Charging a capacitor

I. Fundaun, C. Reese, and H. H. Soonpaa
Physics Department, University of North Dakota, Grand Forks, North Dakota 58202

(Received 8 October 1991; accepted 2 May 1992)

In this Journal, Heinrich described a lecture-room demonstration of stepwise charging of capacitors.[1] He showed that when a capacitor in an RC circuit is charged to a final voltage in steps, then the smaller the voltage steps are, the less energy is dissipated in the resistor. In the limit of an infinite number of infinitesimally small voltage steps there is no dissipation (no entropy change). Heinrich's emphasis was on entropy, and fittingly he measured the heat generated in a resistor in series with the capacitor. This experiment would be a welcome addition to sophomore physics laboratory experiments, but in the form presented by Heinrich it is not very suitable. The most undesirable features are the noise and drift problems when high amplifications of thermocouple voltages are involved, and the long time needed for resetting between runs. It takes minutes before the difference between the temperatures of the resistor and the temperature reference disk becomes small enough to permit another run. Also, the calculation of the energy dissipated in the resistor is cumbersome.

We have built a circuit, which measures the energy dissipated in the resistor by integrating the power in the resistor over a period of time, typically a few seconds. Voltage drop across the resistor is squared by a multiplier, and the output is integrated by a simple operational amplifier integrator. Thus the energy dissipated in the resistor is proportional to the difference between initial and final output voltages of the operational amplifier.

The circuit in Fig. 1(a) consists of the capacitor C_{I} to be charged through a resistor R_{I}, the Analog Devices AD633 multiplier (squarer), and the integrator. The multiplier squares the input voltage and divides it by 10; $V_{\mathrm{out}} = (V_{\mathrm{in}})^2/10$. The following equations apply to a one-step charging of the capacitor.

Initially point A is grounded to discharge capacitor C_{I}, and the voltage at E is set to zero by temporarily shorting capacitor C_{II}. At time $t=0$ point A is connected to voltage V_{in}.

At point B:

$$V_{\mathrm{B}} = V_{\mathrm{in}} \exp(-t/R_{\mathrm{I}}C_{\mathrm{I}}). \tag{1}$$

At point C:

$$V_{\mathrm{C}} = (V_{\mathrm{in}}^2/10) \exp(-2t/R_{\mathrm{I}}C_{\mathrm{I}}). \tag{2}$$

At point D: Potential is at virtual ground, the noninverting input being grounded. Current into D is given by

$$i = V_{\mathrm{C}}/R_{\mathrm{II}} = (V_{\mathrm{in}}^2/10R_{\mathrm{II}}) \exp(-2t/R_{\mathrm{I}}C_{\mathrm{I}}). \tag{3}$$

The current deposits a charge Q into capacitor C_{II}:

$$Q = \int i\,dt = -\left(\frac{1}{2}\right)\left(\frac{V_{\mathrm{in}}^2}{10}\right)\left(\frac{R_{\mathrm{I}}C_{\mathrm{I}}}{R_{\mathrm{II}}}\right)\left[1-\exp\left(\frac{-2t}{R_{\mathrm{I}}C_{\mathrm{I}}}\right)\right]. \tag{4}$$

At point E:

$$V_{\mathrm{out}} = \frac{Q}{C_{\mathrm{II}}} = -\left(\frac{1}{2}\right)\left(\frac{V_{\mathrm{in}}^2}{10}\right) R_{\mathrm{I}}C_{\mathrm{I}}/R_{\mathrm{II}}C_{\mathrm{II}}; \text{ when } t \gg R_{\mathrm{I}}C_{\mathrm{I}}. \tag{5}$$

Energy dissipated in resistor R_{I}:

$$U = \left(\frac{1}{2}\right) V_{\mathrm{in}}^2 C_{\mathrm{I}} = -10 V_{\mathrm{out}} R_{\mathrm{II}} C_{\mathrm{II}}/R_{\mathrm{I}}. \tag{6}$$

In a typical case of $V_{\mathrm{in}} = 5$ V, $R_{\mathrm{I}} = 1$ kΩ, $C_{\mathrm{I}} = 3300$ μF, $R_{\mathrm{II}} = 1$ MΩ, and $C_{\mathrm{II}} = 1$ μF; the output voltage $V_{\mathrm{out}} = -4.125$ V. In Fig. 1(b)–(d) sources of voltage applied at point A of Fig. 1(a) are shown. In Fig. 1(b) the potential divider can be continuous (potentiometer), or it can consist of a switch with discrete individual resistors (six 1-kΩ resistors in our case). Figure 1(c) shows a simple ramp generator, and Fig. 1(d) shows a constant current device with an LM334 constant current source. In Fig. 2 recorder traces of the energy dissipated in resistor R_{I} are shown. Just as in Heinrich's paper[1] one may notice that the energy dissipated when charging in equal voltage steps is

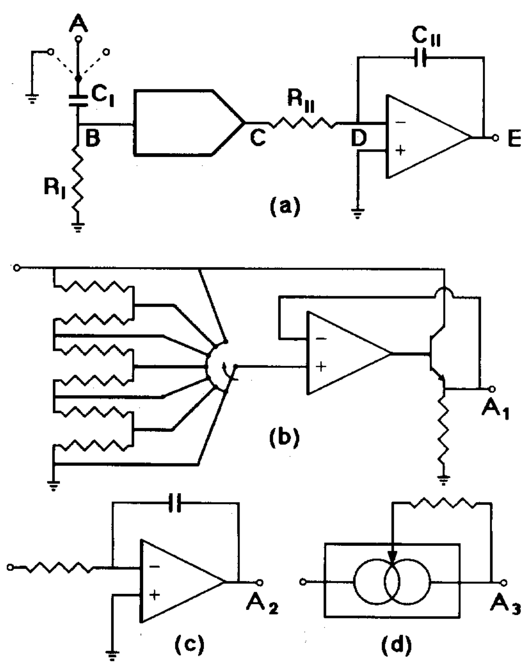

Fig. 1. (a) The multiplier–integrator circuit. (b) Variable voltage source. (c) Ramp generator for constant dV/dt. (d) LM334 current pump.

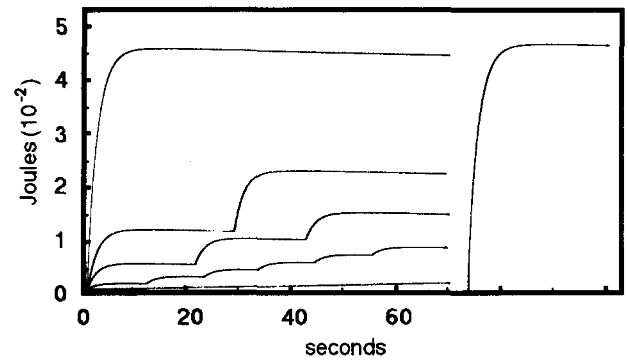

Fig. 2. Energy dissipated in $R_{\rm I}$ vs time.

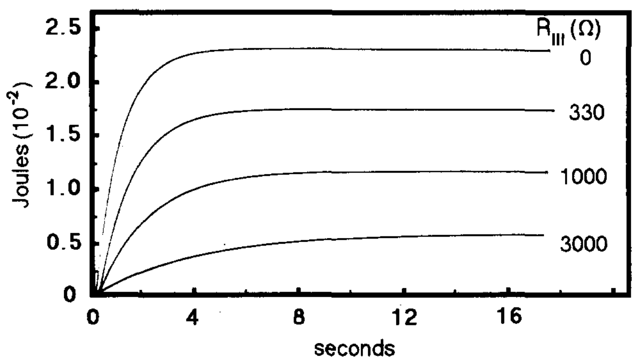

Fig. 3. Energy dissipated in $R_{\rm I}$ (1 kΩ) vs time when $C_{\rm I}$ is charged from capacitor of equal capacitance through different resistors $R_{\rm III}$.

inversely proportional to the number of steps. The graph on the right side of Fig. 2 represents the energy released when the capacitor is discharged to ground through switch SW1. The almost horizontal line at the bottom represents continuous charging by $i=0.1$ mA or $dV/dt=0.03$ V/s.

This energy meter allows many improvisations. An often asked question is: "What happens to the energy when a capacitor of capacitance C and potential difference V is connected to another capacitor of equal capacitance C and zero potential difference?" We know that charge will be conserved, but some energy will be lost. Initially $U_i=Q^2/2C$. After contact has been made, the total energy in the two capacitors will be $U_f=2[(Q/2)^2/2C]=U_i/2$. One half of the energy is lost. Energy loss does not depend upon the way the connection of the capacitors is performed. They may be connected by resistors of any value, including bare wires. Also, more than one resistor may be used. To demonstrate that the energy loss takes place in resistors we used the circuit of Fig. 1(a). Capacitor $C_{\rm III}$ $(=C_{\rm I})$ was charged to 5 V with one lead grounded. The 5 V lead was connected to SW1 through a resistor $R_{\rm III}$. When the switch was closed, current flowed from $C_{\rm III}$ through $R_{\rm III}$ through $C_{\rm I}$ (displacement current) through $R_{\rm I}$ to ground. In the final state there is no current and both capacitors have equal voltages and equal charges. In Fig. 3 the energy loss in $R_{\rm I}$ (1 kΩ) is shown. When $R_{\rm III}=0$ all the loss takes place in $R_{\rm I}$, and the value of 0.023 J is one-half of the 0.046 J shown in Fig. 2 for the fully charged capacitor. When $R_{\rm III}=R_{\rm I}=1$ kΩ, equal amounts of energy, 0.011 J, are dissipated in $R_{\rm III}$ and $R_{\rm I}$. One can also measure energy dissipated in $R_{\rm III}$ by connecting the multiplier across $R_{\rm III}$ (AD633 can be operated with both inputs off ground).

Another application of our energy meter is the measurement of capacitances. Equation (5) applies to a final (nontransient) situation. The output voltage $V_{\rm out}$ is directly proportional to the capacitance of an unknown capacitor $C_{\rm I}$.

Although a recorder is the most convenient device for displaying the results, one can also use a voltmeter or an oscilloscope. In case there are not enough recorders to provide one for each work station, students can get acquainted with the experiment through the use of voltmeters and oscilloscopes, and take turns recording the results on recorders. The key part of the circuit is the multiplier, which can be purchased for less than \$4 (AD633 from Analog Devices, Inc.). All other components of the circuit in Fig. 1 cost less than one dollar each. For the integrator operational amplifier we found the inexpensive JFET input LF351 to be adequate. Any laboratory power supply with ±12 V can be used. The quiescent current is only 10 mA, and the maximum current when the switch is first closed is equal to $V_{\rm in}/R_{\rm I}$, 5 mA in a typical case.

[1]F. Heinrich, "Entropy change when charging a capacitor: A demonstration experiment," Am. J. Phys. **54**, 742–744 (1986).

THE ENERGY BILL OF RIGHTS

Some proposals face technical obstacles; others collide with popular sentiments, like the apparently widely held belief that the right to speed is part of the Bill of Rights.

Matthew L. Wald, "U.S. Energy Options, U.S. Habits," New York Times, 24 September, 1990, p. 1.