Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. Figure 8 illustrates this graphically.

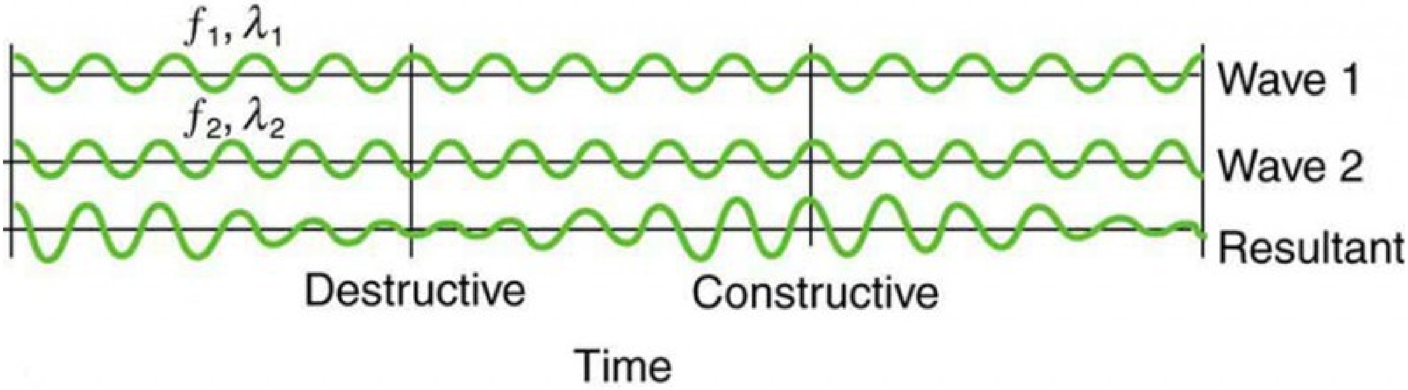

Figure 8. Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. For our example the two frequencies are 5000Hz and 4992.17Hz with a resulting frequency of 7.83Hz. This wave fluctuates in amplitude, or *beats*, with a frequency called the *beat frequency*. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as $x=X\cos(2\pi tT)=X\cos(2\pi ft) x=X\cos(2\pi tT)=X\cos(2\pi ft)$, where $f=1T f=1T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant $x = x_1 + x_2$. More specifically, $x = X\cos(2\pi f_1 t) + X\cos(2\pi f_2 t)$.

Using a trigonometric identity, it can be shown that $x = 2X\cos(\pi f_B t)\cos(2\pi f_{ave} t)$, where $f_B = |f_1 - f_2|$ is the beat frequency, and f_{ave} is the average of f_1 and f_2. **These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_B.** The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency f_{ave}. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.