**Generator Dynamic Modelling with FEMM**

This paper presents a method for dynamic modelling of a generator in FEMM where the output voltage and current are automatically calculated throughout the LUA steps.

It is assumed that a LUA controlled simulation is set up using small angular increments of the rotor. Calculation of the open circuit voltage is easily achieved by ascribing a time increment $\Delta t$ to each step, obtaining the change of flux linkage $\Delta\Phi$ at each step, then

$$V = -\frac{\Delta\Phi}{\Delta t}$$

(Note it is assumed that FEMM provides flux linkage values so all flux values used in this report are assumed to be flux linkage)
The problem comes in loading that voltage with a resistor where the load current by Lenz's law creates a flux which opposes that flux change $\Delta\Phi$. That then reduces the flux change which changes the value of *V*. We need a method of calculating the correct voltage and current "on the fly". It may be expected that this calculation will use the L/R time constant of the coil and load, and that brings in a further complication, the L value may also change with rotor movement.

The method adopted here needs two FEMM runs per rotor step. In the first run the coil current is set to zero: this allows the unloaded flux to be recorded, also the unloaded torque. By subtracting from that flux the previous value of unloaded flux we get the unloaded flux change $\Delta\Phi_m$ that is independent of the current, and due only to the rotor movement. Using that value, the known value of coil inductance and the Lenz flux $\Phi_L$ at the previous step, we can calculate the expected rate of change of Lenz flux $d\Phi_L/dt$. That can be converted to an expected value of flux change $\Delta\Phi_L$ which can be added to $\Phi_L$ to get a new value of Lenz flux. That new flux value and the inductance give us the expected current needed to produce it. FEMM can then be run again with that new value of current, and results recorded. Finally the FEMM results can be used to recalculate inductance ready for the next step.

**Math background.**

Let the flux $\Phi$ in the coil be the sum of two fluxes, the input flux from the magnet $\Phi_M$ and the Lenz flux from the load current $\Phi_L$.

$$\Phi = \Phi_M + \Phi_L \qquad (1)$$

The voltage *V* across the load is then

$$V = -N\frac{d\Phi}{dt} = -N\left(\frac{d\Phi_M}{dt} + \frac{d\Phi_L}{dt}\right) \qquad (2)$$

This drives current *I* through the load *R*

$$I = \frac{V}{R} \qquad (3)$$

The Lenz flux $\Phi_L$ from this current is then

$$\Phi_L = LI \qquad (4)$$

where *L* is the coil inductance. Combining (2), (3) and (4) gives

$$\Phi_L = -\frac{L}{R}\left(\frac{d\Phi_M}{dt} + \frac{d\Phi_L}{dt}\right) \qquad (5)$$

which can be arranged into

$$\frac{d\Phi_L}{dt} = -\frac{d\Phi_M}{dt} - \frac{R}{L}\Phi_L \qquad (6)$$

By running FEMM with zero current at each rotor step, $\frac{d\Phi_M}{dt}$ can be obtained from the change of $\Phi_M$ from the previous step divided by the step time

$$\frac{d\Phi_M}{dt} = \frac{\Phi_{Mnew} - \Phi_{Mold}}{\Delta t} \qquad (7)$$

The second term in (6) is also readily available using the previous value of $\Phi_L$. Hence $\frac{d\Phi_L}{dt}$ is evaluated. A new value for $\Phi_L$ is then calculated

$$\Phi_{Lnew} = \Phi_{Lold} + \frac{d\Phi_L}{dt}\Delta t$$

Current needed to get this expected value of $\Phi_{Lnew}$ can then be got from (4), and this is the correct value of load current. FEMM is then run again at this load current and the results recorded. The actual change of flux from the first unloaded run, $\Phi_{Lactual}$, as given by FEMM, is then used in (4) to get a new value of inductance ready for the next rotor increment. That actual flux is also used as the old $\Phi_L$ value for the next rotor step.

There are some start values needed. The coil inductance $L$ can be got from two FEMM runs with the rotor stationary by putting an arbitrary value of current through the coil on the second run, obtaining the flux change then calculating $L$ from

$$L = \frac{\Delta\Phi}{I} \qquad (9)$$

The initial Lenz flux $\Phi_L$ needs some previous full runs to get right. The initial wrong value (perhaps zero on the first run) gradually gets taken away by the L/R time constant exponential decay, so eventually yielding the correct start value at the end of a full cycle. For light loading where the *L/R* time constant is very short just one run will do, but for heavy loading which involves a large phase shift it may be necessary to run through several full cycles before the phase shift settles down.