On Electrodynamic Formulae for the Marinov Generator

1. Introduction

Classical electrodynamics as currently taught gives the force **F** acting on an electric charge q (such as an electron where q is negative) as

$$\mathbf{F} = q\mathbf{E} \tag{1}$$

where **E** is the electric field in which the charge is immersed (here we use bold characters to represent vectors). **E** can come from a variety of sources and is generally expressed by

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} \tag{2}$$

where ϕ is the scalar electric potential, **A** is the magnetic vector potential, **v** is the velocity of q and **B** is the magnetic field, all at the position of q. This formula will be found in any college text-book teaching electromagnetic theory. The first term is the electrostatic or quasi-static *Coulomb* field from nearby charges. The second term is the so-called *transformer induction* where **A** and the resultant **E** form circles around the transformer core, and it is this **E** field that drives the conduction electrons inside the conductors forming the transformer secondary coil. Since that **A** field is related to the magnetic flux Φ within the core, the flux must change with time in order to create that **E** field; unfortunately the presence of that **E** field is not usually taught in electrical engineering, it is hidden behind the formula expressing volts-per-turn as

$$V = -\frac{\partial \Phi}{\partial t} \tag{3}$$

where Φ is the magnetic flux within the core. The missing formula that is generally not taught in electrical engineering links the vector potential **A** to the flux Φ by stating that for any closed circuit that encircles the core the line-integral of **A** is exactly equal to Φ, hence the second term in (2) leads directly to (3).

The third term is the so-called *motional induction* that applies to either (a) generators where **v** is the movement of the conductor whence **E** yields the voltage induced into the conductor or (b) motors where **v** is the drift velocity of the conduction electrons within the conductor and **E** yields the sideways force applied to those electrons that then move sideways using the Coulomb force of attraction to drag the lattice of positive ions with them. The vector product $\mathbf{v}\times\mathbf{B}$ is usually taught by means of Fleming's RH and LH rules with load current taking the place of induced **E** or drive current taking the place of drift velocity.

There have been various papers written about the possibility of another **E** field that should be added to (2), prompted by the work of the late Stephan Marinov, such that (2) now becomes

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} - \nabla(\mathbf{v}\cdot\mathbf{A})\,. \tag{4}$$

This new term $\nabla(\mathbf{v}\cdot\mathbf{A})$ is the gradient of the scalar product of the velocity **v** with the magnetic vector potential **A**, that scalar being the component of **A** that is tangential to the velocity multiplied by the velocity magnitude. This scalar $(\mathbf{v}\cdot\mathbf{A})$ is something that in earlier times was referred to as *electro-kinetic potential*. That potential has long been discarded from modern theories, but is resurrected to account for some of Marinov's experiments. In the presence of a static **A** field that is not spatially uniform it seems to fit the bill of a time-changing **A** as "seen" by a moving electron.

Marinov's work was a novel form of electric motor that supposedly developed force along the conductor in the form of a slip-ring, but it is seriously disputed by others who have tried replications and either measured zero torque or more confusingly positive or negative torque depending on where the electrical contacts are made to the slip-ring. The new term being the gradient of a scalar potential, just like the Coulomb field, when integrated around a closed circuit produces zero voltage induction, hence little attention has been paid to the generator version of Marinov's experiment. In this present paper we look at the derivation of (4) to question some of the assumptions made, and arrive at a different formula. We also question how Marinov's longitudinal induction, i.e. forces on the conduction electrons acting *along* the filamentary conductor, can be expected to translate into a measurable force on the whole conductor to act as a motor, whereas in fact it should simply appear as an internal **E** field affecting a measurement of conductance or resistance.

2. The Marinov Motor/Generator

The Marinov motor used a slip-ring rotor that passes around a pair of parallel magnetized cores (either permanent magnets or electro-magnets) that are long and have opposite directions of magnetization. Marinov actually used a cylindrical bar magnet that was split into two then one half reversed to create two magnets each having semi-circular cross section. He used a mercury-filled circular channel as the slip-ring with two electrical contacts penetrating the surface at diametrically opposite positions, and noted circular movement of the mercury when current was applied. Such a scheme is depicted in Figure 1 taken from one of Marinov's publications

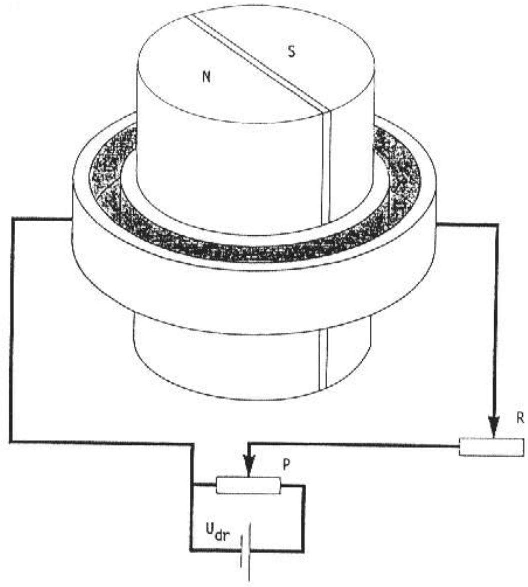

Figure 1. The Original Marinov Motor

Other forms of the motor used two parallel rod magnets connected to appear as an elongated magnetized toroidal core, Figure 2.

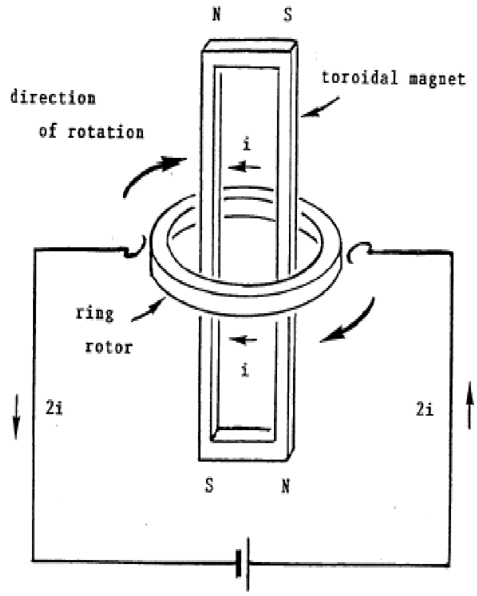

Figure 2. Marinov Motor

It is this form that has been investigated by various people.

3. Derivation of Equation (4)

Several authors start on the basis that a charge q with mass m within a magnetic vector potential $\mathbf{A}$ has both a mechanical momentum $m\mathbf{v}$ and another electro-kinetic momentum $q\mathbf{A}$. The latter is a form of *hidden* momentum, *hidden* because it is not related to velocity. To many schooled in classical mechanics the idea of a body possessing momentum that is not related to is velocity is hard to grasp. However it should be realized that electro-dynamic or electro-kinetic forces are transmitted to a body via photons or sub-photons, and these invisible particles travelling at light velocity do carry momentum. Thus the so-called *hidden* momentum could arise from some form of *supplied* momentum, supplied by interaction with those invisible space particles. It can be argued that space is filled with such virtual particles arriving from all directions, and all our measured forces on matter come from subtle changes in the average effect of momentum exchange taking place at both absorption and re-emission of those space particles.

Now it is possible to relate the rate-of-change of the *total* momentum to an applied force **F**, such as the Coulomb force from nearby charge as expressed by the presence of an electric field $\mathbf{E} = -\nabla\phi$ as

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v} + q\mathbf{A}) = -(\nabla\phi)q \tag{5}$$

This is a classical inertial effect where **F** is the force that must be *applied* to create the change of momentum. For a particle with fixed q (like an electron) within a constant **A** field its hidden momentum $q\mathbf{A}$ is constant, its time derivative is zero so it disappears from (5), then **F** is simply the classical value needed to achieve the change of velocity for a given mass. However it should be recognised that if $q\mathbf{A}$ changes with time then this changing momentum can *create* a force so that now a new force value **F'** is needed for the same change of velocity at the given mass. It makes sense to move $q\mathbf{A}$ over to the RH side of (5) where we obtain for the new force

$$\mathbf{F}' = \frac{d}{dt}(m\mathbf{v}) = -(\nabla\phi)q - \frac{d}{dt}(q\mathbf{A}) \tag{6}$$

Dividing by q gives the total effective **E** field as

$$\mathbf{E} = \frac{\mathbf{F}'}{q} = -\nabla\phi - \frac{d\mathbf{A}}{dt} \tag{7}$$

It is important now to realise that instead of the partial differential of **A** that applies to it's change with time at a fixed point in space, one should use the full differential $D\mathbf{A}$ that also takes account of how **A** changes with time as the test point moves through space containing a non-uniform **A** field. We then get

$$\mathbf{E} = -\nabla\phi - \frac{D\mathbf{A}}{Dt} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t} - (\mathbf{v}\cdot\nabla)\mathbf{A} \tag{8}$$

which contains both the partial differential and the so called *convective term* used in fluid dynamics. Then using a well known vector relationship

$$\nabla(\mathbf{v}\cdot\mathbf{A}) = \mathbf{v}\times\nabla\times\mathbf{A} + (\mathbf{v}\cdot\nabla)\mathbf{A} + \mathbf{A}\times\nabla\times\mathbf{v} + (\mathbf{A}\cdot\nabla)\mathbf{v} \tag{9a}$$

that can be rearranged into

$$(\mathbf{v}\cdot\nabla)\mathbf{A} = \nabla(\mathbf{v}\cdot\mathbf{A}) - \mathbf{v}\times\nabla\times\mathbf{A} - \mathbf{A}\times\nabla\times\mathbf{v} - (\mathbf{A}\cdot\nabla)\mathbf{v} \tag{9b}$$

we find that (8) can be expressed as

$$E = -\nabla\phi - \frac{\partial A}{\partial t} - \nabla(\mathbf{v}\cdot\mathbf{A}) + \mathbf{v}\times\nabla\times\mathbf{A} + \mathbf{A}\times\nabla\times\mathbf{v} + (\mathbf{A}\cdot\nabla)\mathbf{v} \tag{10}$$

Most authors then go on to say that for a charge q moving at velocity **v** the del operator ∇ does not act on **v** hence the last two terms can be ignored. Then since $\mathbf{v}\times\nabla\times\mathbf{A} = \mathbf{v}\times\mathbf{B}$ we get back to (4). Dennis Marvel in a private correspondence with the author puts the point more succinctly saying

> "This point is worth a little discussion. Vector operators involving spatial derivatives act on vector *fields*, not on isolated vectors. The velocity *field* of a single particle has meaning only in a Lagrangian, as opposed to an Eulerian, coordinate frame and takes on a uniform value throughout space. At each point, the velocity field is equal to the instantaneous velocity of the isolated particle: Velocity arrows positioned throughout space track the particle's local time-varying velocity vector like an enormous landscape of weather vanes. This picture arises from the portability of vectors. The particle's velocity *relative* to another particle has meaning only if its velocity vector can be translated to the other particle's position for subtraction under the triangle rule. Hence the exclusive use of Lagrangian coordinates in the evaluation of differential vector operators acting on the velocity or acceleration of an individual particle seems to be the only policy that makes sense."

The intention of eliminating all of terms where del operates on **v** is to remove any possible forces induced from the **A** field by a *changing* velocity since the source is simply the change of momentum $q\mathbf{A}$ that is independent of velocity. However the full $\nabla(\mathbf{v}\cdot\mathbf{A})$ term in (4) does still contain velocity derivatives, so it is necessary to use a truncated version where those velocity derivatives are suppressed. The subscript $_A$ is used to identify this truncated version as $\nabla_A(\mathbf{v}\cdot\mathbf{A})$ so that (4) becomes

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} - \nabla_A(\mathbf{v}\cdot\mathbf{A}). \tag{11}$$

Appendix A examines all the Cartesian components to show that the convective term $-(\mathbf{v}\cdot\nabla)\mathbf{A}$ can be replaced by $\mathbf{v}\times\mathbf{B} - \nabla_A(\mathbf{v}\cdot\mathbf{A})$, and this throws up some interesting observations.

4. Re-look at Equation (5)

For this exercise we will stay with the Marinov slip-ring that lend itself to the use of cylindrical components of the convective term $(\mathbf{v}\cdot\nabla\mathbf{A})$ thus ensuring that no components are discarded unnecessarily.

In the cylindrical coordinate system the convective operator $(\mathbf{v}\cdot\nabla)\mathbf{A}$ is given by

$$\begin{aligned}(\mathbf{v}\cdot\nabla)\mathbf{A} = \mathbf{a}_r&\left[v_r\frac{\partial A_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial A_r}{\partial\theta}-\frac{v_\theta A_\theta}{r}+v_z\frac{\partial A_r}{\partial z}\right]\\ +\mathbf{a}_\theta&\left[v_r\frac{\partial A_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{v_\theta A_r}{r}+v_z\frac{\partial A_\theta}{\partial z}\right]\\ +\mathbf{a}_z&\left[v_r\frac{\partial A_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial A_z}{\partial\theta}+v_z\frac{\partial A_z}{\partial z}\right]\end{aligned} \tag{11}$$

where $\mathbf{a}_r$, $\mathbf{a}_\theta$ and $\mathbf{a}_z$ are the unit vectors. Note that there are no spatial derivatives of v to be discarded. At first sight this supports the view that any terms on the RH side of (10) that include velocity derivatives should be discarded, but we will now show that *the procedure that simply eliminates terms where ∇ operates on v does not fully do that*. If we take the disputed $\nabla(\mathbf{v}\cdot\mathbf{A})$ term in (4) that came from this procedure, its cylindrical coordinates are given by

$$\begin{aligned}\nabla(\mathbf{v}\cdot\mathbf{A}) = \mathbf{a}_r&\left[\frac{\partial}{\partial r}(v_rA_r+v_\theta A_\theta+v_zA_z)\right]\\ +\mathbf{a}_\theta&\left[\frac{1}{r}\frac{\partial}{\partial\theta}(v_rA_r+v_\theta A_\theta+v_zA_z)\right]\\ +\mathbf{a}_z&\left[\frac{\partial}{\partial z}(v_rA_r+v_\theta A_\theta+v_zA_z)\right]\end{aligned} \tag{12}$$

Note that all the spatial derivatives apply to the product of v and A components where we must apply the differentiation product rule, for example

$$\frac{\partial}{\partial\theta}(v_\theta A_\theta) = v_\theta\frac{\partial A_\theta}{\partial\theta}+A_\theta\frac{\partial v_\theta}{\partial\theta} \tag{13}$$

which now includes the spatial velocity derivative $\frac{\partial v_\theta}{\partial\theta}$. *Somehow velocity derivatives have crept in*. Other authors have used a modified version of $\nabla(\mathbf{v}\cdot\mathbf{A})$ with the velocity derivatives removed as

$$\begin{aligned}\nabla_A(\mathbf{v}\cdot\mathbf{A}) = \mathbf{a}_r&\left[v_r\frac{\partial A_r}{\partial r}+v_\theta\frac{\partial A_\theta}{\partial r}+v_z\frac{\partial A_z}{\partial r}\right]\\ +\mathbf{a}_\theta&\left[\frac{v_r}{r}\frac{\partial A_r}{\partial\theta}+\frac{v_\theta}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{v_z}{r}\frac{\partial A_z}{\partial\theta}\right]\\ +\mathbf{a}_z&\left[v_r\frac{\partial A_r}{\partial z}+v_\theta\frac{\partial A_\theta}{\partial z}+v_z\frac{\partial A_z}{\partial z}\right]\end{aligned} \tag{14}$$

The futility of using the vector relationship (9) and then discarding some of its terms is borne out when we calculate the induction for the Marinov generator using firstly (4) and then again using (8). They should give the same results.

We can choose the coordinates such that the slip-ring lies in the r-θ plane with its rotation axis along the z direction, then we get for the ring movement $v_r = 0$, $v_z = 0$ and with the ring symmetrical to the magnets $A_z = 0$. Only the $\mathbf{a}_\theta$ component in (12) can induce voltage into the slip ring, hence we are left with

$$\mathbf{E} = -\nabla(\mathbf{v}\cdot\mathbf{A}) = -\mathbf{a}_\theta\left(\frac{v_\theta}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{A_\theta}{r}\frac{\partial v_\theta}{\partial\theta}\right) \tag{14}$$

Although the slip ring velocity v_θ is constant, for any induced current the electrons that flow via the brushes endure a change of velocity there, so the second term in (14) cannot be ignored. If (14) is valid then any voltage induced across the slip-ring from the first term is negated by the electron velocity changes at both brush contacts in the second term, i.e. (14) tells us *the Marinov generator will not work*. If we compare this with the result from (8) using the $\mathbf{a}_\theta$ component in (11) we get

$$\mathbf{E} = -(\mathbf{v}\cdot\nabla\mathbf{A}) = -\mathbf{a}_\theta\left(\frac{v_\theta}{r}\frac{\partial A_\theta}{\partial\theta} + \frac{v_\theta A_r}{r}\right) \qquad (15)$$

We see this differs from (14) in its second term. Thus the application of the vector identity (9b) plus the attempted elimination of velocity derivatives has left us with two different formulae for the same thing, and that can't be right. The direct application of the convective term in (8) without going through the vector expansion and elimination procedures yields a formula that tells us *the Marinov generator will work*.

5. Discussion

If we look again at (5) which is the basis for developing (4) we immediately see the disconnect between mechanical momentum $m\mathbf{v}$ and electro-kinetic momentum $q\mathbf{A}$ in that $q\mathbf{A}$ is independent of velocity. Certainly a change of velocity can induce a force via the first term, that force being related to the velocity rate-of change (i.e. acceleration) of mass, but in the second term it is the rate-of-change of $\mathbf{A}$ (and/or possibly q in certain circumstances) that induces the force. In a spatially non-uniform $\mathbf{A}$ field the time rate-of-change of $\mathbf{A}$ as seen by the moving charge is related to $\mathbf{v}$, *not to the rate-of-change of* $\mathbf{v}$. Thus the second term in (14) cannot be correct. If we look at the momentum $e\mathbf{A}$ of each electron as it moves at trivial drift velocity along the wires, then passes onto the slip ring at significant velocity and so on, in a uniform $\mathbf{A}$ field we see that everywhere the $e\mathbf{A}$ momentum doesn't change, *even over the electron acceleration or deceleration regions at the brushes*, figure 3.

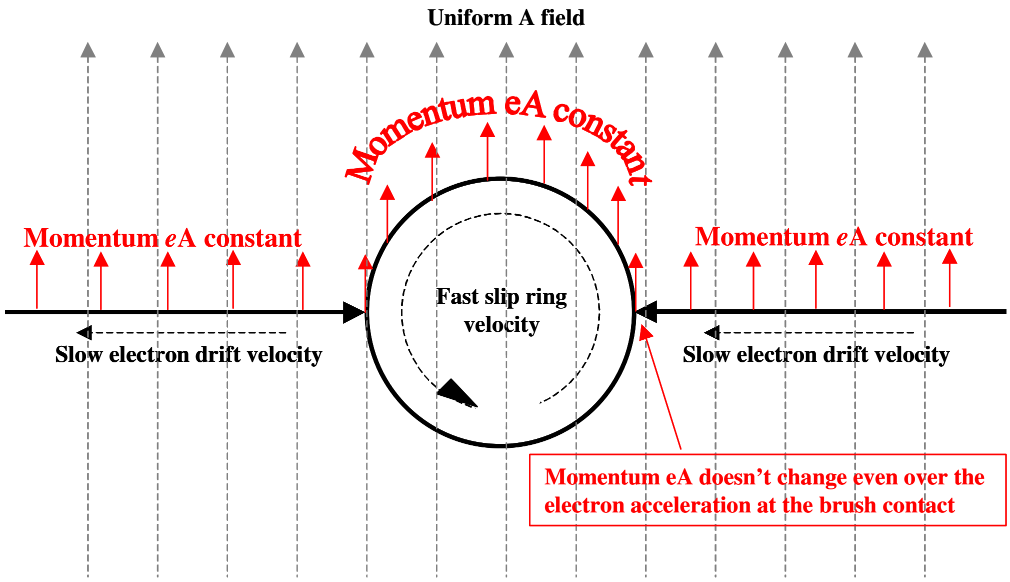

Figure 3. Electro-kinetic momentum on electrons

This should tell us that (14) is invalid and we need to use (15). So we will stick with (8) as the general defining equation leading to (15) for the slip ring set up. Now $v_\theta = \omega r$ where ω is the angular velocity and r is the fixed radius of the slip ring, so this now becomes

$$\mathbf{E}_\theta = -\mathbf{a}_\theta\omega\left[\frac{\partial A_\theta}{\partial\theta} + A_r\right] \qquad (16)$$

In a uniform $\mathbf{A}$ field, let's say $A_\theta = A\cos\theta$, then $\frac{\partial A_\theta}{\partial\theta} = -A\sin\theta$ and since $A_r = A\sin\theta$ the two terms cancel out, $E_\theta = 0$ and there is no induction. (Interestingly both (14) and (15) predict zero induction for this case but for different reasons associated with their second terms.) For a spatially non-uniform **A** field such as that near magnetized material the voltage V induced across the slip ring is given by

$$V = -\omega r\int_0^{\pi}\left(\frac{\partial A_\theta}{\partial \theta} + A_r\right)d\theta = -v_\theta\left[(A_\theta)_\pi - (A_\theta)_0\right] - v_\theta\int_0^{\pi} A_r.d\theta \qquad (17)$$

It is possible to arrive at **A** field configurations where the last term integrates to zero or, unlike that of a uniform **A** field where it negates the first terms, adds to the overall induction. Take for example four magnetised rods that pass through a slip ring where alternate rods carry flux in opposite directions. Figure 4 shows the **A** field and included there is that half of the slip ring over which the integration (17) is carried out (integration over the other half yields the same voltage and polarity). The slip ring is 200mm in diameter and the flux density in each rod is 1 Tesla.

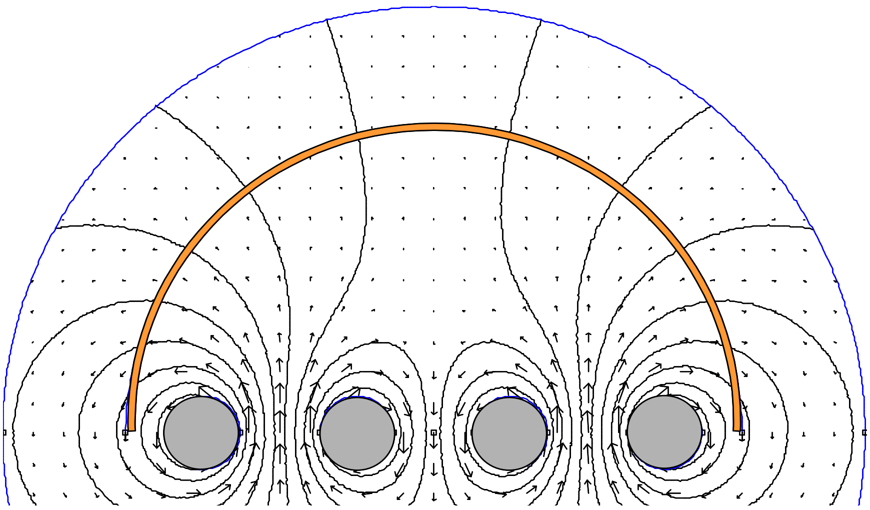

Figure 4. Slip-ring around four rods.

The next figure show the A_θ and the A_r values going CCW around the half slip ring.

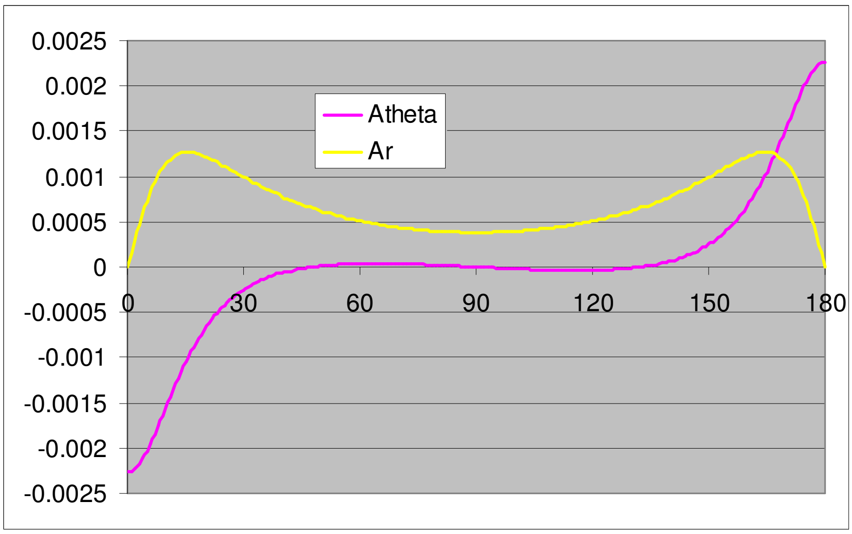

Figure 5. A_θ and A_r values

It is seen that both $\frac{\partial A_\theta}{\partial \theta}$ and A_r values are positive, unlike the uniform field case where they are of opposite polarity and cancel out. Using the device as a generator, with the 200mm diameter slip ring rotating at 1000RPM the induced voltage calculates at 70.7mV which although low could be usefully employed.

6. The Marinov Motor/Generator

For Marinov's split magnet Figure 6 shows the A field lines in the plane of the rotor. The red line depicts a half rotor along which radial A_r and tangential A_θ components are determined.

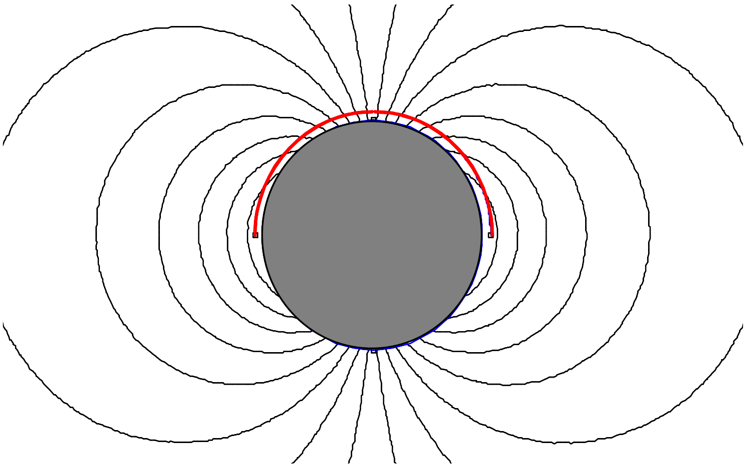

Figure 6 A field lines

Results are shown in Figure 7 assuming that the magnets have a flux density of 1 Tesla.

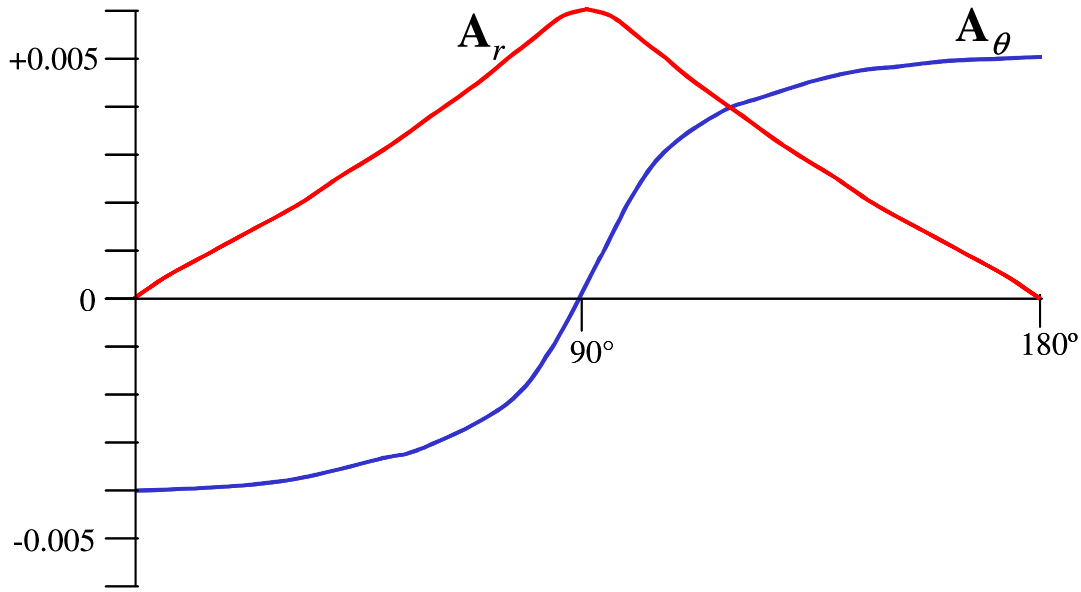

Figure 7. Radial and Tangential A-field components

Here it can be seen that both $\frac{\partial A_\theta}{\partial \theta}$ and A_r are positive thus they are additive in (16).

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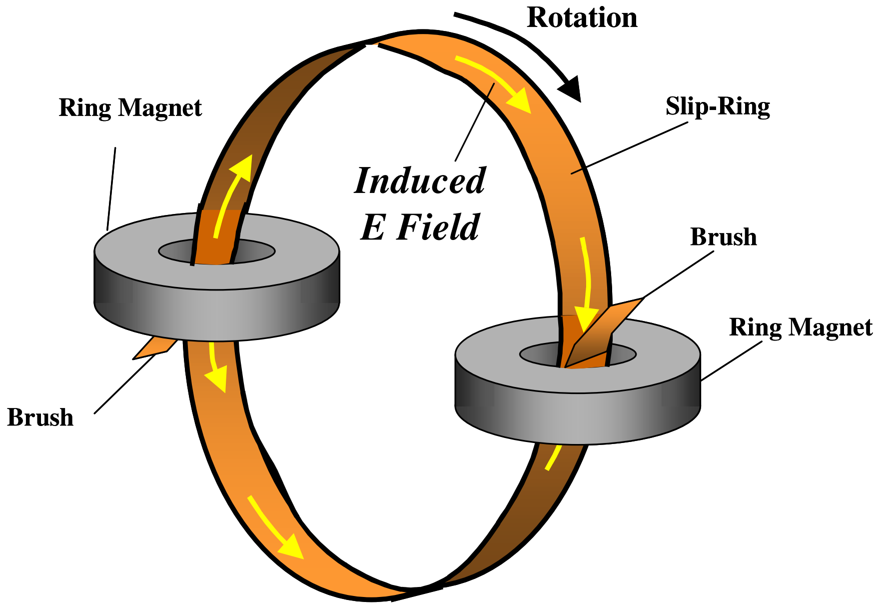
Figure 8 Using two ring cores



A typical A field plot for this configuration is shown in figure 9

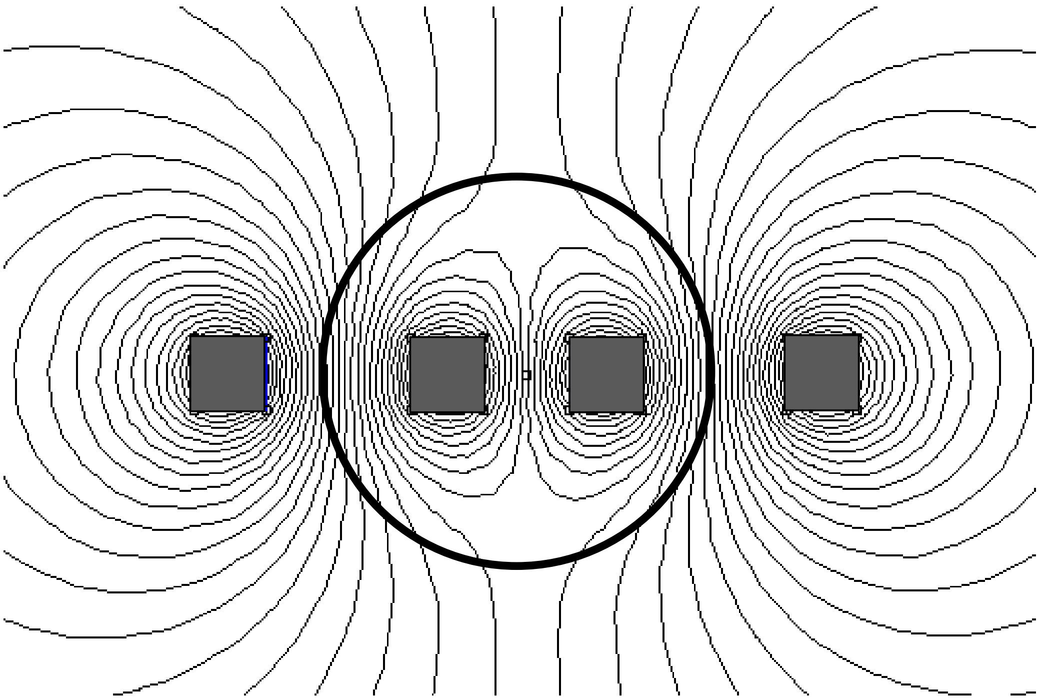


Figure 9. A-field for two ring magnets

Appendix.

The Cartesian components of $(\mathbf{v}\cdot\nabla)\mathbf{A}$ are:

$$(\mathbf{v}\cdot\nabla)\mathbf{A} = \begin{array}{l} \mathbf{a}_x\left[\overline{v_x\dfrac{\partial A_x}{\partial x}} + \boxed{v_y\dfrac{\partial A_x}{\partial y} + v_z\dfrac{\partial A_x}{\partial z}}\right] \\ \mathbf{a}_y\left[\boxed{v_x\dfrac{\partial A_y}{\partial x}} + \overline{v_y\dfrac{\partial A_y}{\partial y}} + \boxed{v_z\dfrac{\partial A_y}{\partial z}}\right] \\ \mathbf{a}_z\left[\boxed{v_x\dfrac{\partial A_z}{\partial x} + v_y\dfrac{\partial A_z}{\partial y}} + \overline{v_z\dfrac{\partial A_z}{\partial z}}\right] \end{array} \tag{A1}$$

where the overbars denote the longitudinal components parallel to the velocity and the coloured rectangles denote the transverse components.

The Cartesian components of $\mathbf{v}\times\mathbf{B}$ are:

$$\mathbf{v}\times\mathbf{B} = \begin{array}{l} \mathbf{a}_x\left[v_y\left(\dfrac{\partial A_y}{\partial x} - \boxed{\dfrac{\partial A_x}{\partial y}}\right) - v_z\left(\boxed{\dfrac{\partial A_x}{\partial z}} - \dfrac{\partial A_z}{\partial x}\right)\right] \\ \mathbf{a}_y\left[v_z\left(\dfrac{\partial A_z}{\partial y} - \boxed{\dfrac{\partial A_y}{\partial z}}\right) - v_x\left(\boxed{\dfrac{\partial A_y}{\partial x}} - \dfrac{\partial A_x}{\partial y}\right)\right] \\ \mathbf{a}_z\left[v_x\left(\dfrac{\partial A_x}{\partial z} - \boxed{\dfrac{\partial A_z}{\partial x}}\right) - v_y\left(\boxed{\dfrac{\partial A_z}{\partial y}} - \dfrac{\partial A_y}{\partial z}\right)\right] \end{array} \tag{A2}$$

where there are no longitudinal components. It is seen that only half the transverse components (as indicated by the coloured rectangles) get carried over from (A1)

The Cartesian components of $\nabla(\mathbf{v}\cdot\mathbf{A})$ are:

$$\nabla(\mathbf{v}\cdot\mathbf{A}) = \begin{array}{l} \mathbf{a}_x\left[v_x\dfrac{\partial A_x}{\partial x} + v_y\dfrac{\partial A_y}{\partial x} + v_z\dfrac{\partial A_z}{\partial x} + A_x\dfrac{\partial v_x}{\partial x} + A_y\dfrac{\partial v_y}{\partial x} + A_z\dfrac{\partial v_z}{\partial x}\right] \\ \mathbf{a}_y\left[v_x\dfrac{\partial A_x}{\partial y} + v_y\dfrac{\partial A_y}{\partial y} + v_z\dfrac{\partial A_z}{\partial y} + A_x\dfrac{\partial v_x}{\partial y} + A_y\dfrac{\partial v_y}{\partial y} + A_z\dfrac{\partial v_z}{\partial y}\right] \\ \mathbf{a}_z\left[v_x\dfrac{\partial A_x}{\partial z} + v_y\dfrac{\partial A_y}{\partial z} + v_z\dfrac{\partial A_z}{\partial z} + A_x\dfrac{\partial v_x}{\partial z} + A_y\dfrac{\partial v_y}{\partial z} + A_z\dfrac{\partial v_z}{\partial z}\right] \end{array} \tag{A3}$$

which include spatial derivatives of velocity. We are only interested in time variations of **A** since it is only these that induce an **E** field. If we denote (A3) with the derivatives of velocity suppressed as $\nabla_A(\mathbf{v}\cdot\mathbf{A})$ then its components are:

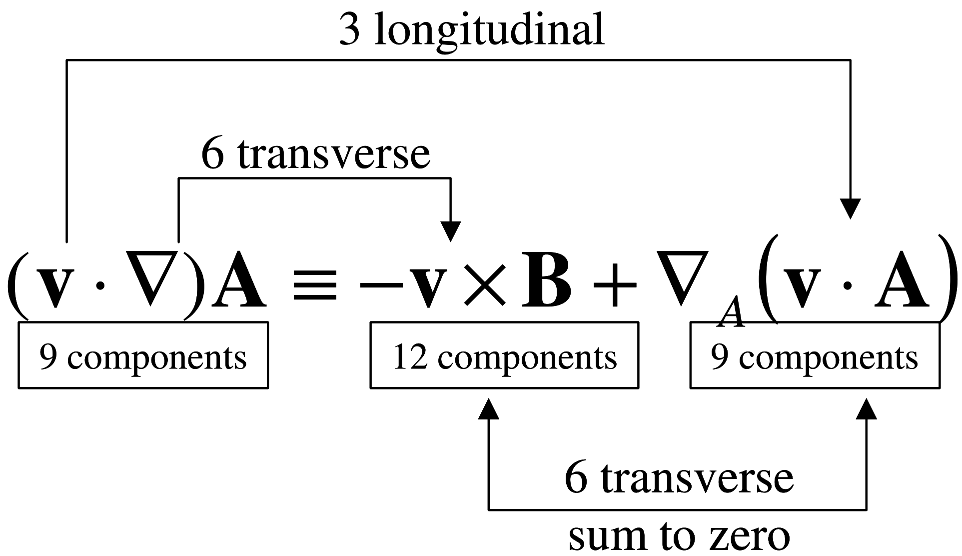
$$\nabla_A(\mathbf{v}\cdot\mathbf{A}) = \begin{array}{l} \mathbf{a}_x\left[\overline{v_x\dfrac{\partial A_x}{\partial x}} + v_y\dfrac{\partial A_y}{\partial x} + v_z\dfrac{\partial A_z}{\partial x}\right] \\ \mathbf{a}_y\left[v_x\dfrac{\partial A_x}{\partial y} + \overline{v_y\dfrac{\partial A_y}{\partial y}} + v_z\dfrac{\partial A_z}{\partial y}\right] \\ \mathbf{a}_z\left[v_x\dfrac{\partial A_x}{\partial z} + v_y\dfrac{\partial A_y}{\partial z} + \overline{v_z\dfrac{\partial A_z}{\partial z}}\right] \end{array} \tag{A4}$$

where the overbars represent the longitudinal components carried over from (A1). The remaining terms account for the transverse components in (A2) that were not carried over from (A1)

Examination of the terms in (A1), (A2) and (A4) shows that we can create the identity

$$(\mathbf{v}\cdot\nabla)\mathbf{A} \equiv -\mathbf{v}\times\mathbf{B} + \nabla_A(\mathbf{v}\cdot\mathbf{A}) \tag{A5}$$

However there is a subtlety in this equivalence that is not generally appreciated. The convective term on the LH side has 9 components, 3 longitudinal and 6 transverse, see (A1). The first term on the RH side of (A5) has 12 components all of which are transverse, see (A2), but only 6 of these are inherited from the LH side. The last term on the RH side has 9 components, 3 longitudinal that are inherited from the LH side and 6 transverse that account for the "missing" 6. The situation is summed up in the following figure.



Note that the 6 transverse components shared by the two RH side terms sum to zero, as they must do for the equivalence to hold. If the LH side is the true defining formula then any calculations using only the $\mathbf{v}\times\mathbf{B}$ term on the RH side must give the wrong result since it includes components that would be negated by the last term. This is easily demonstrated in the Faraday disc homopolar generator where the convective $(\mathbf{v}\cdot\nabla)\mathbf{A}$ formula (or its full equivalent discussed here) yields half the output voltage as calculated using just the $\mathbf{v}\times\mathbf{B}$ motional induction. That $\mathbf{v}\times\mathbf{B}$ motional induction has been in use for so long that it beggars belief that it consistently yields incorrect results, hence this calls into question the whole validity of (A5).