductor 1 and an inner conductor 2, terminating in an absorptive load 3, represented schematically by a resistor. A power supply 4 transmits power of appropriate frequency along the line. The resulting electromagnetic field in the insulating fuel medium 5, which comprises the dielectric separating inner and outer conductors of the coaxial transmission line, causes beta decays to be induced in the fuel. The energy generated within the fuel 5, and the energy transmitted to the absorptive load 3, are transferred to a coolant fluid 6, which runs a conventional system of turbines 7, and generators 8, to produce the electric power output. A heat dump 9 is provided in the working fluid system in order to complete the thermal cycle. If the generators 8 are omitted, the power plant can be used to provide mechanical energy rather than electrical energy.

To represent the fact that the system could consist of any number of coaxial transmission lines as just described, Fig. 2 shows two such assemblies.

For ease of representation, the transmission line in Fig. 2 is shown with a larger diameter-to-length ratio than is likely to be used in practice. Also for ease of representation, the transmission line is shown straight, whereas in practice it may be employed in a coiled configuration with coils one or more layers deep, with axis of coiling horizontal, vertical, or at any other orientation. Other space-saving configurations other than coiling may also be used.

An alternative configuration would have the transmission line consist of a grid of conductors embedded in a natural mineral deposit containing the fuel material, where this natural deposit is protected, by nature or by design, from developing inadvertent conductivity paths.

In the theoretical treatments given above, most of the work was done in a Gaussian system of units, with $\hbar = c = 1$. A change is now made to the SI, or Mks, system of practical electromagnetic units.

A circular cylinder transmission line operating in the simplest TEM mode has electric and magnetic fields given by

$$\vec{E} = \hat{\rho}\,\frac{C}{\rho}\cos(\omega t - kz) \tag{55}$$

$$\vec{B} = \hat{\phi}\,\sqrt{\mu\varepsilon}\,\frac{C}{\rho}\cos(\omega t - kz) \tag{56}$$

expressed in ρ, ϕ, z cylindrical coordinates, and with permittivity ε and permeability μ relating to the dielectric material contained between the inner and outer conductors. The amplitude factor C contained in Eqs. (55) and (56) can be related to the intensity parameter z. It is convenient to use a mean intensity

parameter $\bar{z}$, where the mean is obtained by averaging over the volume of the dielectric in the transmission line. From Eqs. (46) or (47), z is proportional to $1/\rho^2$, and

$$\left(\frac{1}{\rho^2}\right)_{aver.} = \frac{2\ell n(\rho_o/\rho_i)}{\rho_o^2-\rho_i^2} \quad ,$$

where ρ_o and ρ_i are, respectively, the inner radius of the outer conductor and the outer radius of the inner conductor. The end result is that $\bar{z}$ is

$$\bar{z} = \left(\frac{eR_oC}{\hbar\omega}\right)^2 \frac{2\ell n(\rho_o/\rho_i)}{(\rho_o^2-\rho_i^2)} \qquad (57)$$

or

$$C = \bar{z}^{-\frac{1}{2}} \frac{\hbar\omega}{eR_o} \left[\frac{\rho_o^2-\rho_i^2}{2\ell n(\rho_o/\rho_i)}\right]^{\frac{1}{2}} . \qquad (58)$$

The transmission line is presumed to terminate in a non-reflecting absorptive load. This means that the simple propagating plane wave character of the fields is unaffected. It also means that the power transmitted along the line can be converted to thermal power, which adds to that arising from induced beta decay. Thus, a portion of the power employed to operate the device can be recovered.

The power transmitted along a coaxial transmission line is

$$U = \pi(\varepsilon/\mu)^{\frac{1}{2}} C^2 \ell n(\rho_o/\rho i) \quad ,$$

which becomes

$$U = 2\pi^3(\hbar/eR_o)^2(\varepsilon/\mu)^{\frac{1}{2}}\bar{z}\nu^2\rho_o^2(1-\rho_i^2/\rho_o^2) \qquad (59)$$

when Eq. (58) and $\omega=2\pi\nu$ are used. The transmission lines considered will not be long enough for attenuation along the line to be an important factor.

Output power from the coaxial system is just average power density times fuel volume, or, from Eq. (43),

$$P = E\, W_{ind}\, \rho\, \pi\, \rho_o^2(1-\rho_i^2/\rho_o^2)\ell \ . \qquad (60)$$

In Eq. (60), E is the usable energy released per beta decay nucleus expressed in joules (not in MeV as in Eq. (43)), ρ without subscript is the density of beta decay nuclei, and ℓ is the length of the transmission line. Equations (59) and (60) make clear that input and output powers have the same dependence on the radius of the transmission line, but output power is proportional to the length of the line. This suggests the use of long lines, which may be coiled into compact arrays. Total power output of a single plant need not come from a single transmission line, but could be the summed contributions of a number of long, coiled lines.

For example, for ^{113}Cd at $\bar{z} = 3.394$, as in Eq. (28), a choice of 300 MW thermal power for P in Eq. (60) at a length of 10^4m gives a value for ρ_o of 1.68m. Then, with these parameters and a choice of ν = 27.4kHz, the input power from Eq. (59) is 33.3MW, or P/9. The total thermal power available to the heat exchange medium is P+U, so in this example this total power is ten times the input power.

As another example suppose the fuel is potassium hydroxide, enriched to 90% in ^{40}K. If this system is operated at $\overline{z}$ = 18.647 (see Eq. (32)), a choice of 2.5 x 10^9W thermal power for P in Eq. (60) at a length of 10^4m gives a value for ρ_o of 0.481m. With these parameters, and a choice of ν = 110kHz, the input power from Eq. (59) is 2.8 x 10^8W, or P/9. As in the Cd example above, the total power P+U is ten times the input power.

2. Coaxial Resonant Cavity System.

Another embodiment of this invention employs the electromagnetic field existing in a resonant coaxial cavity excited in lowest TEM mode. The cavity is just like the coaxial transmission line treated above, except that it is terminated by reflectors at a length equal to an integer number of half wavelengths. As with the transmission line, the fuel constitutes the dielectric medium contained between the inner and outer conductors of the coaxial cavity. The nuclear radiations emitted by the fuel are converted to thermal energy by being stopped within the fuel and/or surrounding materials. This thermal energy is then converted to mechanical and/or electrical energy in the conventional manner.

Figure 3 shows a system based on a coaxial resonant cavity consisting of an outer conductor 1 and an inner conductor 2. A power supply 4 provides the power necessary to sustain an electromagnetic field which is resonant in the cavity. This electromagnetic field induces beta decays to occur in the insulating fuel medium 5, which comprises the dielectric separating the inner and outer conductors of the coaxial cavity. The energy generated within the fuel

medium, as well as the energy occurring as wall losses in the cavity, are transferred to a coolant fluid 6, which runs a conventional system of turbines 7, and generators 8, to produce the electric power output. A heat dump 9 is provided in the working fluid system in order to complete the thermal cycle. Direct mechanical output can be provided in place of electrical output if the generators 8 are omitted.

To represent the fact that the system could consist of any number of coaxial cavities as just described, Fig. 3 shows two such assemblies.

For ease of representation, the resonant cavity in Fig. 3 is shown with a larger diameter-to-length ratio than is likely to be used in practice. Also for ease of representation, the cavity is shown straight, whereas in practice it may be employed in a coiled configuration with coils one or more layers deep, with axis of coiling horizontal, vertical, or at any other orientation. Other space-saving configurations other than coiling may also be used.

An alternative configuration would have the resonant cavity consist of a grid of conductors embedded in a natural mineral deposit containing the fuel material, where this natural deposit is protected, by nature or by design, from developing inadvertent conductivity paths.

The coaxial cavity considered here is taken to be the same as the coaxial transmission line treated above, but with the length specified to be an integer multiple of half a wavelength, and with both ends closed by reflectors. Equations (55) and (56) are

replaced by

$$\vec{E} = \hat{\rho}\,\frac{C'}{\rho}\,\sin kz\,\cos\omega t \qquad (61)$$

$$\vec{B} = \hat{\Phi}\,\sqrt{\mu\varepsilon}\,\frac{C'}{\rho}\,\cos kz\,\cos\omega t\ . \qquad (62)$$

The length of the cavity is given in terms of field frequency ν by

$$\ell = n/2\sqrt{\mu\varepsilon}\ \nu \quad , \qquad (63)$$

where n is the number of half wavelengths within the cavity. In the above equations, k is defined by $k = \omega\sqrt{\mu\varepsilon}$.

The intensity parameter can be averaged radially as it was in the transmission line, but an axial averaging is also appropriate. This axial averaging is complicated by the fact that induced beta decay occurs under plane-wave-like conditions where $|\vec{E}| = c|\vec{B}|/\varepsilon_r^{\frac{1}{2}}$. This is true throughout a transmission line, but a cavity has regions where $|\vec{E}| > c|\vec{B}|/\varepsilon_r^{\frac{1}{2}}$ and regions where the opposite is true. When axial averaging is done in a cavity under the premise that the governing field amplitude is always the smaller of the local values of $|\vec{E}|$ or $c|\vec{B}|/\varepsilon_r^{\frac{1}{2}}$, then the combined axial and radial average is

$$\overline{z} = \left(1 - \frac{2}{\pi}\right)\left(\frac{eR_oC'}{\hbar\omega}\right)^2\ \frac{2\ln(\rho_o/\rho_i)}{(\rho_o^2 - \rho_i^2)} \qquad (64)$$

or

$$C' = \bar{z}^{1/2}\left(1-\frac{2}{\pi}\right)^{-1/2}\frac{\hbar\omega}{eR_0}\left[\frac{\rho_0^2-\rho_i^2}{2\ln(\rho_0/\rho_i)}\right]^{1/2} \qquad (65)$$

In this case input power is just the loss in the cavity walls. This power loss is

$$U = \frac{n\pi^2}{4}\left(\frac{\varepsilon}{\mu}\right)^{1/2}\frac{\delta}{\rho_0}C'^2\left(1+\frac{\rho_0}{\rho_i}+\frac{4\rho_0}{l}\ln\frac{\rho_0}{\rho_i}\right). \qquad (66)$$

The last term in the final parenthesis in Eq. (66) gives the losses in the end walls of the cavity. These are negligible for parameters of interest here, so this term is dropped hereafter. The quantity C' in Eq. (66) is given by Eq. (65), so the cavity equivalent of Eq. (59) for the transmission line is

$$U = \frac{n\pi^5}{(\pi-2)}\left(\frac{\varepsilon}{\mu}\right)^{1/2}\left(\frac{\hbar}{eR_0}\right)^2\delta\bar{z}\nu^2\rho_0\frac{(1-\rho_i^2/\rho_0^2)(1+\rho_0/\rho_i)}{2\ln(\rho_0/\rho_i)}. \qquad (67)$$

The factor δ in Eqs. (66) and (67) is the skin depth, given by

$$\delta = (2/\mu\omega\sigma)^{\frac{1}{2}}, \qquad (68)$$

where σ is the conductivity of the cavity wall material. If this material is copper, then Eq. (68) is

$$\delta = (6.61 \times 10^{-2})\nu^{-\frac{1}{2}}. \qquad (69)$$

This implies a $\nu^{5/2}$ dependence on frequency in Eq. (67).

Output power is still expressed by Eq. (60) with the important proviso that the $\bar{z}$ employed in the evaluation of W_{ind} in Eq. (60) comes from Eq. (64) and

not from Eq. (57). Equation (64) takes into consideration the spatially periodic decline to zero of the fields within the cavity. With the same set of assumptions employed for the transmission line example with ^{113}Cd, one obtains again $\rho_o = 1.68m$ for $\ell = 10^4 m$ with $\bar{z} = 3.394$ and $P = 3 \times 10^8 W$. A calculation of input power now requires an assumption for n. If n=10, then $\nu = 75kHz$ and $U = 3 \times 10^6 W$. In the case of a cavity, output thermal power is then about 100 times input power. For the same set of assumptions as employed for the transmission line ^{40}K example, one obtains again $\rho_o = 0.481m$ for $\ell = 10^4 m$ with $\bar{z} = 18.647$ and $P = 2.5 \times 10^9 W$. If $n = 20$, then $\nu = 130kHz$ and $U = 2.5 \times 10^7 W$.

3. Other Transmission Lines and Resonant Systems

The results presented above apply also to other traveling wave and standing wave transmission line arrangements. By other transmission lines are meant, for example, the two-wire, four-wire, or coaxial cage transmission lines, any of the possible strip-line configurations, or any other transmission line arrangement that can support a simple TEM mode. In such cases with a traveling-wave TEM mode, the electric and magnetic fields will be essentially those stated in Eqs. (55) and (56). For terminations so arranged as to provide resonance, then Eqs. (61) and (62) are applicable.

4. Other Configurations

Although transmission lines represent a very convenient way to provide plane-wave-like electromagnetic fields of large enough intensity to

induce beta decay, other field-producing configurations can also be used. The fields in such cases will not be strictly simple TEM such as provided by plane waves and transmission lines, but nevertheless some proportion of the total fields produced can be of that nature. For example, the fields in close proximity to a long cylinder carrying an alternating current, or the fields inside a large-diameter solenoid carrying alternating current, or the fields in close proximity to a large, hollow torus carrying alternating current in the azimuthal direction, will all possess components that can be employed to induce beta decay.

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C L A IM S

1. A process for inducting beta decay transitions of atomic nuclei comprising providing a medium which includes atomic nuclei that have forbidden beta decay transitions in which the initial and final nuclear states do not have the same intrinsic parity or have total angular momenta which differ by more than one quantum unit of angular momentum, and applying to the medium an electromagnetic field which has an intensity sufficient to provide the angular momentum or intrinsic parity necessary to overcome the forbiddenness of the beta decay transitions of the atomic nuclei, thereby to induce the beta decay transitions.

2. A process for obtaining useful energy comprising the steps of:

providing a fuel medium which includes atomic nuclei that have forbidden beta decay transitions;

applying an electromagnetic field to the fuel medium, the field having an intensity sufficient to overcome the forbiddenness of beta decay transitions of the nuclei; and,

capturing nuclear emissions caused by beta decay transitions of the nuclei and recovering useful energy therefrom.

3. A process for reducing the halflife of nuclear waste products that include atomic nuclei that have forbidden beta decay transitions, comprising applying an electromagnetic field to the waste products, the field having an intensity sufficient to overcome forbiddenness of beta decay transitions of the nuclei thereby to enhance beta decay with the release of nuclear emissions from the nuclear waste products.

4. A process according to any one of the preceding claims, in which the electromagnetic field is an alternating electromagnetic field of frequency ν and magnetic induction B, with B/ν at least of the order of 10^{-10} teslas per hertz.

5. A process according to any one of the preceding claims, in which the density ρ of the atomic nuclei is at least of the order of $10^{14}/EW_{ind}$ nuclei per cubic metre, where E is the decay energy in MeV involved in the beta decay of a single nucleus, and W_{ind} is the total induced transition probability in sec_{-1} for the beta decay.

6. A process according to any one of the preceding claims, in which the atomic nuclei are selected from a group consisting of ^{90}Sr, ^{137}Cs, ^{48}Ca, ^{87}Rb, ^{40}K, ^{50}V, ^{113}Cd, ^{115}In, ^{96}Zr, ^{85}Kr, ^{99}Tc, ^{135}Cs and ^{129}I.

7. An apparatus for inducing beta decay transitions of atomic nuclei by a method in accordance with any one of the preceding claims, the apparatus comprising a medium (5) which includes atomic nuclei that have forbidden beta decay transitions in which the initial and final nuclear states do not have the same intrinsic parity or have total angular momenta which differ by more than one quantum unit of angular momentum, field producing means (1, 2) for producing an electromagnetic field in the medium and means (4) for energising the field producing means to establish the field at an intensity sufficient to provide the angular momentum or intrinsic parity necessary to overcome the forbiddenness of the beta decay transitions of the atomic nuclei.

8. An apparatus according to claim 7, which also comprises means (6, 7, 8) for collecting the energy of nuclear emissions caused by beta decay of the atomic nuclei.

9. An apparatus according to claim 7 or 8, in which the field producing means comprises a transmission line (1, 2, Figure 2) or a resonant cavity (1, 2, Figure 3) and in which the fuel medium is located within the transmission line (1, 2, Figure 2) or resonant cavity (1, 2, Figure 3).

10. An apparatus according to claim 9, in which the transmission line or resonant cavity is formed by a pair of coaxial conducting cylinders (1, 2) and in which the medium (5) is located in the annular region between the inner and outer conducting cylinders (1, 2).

11. An apparatus according to any one of claims 7 to 10, in which the energising means (4) comprises an alternating current source.

12. An apparatus according to any one of claims 7 to 11, in which the medium (5) is an electrically insulating material containing the atomic nuclei.

13. An apparatus according to claim 12, in which the insulating material is a dielectric liquid.

14. An apparatus according to any one of the preceding claims, in which the medium (5) is a fluid and acts as a heat transfer fluid to transfer heat away from the field producing means (1, 2).

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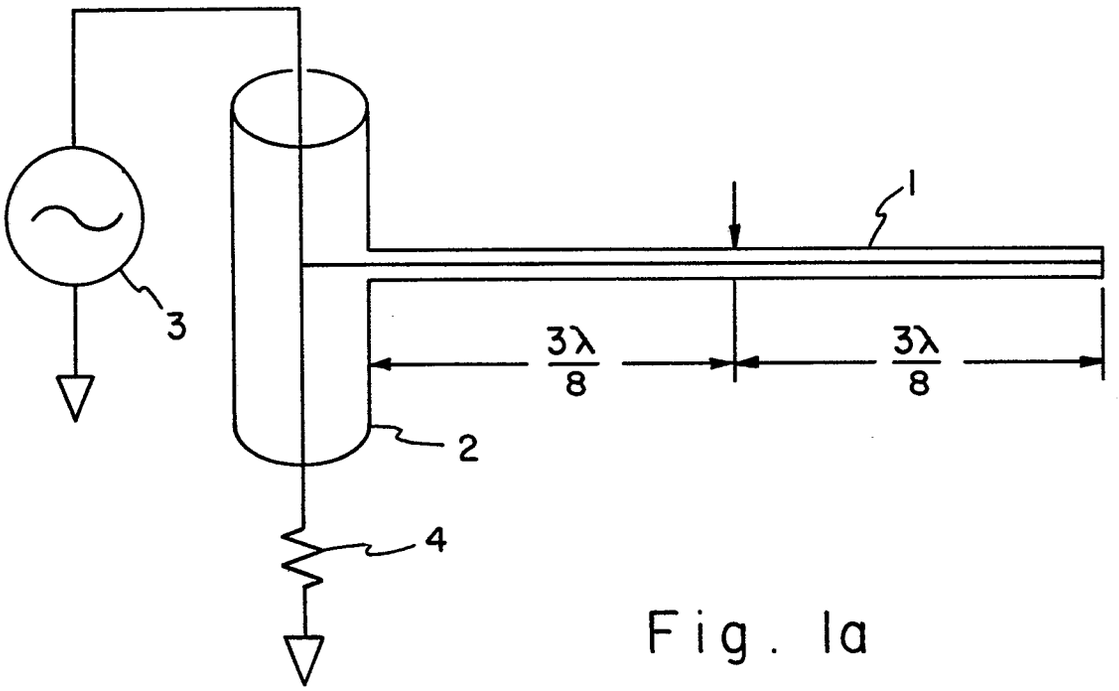

Fig. 1a

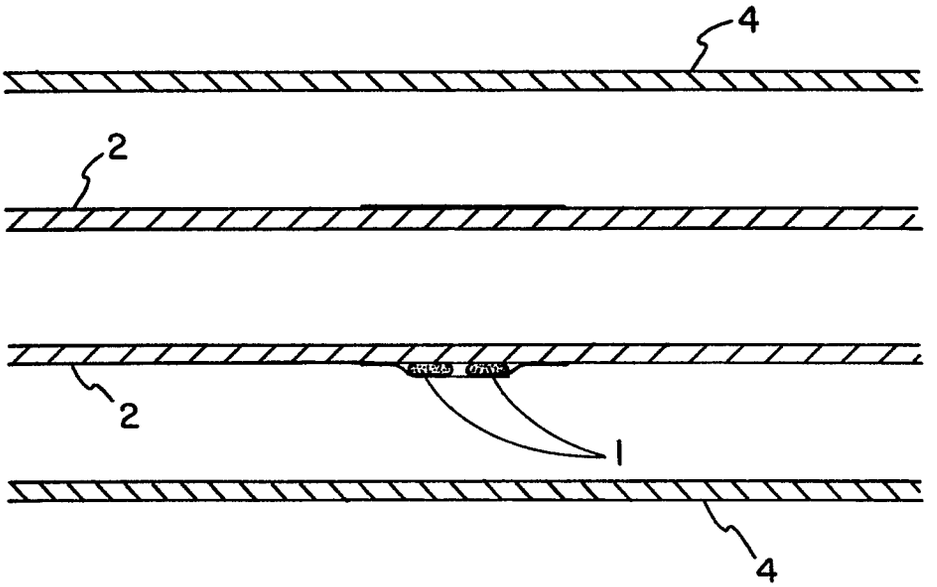

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Fig. 1b

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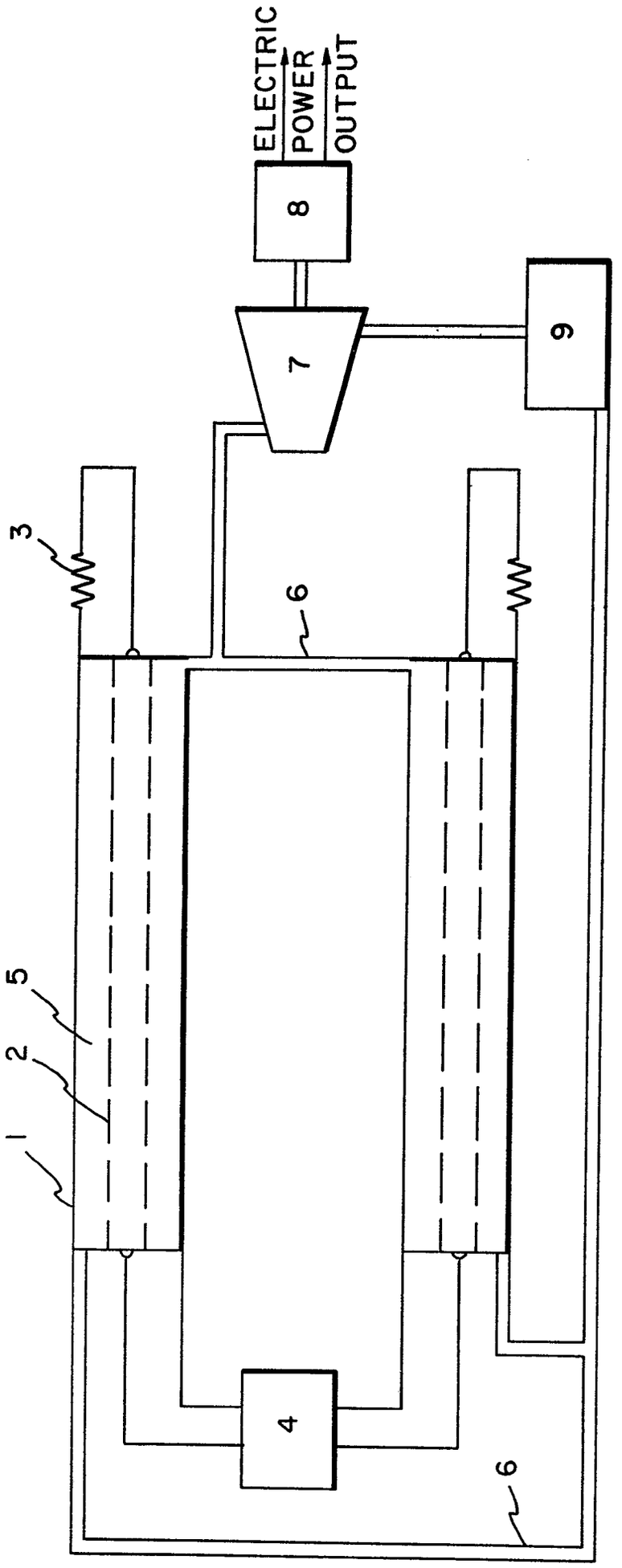

Fig. 2

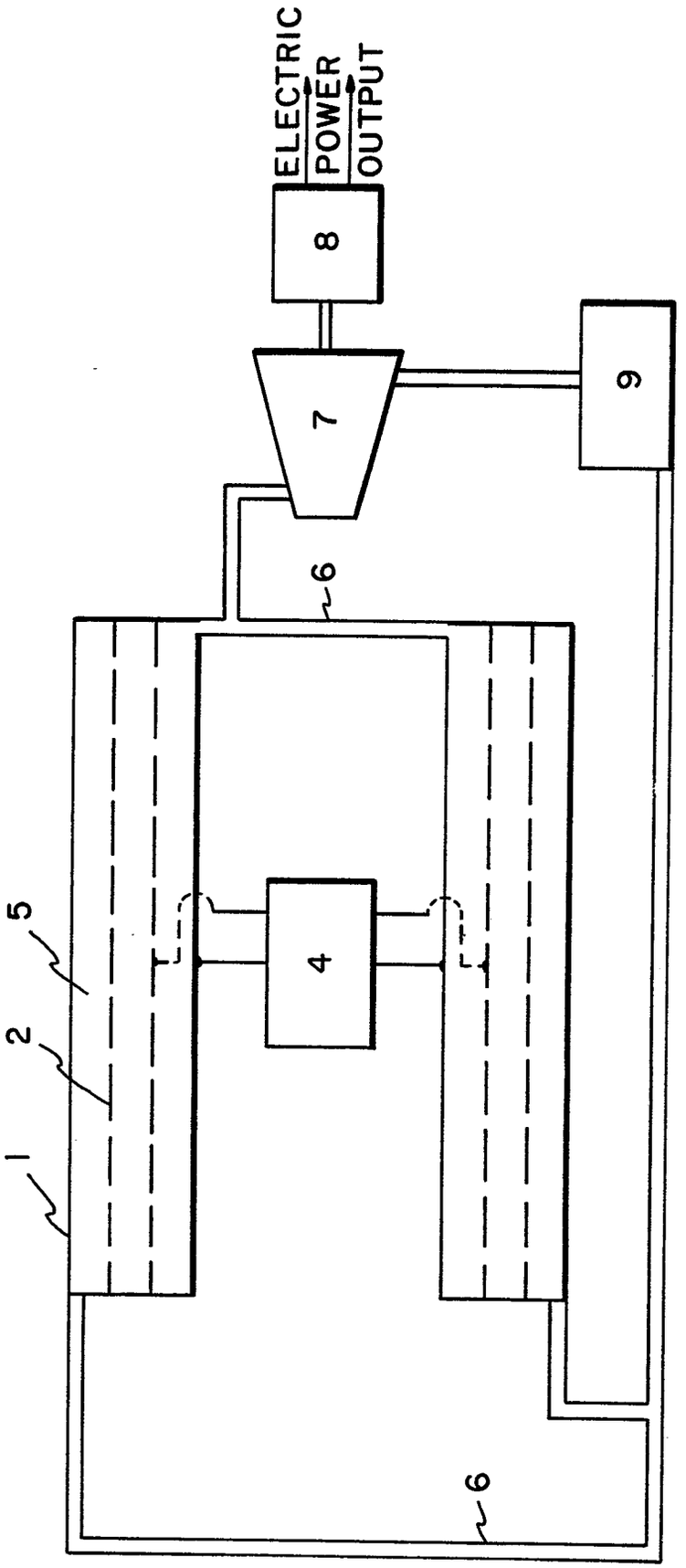

Fig. 3

European Patent Office

EUROPEAN SEARCH REPORT

Application number 0099946

EP 82 30 3910

DOCUMENTS CONSIDERED TO BE RELEVANT

Category	Citation of document with indication, where appropriate, of relevant passages	Relevant to claim	CLASSIFICATION OF THE APPLICATION (Int. Cl. ³)
A	NUCLEAR INSTRUMENTS AND METHODS, vol. 141, no. 3, 15th March 1977, pages 429-432, North-Holland Publishing Co.; M.BURNS et al.: "Laser induced nuclear orientation effects". *The whole article*	1	G 21 G 1/12
A	GB-A- 802 971 (VERSCHRAEGHEN) *Claim 1*	1,3	
A	GB-A-1 147 585 (USAEC) *Claim 1*	1,3	
A	DE-A-2 249 429 (GESELLSCHAFT FÜR KERNFORSCHUNG)		
			TECHNICAL FIELDS SEARCHED (Int. Cl. ³)
			G 21 G 1/00 G 21 F 9/00 G 21 K 1/00

The present search report has been drawn up for all claims

Place of search	Date of completion of the search	Examiner
THE HAGUE	08-03-1983	NICOLAS H.J.F.

CATEGORY OF CITED DOCUMENTS

X : particularly relevant if taken alone
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