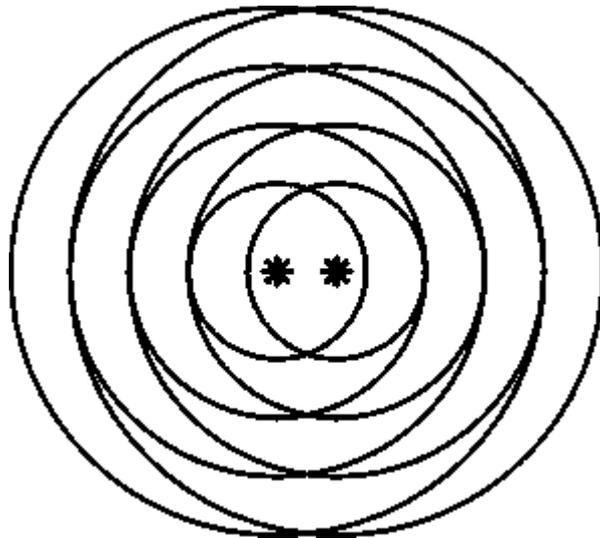


János Vajda

**VIOLATION OF THE
LAW OF ENERGY CONSERVATION
IN WAVE FIELDS**



Title of the original:
AZ ENERGIATÉTEL SÉRÜLÉSE HULLÁMTEREKBEN

As a sign of my gratitude and respect for the Faculty of Electric Engineering at the University of Technical Sciences, in memory of my parents, and to my native Village Nagyharsány.

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Preface

The possibility of producing excess energy is a much-disputed question nowadays. Does the law of energy conservation – as a postulate – have general validity, or can it be violated under certain circumstances? In order to make a correct conclusion about this matter, instead of subjective prejudices, it is preferable to objectively examine the subject within the context of the known laws of physics, thus maintaining consistency with the practice.

The objective of this study is to examine whether it is possible to violate the generally accepted law of energy conservation in the field of electromagnetic waves.

The discussion of energy correlations in the field of electromagnetic waves does not imply the restriction of this problem to a narrow area, since the laws of wave propagation cover a wide spectrum, from sound waves to X-rays, as well as the Compton effect and the de Broglie waves related to the particles. Thus they are applicable in a fairly wide area of physics. Consequently, the question of energy conservation, aside from its theoretical significance also has a practical importance, owing to the energy demand of present and future, as well as to the protection of the environment (which is not negligible). Therefore it may interest not only theoretical experts and physicists, but also professional teachers and students, and those dealing with practical problems.

The composition of this material serves to illuminate this problem from multiple points of view, together with the mathematical demonstrations required for the exact discussion. From the side of the reader it presumes the knowledge of higher mathematics, theoretical electronics, and antenna theory, but the essence will be clear even without these. The aim of the detailed mathematical demonstrations used in certain sections is also to promote a better perspective and understanding.

This summary material is the result of many years of work (under circumstances, which could not be called ideal). For that very reason I express my gratitude to all those who helped me in this work, especially to the head oculist Dr. Zoltán Vass at the hospital in Baja and his colleagues, who made the writing of this material possible by their selfless healing work, as well as to the publisher of this study.

Budapest 1998

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Introduction

The natural sciences – specifically physics – derive their laws by observing the phenomena and occurrences in practice through experimentation and measurements. In many cases we can derive new laws directly from calculations using the already known facts, omitting the expensive experiments and measurements. Of course, the laws derived in this way should be confirmed by empirical evidence.

It happens even in the area of natural sciences that in certain cases we have to rely on axioms and postulates. In certain cases we can make an attempt to re-examine a postulate, which might be specially justified in such cases, when the phenomena and occurrences contradict or seemingly contradict them. The law of energy conservation is such a postulate, and its examination is the very purpose of this study (within the field of electromagnetic waves). A well grounded justification is provided for performing such calculations (analysis) – in accordance with these objectives – by the following empirical phenomena.

As it is already known, based on the superposition of electromagnetic fields, the power (or energy) density which can be measured in a certain point of geometrical space is proportional to the square of the vectorial sum of the electromagnetic field intensity vectors present at that point. However, according to the law of energy conservation, this resultant power is supposed to be equal to the sum of each single wave's power – namely equal to the sum of the square of each single field intensity vector present at that point. We can put this into mathematical form in the following manner e.g. with the vectors of electric field intensities, but it is also valid in the same way for the vectors of magnetic field intensities. Let us signify the individual electric field intensity vectors with $\vec{E}_1, \vec{E}_2, \vec{E}_3, \dots, \vec{E}_n$, then we can write with the “k” proportionality coefficient that:

$$0 \leq k \left(\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n \right)^2 \geq k \left(\vec{E}_1^2 + \vec{E}_2^2 + \vec{E}_3^2 + \dots + \vec{E}_n^2 \right)$$

Experience law of energy conservation

From this we can conclude that in the case of electromagnetic fields, the law of energy conservation can be violated at certain points of geometrical space.

We can arrive to the same conclusion by observing the radiant characteristics of the antennas. Let there be given two identical antennas oriented in the same direction, both are fed from the same generator with identical frequencies, identical phase, and identical powers. If the two antennas radiate with identical polarization, then the individual field intensities in far-space, created by each of the antennas in the main direction of radiation will be summed up – namely the resultant field intensity will be double – and therefore the power density will be four times the field intensity and power density created by one single antenna at that point respectively. On the other hand, according to the law of energy conservation, the individual powers are supposed to add together. Thus the resultant power density supposed to be only the double the value of the power density created by each antenna at that point. Consequently, in this case in the main direction of radiation of the two antennas the law of energy conservation will be violated. However, if the two antennas radiate with identical polarization, but with a phase difference of 90°; or with identical phase, but with perpendicular polarization to each other, then the law of energy conservation will remain valid (not only in the primary direction, but in all directions).

In connection with the above discussion, the “usual” argument to support the inviolability of the law of energy conservation is that the local deviations (in \pm direction) appearing at the individual points (or regions) in the geometrical space – resulting from the rules of superposition and interference –

will mutually neutralize each other within the whole (closed) volume of space, and therefore the law of energy conservation will not be violated. This seems to be an attractive idea, but – as it will come to light later – it is not without merit to verify the general validity of this statement with calculations.

To support the justifiability of the examination of the law of energy conservation, we can also mention the following empirical fact. In the case of the “double T” or “magic T” well known in the microwave technology as an eight-pole, if we connect signals with identical frequency, phase, and amplitude to the input gates, then each of the input powers fed into the input gates will be halved towards the output gates. But towards one of the output gates the two field intensities will be in counter-phase and they will “extinguish” each other, therefore the output power will be zero. The other two halves of power from the input gates proceed towards the other output gate with identical field intensities and they will be added together. Therefore on this output gate we can measure a value of power identical with the total input power as a result of the two single halves of input power, namely the half of the total input power. On the basis of this measurement result we can justifiably ask: how can this phenomenon be explained without the violation of the law of energy conservation? Namely, that on one of the output gates the total input power appears as the effect of the half of the total input power. And where did the other half of the total input power disappear (since the output power of the other output gate happens to be zero)?

On the basis of the empirical facts described above we can justifiably begin performing the calculations (analysis) in accordance with our object. During the calculations we will assume that the reader has a general knowledge of the related theory of electromagnetism, vector analysis, and antenna theory (which can be found in the cited summary literature), therefore we will not go into the detailed description of the laws, correlations, terms, and methods applied below.

The notations used in the calculations are also as customary, and their meanings will appear in the text or in the illustrations. Generally it is sufficient to perform the examinations for the case of two waves, and then based on this we can also set up the correlations valid for the cases with more than two waves (radiation).

In the analysis we will use the expressions and approximations customary to this subject. So, for example in the case of wave propagation we assume a lossless and linear free space, and in the case of wave- (or radiation-) sources we assume that the total power of the sources will be radiated out without any loss. Occasionally we will use approximations that are accepted for the propagation of waves in far space. At certain points the derived results will be interpreted also for cases, where the expressions and approximations will be modified. For the calculations the wave radiating sources are assumed to be antennas, but this does not limit the validity of the derived results, since we will apply their radiant characteristics (parameters) with the correlations valid for them (and in accordance with practice).

In Summary: the task we have set as an aim of this paper is to determine the power- and energy correlations of the electromagnetic spherical waves in the field of radiation, radiated – simultaneously – into free space from several sources; together with the comparative evaluation of the radiated powers versus the radiant power of the resultant wave, in order to determine whether the law of energy conservation can be violated or not, and if yes then under what circumstances. The analysis of the problem has been performed from multiple points of view according to the points listed in the contents, including the Maxwell equations, and describing a few numerical (practical) calculation results as well.

1. The power and energy of the electromagnetic waves in vectorial form

We can determine the power- and energy correlations of waves spreading in free space based on figure 1. As is known, for the value of power $P_s = P_{out}$ radiated through the closed surface A by an antenna (source of radiation) placed within the volume V at point O , which is fed with power P_{in} , we can write the following (independently of the shape of surface A):

$$P_s = P_{out} = \oint_A \vec{S} d\vec{A} = \oint_A (\vec{E} \times \vec{H}) d\vec{A} = P_{in}$$

Namely at the inputs we assume that the antenna will radiate out the total input power.

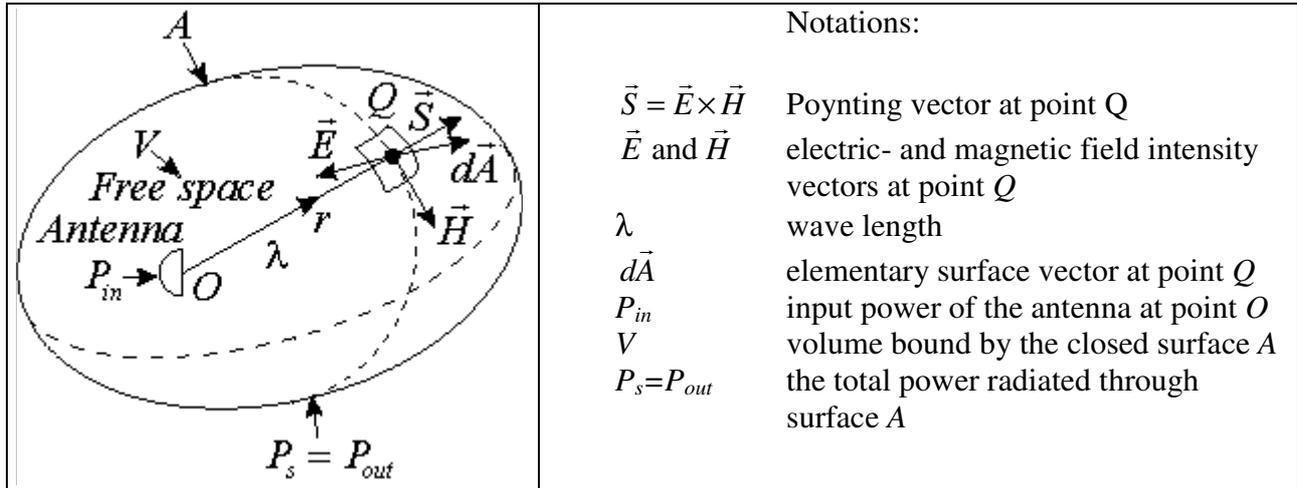


Fig. 1.

Let us place two antennas within volume V at different points O_1 and O_2 , and let us connect to their inputs – not simultaneously – powers P_{in1} and P_{in2} , then using indexes 1 and 2 in the above sense we can write that:

$$P_{s1} = P_{out1} = \oint_A \vec{S}_1 d\vec{A} = \oint_A (\vec{E}_1 \times \vec{H}_1) d\vec{A} = P_{in1} \quad (1.1)$$

$$P_{s2} = P_{out2} = \oint_A \vec{S}_2 d\vec{A} = \oint_A (\vec{E}_2 \times \vec{H}_2) d\vec{A} = P_{in2} \quad (1.2)$$

These equations (in accordance with our reservations) do satisfy the law of energy conservation, since the input powers will appear on the closed surface as radiant powers.

Let us connect now the input powers to the antennas simultaneously.

As is known, in the case of radiation originating from two sources the field intensities will add up in vectorial form, namely the interference of two waves will take place in the space. Thus, for the case of simultaneous radiation we can write (where index r refers to the resultant quantity):

$$\vec{S}_r = (\vec{E}_1 + \vec{E}_2) \times (\vec{H}_1 + \vec{H}_2)$$

From the distributive property of the vectorial multiplication follows that:

$$\vec{S}_r = (\vec{E}_1 \times \vec{H}_1) + (\vec{E}_2 \times \vec{H}_2) + (\vec{E}_1 \times \vec{H}_2) + (\vec{E}_2 \times \vec{H}_1)$$

With the earlier $\vec{S}_1 = (\vec{E}_1 \times \vec{H}_1)$; $\vec{S}_2 = (\vec{E}_2 \times \vec{H}_2)$ notations:

$$\vec{S}_r = \vec{S}_1 + \vec{S}_2 + (\vec{E}_1 \times \vec{H}_2) + (\vec{E}_2 \times \vec{H}_1) = \vec{S}_1 + \vec{S}_2 + \Delta\vec{S} \quad (1.3)$$

where:

$$\Delta\vec{S} = (\vec{E}_1 \times \vec{H}_2) + (\vec{E}_2 \times \vec{H}_1) \quad (1.4)$$

Now knowing \vec{S}_r , the resultant radiant power output $(P_s)_r = (P_{out})_r$ flowing out through the surface A is:

$$(P_s)_r = (P_{out})_r = \oint_A \vec{S}_r d\vec{A} = \oint_A \vec{S}_1 d\vec{A} + \oint_A \vec{S}_2 d\vec{A} + \oint_A \Delta\vec{S} d\vec{A} \quad (1.5)$$

We have seen in the preceding that $\oint_A \vec{S}_1 d\vec{A} = P_{in1}$ and $\oint_A \vec{S}_2 d\vec{A} = P_{in2}$

So we can write the following:

$$(P_s)_r = (P_{out})_r = P_{in1} + P_{in2} + \oint_A \Delta\vec{S} d\vec{A} = (P_{in})_r + \Delta P$$

where the total input power is: $(P_{in})_r = P_{in1} + P_{in2}$

and the ΔP power difference is:

$$\Delta P = \oint_A \Delta\vec{S} d\vec{A} = \oint_A \left[(\vec{E}_1 \times \vec{H}_2) + (\vec{E}_2 \times \vec{H}_1) \right] d\vec{A} \quad (1.6)$$

Applying the Gauss theorem for the value of ΔP we can write that:

$$\Delta P = \int_V \text{div} \Delta\vec{S} dV = \int_V \text{div} \left[(\vec{E}_1 \times \vec{H}_2) + (\vec{E}_2 \times \vec{H}_1) \right] dV \quad (1.7)$$

We can see that the source density of ΔP per unit volume is: $\text{div} \Delta\vec{S}$

The above correlations are valid for the case of simultaneous radiation from two sources. In the case of the simultaneous presence of electromagnetic waves originating from more than two sources (n-sources) we can write that:

$$(\vec{S}_r) = \left(\sum_{k=1}^n \vec{E}_k \right) \times \left(\sum_{k=1}^n \vec{H}_k \right) = \sum_{k=1}^n \vec{S}_k + (\Delta\vec{S})_n \quad \text{where } \vec{S}_k = (\vec{E}_k \times \vec{H}_k)$$

By expounding this equation – similarly as above – we can arrive at the following results:

$$(\Delta P)_n = \oint_A (\Delta\vec{S})_n d\vec{A} \quad \text{where: } (\Delta\vec{S})_n = \sum_{k=1}^n \sum_{m=1}^n (\vec{E}_k \times \vec{H}_m) \quad \text{here } k \neq m$$

$$(P_{sr})_n = (P_{out})_n = \sum_{k=1}^n (P_{in})_k + (\Delta P)_n \quad \text{where: } (P_{in})_k = \oint_A \vec{S}_k d\vec{A} \quad (1.8)$$

$$(\Delta P)_n = \oint_A (\Delta\vec{S})_n d\vec{A} = \int_V \text{div} (\Delta\vec{S})_n dV \quad (1.9)$$

Prior to the evaluation of the correlations derived so far, it is necessary to emphasize that the calculated quantities represent the momentary values in time, therefore the power correlations can be converted directly into energy correlations only after averaging them in time.

Evaluating the derived results we can make the following statements:

It is rather surprising that in the case of two simultaneous waves, in both expressions of \vec{S}_r and $P_{sr} = P_{outr}$ will appear also an additional $\Delta\vec{S}$ and ΔP difference quantity respectively. This fact can be made consistent with the law of energy conservation only in special cases. Namely, in order to protect the law of energy conservation it would be highly irresponsible – and at the same time a mathematical contradiction – to state with general validity (for all cases in time, geometrical configuration and for the electric parameters of the radiating sources at will) that these differential quantities can have only zero value in all cases.

It seems to be advisable to examine the values of $\Delta\vec{S}$ and ΔP for simultaneous waves in some special cases.

1/a. Two perpendicularly polarized waves

With symbolic notation $\vec{E}_1 \parallel \vec{H}_2$ and $\vec{E}_2 \parallel \vec{H}_1$ but as we know $\vec{E}_1 \perp \vec{H}_1$ and $\vec{E}_2 \perp \vec{H}_2$. This condition represents two waves with polarization perpendicular to each other. It can be understood that the circularly polarized wave also belongs to this category, since it is the resultant of two waves with perpendicular polarization to each other and 90° phase shift originating from the same source.

In this case, since the vectorial product of parallel vectors is zero, we get:

$$\Delta\vec{S} = (\vec{E}_1 \times \vec{H}_2) + (\vec{E}_2 \times \vec{H}_1) = 0 \quad \text{thus} \quad \Delta P = \oint_A \Delta\vec{S} d\vec{A} = 0 \quad (1.10)$$

Further, for effective values we can also write that:

$$\Delta\vec{S}_{eff} = \frac{1}{\tau} \int_0^\tau \Delta\vec{S} dt = 0 \quad \text{and so} \quad \Delta P_{eff} = \frac{1}{\tau} \int_0^\tau \Delta P dt = 0$$

where τ signifies the time period.

Thus we can state for this case that the law of energy conservation is valid, when the radiant (output) power or energy is identical with the input power or input energy respectively.

1/b. Waves with different frequencies

In this case the frequencies ($\omega = 2\pi f$) of the generators feeding the radiating sources are different from each other, that is $\omega_1 \neq \omega_2 \neq \omega_3 \neq \dots \neq \omega_n$.

Let us choose the time-function to be sine. In such a case the field intensity vectors of each wave belonging together at the examined point Q , can be written in the following form, where ϕ signifies the phase, \vec{E}_0 and \vec{H}_0 the amplitude vectors – depending only on space coordinates – including the polarization.

$$\vec{E} = \vec{E}_0 \sin(\omega t - \phi) ; \vec{H} = \vec{H}_0 \sin(\omega t - \phi) ; \phi = \omega \frac{r}{c}$$

where r signifies the distance of point Q from the source, and c – the speed of light.

For the case of two waves using these notations we can write that:

$$\vec{S}_1 = (\vec{E}_1 \times \vec{H}_1) = [\vec{E}_{01} \sin(\omega_1 t - \phi_1)] \times [\vec{H}_{01} \sin(\omega_1 t - \phi_1)]$$

and for the index #2 similarly deriving \vec{S}_2 , and by multiplying out the time factors:

$$\vec{S}_1 = (\vec{E}_{01} \times \vec{H}_{01}) \sin^2(\omega_1 t - \phi_1) ; \vec{S}_2 = (\vec{E}_{02} \times \vec{H}_{02}) \sin^2(\omega_2 t - \phi_2)$$

$$\Delta \vec{S} = (\vec{E}_{01} \times \vec{H}_{02}) \sin(\omega_1 t - \phi_1) \sin(\omega_2 t - \phi_2) + (\vec{E}_{02} \times \vec{H}_{01}) \sin(\omega_2 t - \phi_2) \sin(\omega_1 t - \phi_1)$$

$$\Delta \vec{S} = [(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01})] \sin(\omega_1 t - \phi_1) \sin(\omega_2 t - \phi_2)$$

As we have seen earlier, for the resultant of the momentarily radiated powers we get:

$$P_{sr} = P_{outr} = \oint_A \vec{S}_1 d\vec{A} + \oint_A \vec{S}_2 d\vec{A} + \oint_A \Delta \vec{S} d\vec{A} = P_{in1} + P_{in2} + \Delta P$$

The effective (time-average) power can be obtained by integration through τ time period.

$$(P_{sr})_{eff} = (P_{outr})_{eff} = \frac{1}{\tau} \int_{t=0}^{\tau} \oint_A \vec{S}_1 d\vec{A} dt + \frac{1}{\tau} \int_{t=0}^{\tau} \oint_A \vec{S}_2 d\vec{A} dt + \frac{1}{\tau} \int_{t=0}^{\tau} \oint_A \Delta \vec{S} d\vec{A} dt = (P_{in1})_{eff} + (P_{in2})_{eff} + \Delta P_{eff}$$

By putting the time functions into different forms for the calculation of the integrals we get:

$$\sin^2(\omega_1 t - \phi_1) = \frac{1}{2} - \frac{1}{2} \cos(2\omega_1 t - 2\phi_1)$$

$$\sin^2(\omega_2 t - \phi_2) = \frac{1}{2} - \frac{1}{2} \cos(2\omega_2 t - 2\phi_2)$$

$$\sin(\omega_1 t - \phi_1) \sin(\omega_2 t - \phi_2) = \frac{1}{2} \cos[(\omega_1 - \omega_2)t - (\phi_1 - \phi_2)] - \frac{1}{2} \cos[(\omega_1 + \omega_2)t - (\phi_1 + \phi_2)]$$

Further, as is known, the mean value of the following type of integral for one period is:

$$\frac{1}{\tau} \int_{t=0}^{\tau} \cos(\omega t - \phi) dt = \frac{1}{\tau} \int_{t=0}^{\tau} \cos(2\pi \frac{t}{\tau} - \phi) dt = \frac{1}{2\pi} \sin(2\pi \frac{t}{\tau} - \phi) \Big|_{t=0}^{\tau} = 0 \quad \text{where } \tau = \frac{1}{f}$$

Thus after the integration by time we get the effective powers (since the integrals calculated for time periods containing time functions with cosines will be zero):

$$(P_{in1})_{eff} = \frac{1}{2} \oint_A \vec{S}_1 d\vec{A} = \oint_A \frac{\vec{E}_{01} \times \vec{H}_{01}}{2} d\vec{A} = \oint_A (\vec{S}_1)_{eff} d\vec{A} ; (\vec{S}_1)_{eff} = \frac{\vec{E}_{01} \times \vec{H}_{01}}{2}$$

$$(P_{in2})_{eff} = \frac{1}{2} \oint_A \vec{S}_2 d\vec{A} = \oint_A \frac{\vec{E}_{02} \times \vec{H}_{02}}{2} d\vec{A} = \oint_A (\vec{S}_2)_{eff} d\vec{A} ; (\vec{S}_2)_{eff} = \frac{\vec{E}_{02} \times \vec{H}_{02}}{2}$$

$$\Delta P_{eff} = \frac{1}{\tau} \int_{t=0}^{\tau} \oint_A \Delta \vec{S} d\vec{A} dt = 0 \cdot \oint_A [(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01})] d\vec{A} = 0 \quad (1.11)$$

The $(\vec{S}_1)_{eff}$ and $(\vec{S}_2)_{eff}$ (the mean values in time) are the well known Poynting-vectors, for electromagnetic waves with sine or cosine time functions.

It can be understood even without extra proof that we get $\Delta P_{eff}=0$ also in the case of n-pieces of simultaneous waves (in spite of the fact, that the momentary value of $(\Delta\vec{S})_n$ is not necessarily zero). Namely, in each part (partial product) of the $(\Delta\vec{S})_n$ vector with time functions figures a product of sine functions with different frequencies, and as we have seen, the time integral of these calculated for one time period is zero.

To summarize, we can state that in the case of any number of simultaneous waves, but with different frequencies there is no effective power difference, thus the law of energy conservation is valid.

$$(P_{out})_{eff} = (P_{sr})_{eff} = \sum_{k=1}^n (P_{ink})_{eff} \quad (1.12)$$

1/c. Coherent waves with identical frequencies

In this case for the frequencies we can write that $\omega_1=\omega_2=\omega_3=\dots=\omega_n=\omega_0$. Based on the analysis in the previous 1/b passage, the correlations valid for the two waves in this case are:

$$\begin{aligned}\vec{S}_1 &= (\vec{E}_{01} \times \vec{H}_{01}) \sin^2(\omega_0 t - \phi_1) = (\vec{E}_{01} \times \vec{H}_{01}) \left[\frac{1}{2} - \frac{1}{2} \cos(2\omega_0 t - 2\phi_1) \right] \\ \vec{S}_2 &= (\vec{E}_{02} \times \vec{H}_{02}) \sin^2(\omega_0 t - \phi_2) = (\vec{E}_{02} \times \vec{H}_{02}) \left[\frac{1}{2} - \frac{1}{2} \cos(2\omega_0 t - 2\phi_2) \right] \\ \Delta\vec{S} &= \left[(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01}) \right] \sin(\omega_0 t - \phi_1) \sin(\omega_0 t - \phi_2) \quad \phi_1 \neq \phi_2 \text{ since } r_1 \neq r_2.\end{aligned}$$

After integrating by time we get:

$$\begin{aligned}(P_{in1})_{eff} &= \oint_A \frac{\vec{E}_{01} \times \vec{H}_{01}}{2} d\vec{A} = \oint_A (\vec{S}_1)_{eff} d\vec{A} ; (\vec{S}_1)_{eff} = \frac{\vec{E}_{01} \times \vec{H}_{01}}{2} \\ (P_{in2})_{eff} &= \oint_A \frac{\vec{E}_{02} \times \vec{H}_{02}}{2} d\vec{A} = \oint_A (\vec{S}_2)_{eff} d\vec{A} ; (\vec{S}_2)_{eff} = \frac{\vec{E}_{02} \times \vec{H}_{02}}{2}\end{aligned}$$

Since $\cos[-(\phi_1 - \phi_2)] = \cos(\phi_1 - \phi_2)$ for the time factor $\Delta\vec{S}$ we can write that:

$$\sin(\omega_0 t - \phi_1) \sin(\omega_0 t - \phi_2) = \frac{1}{2} \cos(\phi_1 - \phi_2) - \frac{1}{2} \cos[2\omega_0 t - (\phi_1 + \phi_2)]$$

Thus the value of the effective power difference is:

$$\Delta P_{eff} = \oint_A \frac{(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01})}{2} \cos(\phi_1 - \phi_2) d\vec{A} = \oint_A (\Delta\vec{S})_{eff} d\vec{A}$$

If the starting phases of the sources $\phi_{01}=\phi_{02}$ are identical, then for the value of $(\phi_1 - \phi_2)$ (where r_1 and r_2 are the distances of the sources from the point Q) we can write that:

$$(\phi_1 - \phi_2) = \omega_0 \frac{r_1}{c} - \omega_0 \frac{r_2}{c} = \frac{2\pi f_0}{c} (r_1 - r_2) = 2\pi \frac{r_1 - r_2}{\lambda_0} = 2\pi \frac{\Delta r_{1,2}}{\lambda_0}$$

$\lambda_0 = \frac{c}{f_0}$ and $\Delta r_{1,2} = (r_1 - r_2)$ is the distance difference of the two waves.

So in the case of $\phi_{01}=\phi_{02}$ for ΔP_{eff} we can write that:

$$\Delta P_{eff} = \oint_A \frac{(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01})}{2} \cos\left(2\pi \frac{r_1 - r_2}{\lambda_0}\right) d\vec{A}$$

Here the effective value of Poynting vector difference is (after the model of $(\vec{S}_1)_{eff}$ and $(\vec{S}_2)_{eff}$):

$$(\Delta\vec{S})_{eff} = \frac{(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01})}{2} \cos\left(2\pi \frac{r_1 - r_2}{\lambda_0}\right) \text{ if: } \phi_{01} = \phi_{02} \quad (1.13)$$

The cosine factor in ΔP_{eff} will remain under the sign of integral, since it is the function of r_1 and r_2 distances; thus we have to take it into consideration while integrating by surface.

If ϕ_{01} and ϕ_{02} starting phases are not identical at the wave sources, then for the general form of ΔP_{eff}

we get: $\left(\omega_0 \frac{r_1}{c} - \phi_{01}\right) - \left(\omega_0 \frac{r_2}{c} - \phi_{02}\right) = 2\pi \frac{r_1 - r_2}{\lambda_0} - (\phi_{01} - \phi_{02}) = 2\pi \frac{\Delta r_{1,2}}{\lambda_0} - \Delta\phi_{01,2}$

$$\Delta P_{eff} = \oint_A \frac{(\vec{E}_{01} \times \vec{H}_{02}) + (\vec{E}_{02} \times \vec{H}_{01})}{2} \cos\left[2\pi \frac{r_1 - r_2}{\lambda_0} - (\phi_{01} - \phi_{02})\right] d\vec{A} \quad (1.14)$$

Based on the analysis up to this point we can see, that for the case of simultaneous presence of n pieces of waves with identical ω_0 frequency we get:

$$(P_{sn})_{eff} = (P_{outn})_{eff} = \sum_{k=1}^n (P_{ink})_{eff} + (\Delta P_n)_{eff} ; (P_{ink})_{eff} = \oint_A \frac{\vec{E}_{0k} \times \vec{H}_{0k}}{2} d\vec{A} \quad (1.15)$$

$$(\Delta \vec{S}_n)_{eff} = \sum_{k=1}^n \sum_{m=1}^n \frac{\vec{E}_{0k} \times \vec{H}_{0m}}{2} \cos \left[2\pi \frac{r_k - r_m}{\lambda_0} - (\phi_{0k} - \phi_{0m}) \right] ; k \neq m \quad (1.16)$$

$$(\Delta P_n)_{eff} = \sum_{k=1}^n \sum_{m=1}^n \oint_A \frac{\vec{E}_{0k} \times \vec{H}_{0m}}{2} \cos \left[2\pi \frac{r_k - r_m}{\lambda_0} - (\phi_{0k} - \phi_{0m}) \right] d\vec{A} ; k \neq m \quad (1.17)$$

As a reminder it is reasonable to mention that the correlations derived for the effective values now signify a mean value in time, thus intelligibly they are valid also for the energy states with waves without amplitude modulation.

In the case of $\omega_1 = \omega_2 = \omega_3 = \dots = \omega_n = \omega_0$ simultaneous coherent waves we can state the followings:

$(\Delta \vec{S}_n)_{eff} \neq 0$ thus the law of energy conservation will be locally violated, since the zero value at the specific point will appear only in exceptional cases.

Although the $(\Delta P_n)_{eff}$ power difference given at 1.17 can have zero value regarding the closed surface A in spite of $(\Delta \vec{S}_n)_{eff} \neq 0$, but this is possible only in exceptional cases. Namely the integral under 1.17 is the function of several variable quantities (space coordinates, the radiation characteristics of the radiation sources and their polarization, their relative position in space and the starting phases at their input), thus it would be a mathematical contradiction to state generally that the value of this integral should be zero in all cases. From this follows that the law of energy conservation – for the space V bound by the closed surface A – in the case of coherent waves is not generally valid, since depending on the characteristics of the configuration, a mean power- and energy difference may also appear.

This power- or energy difference may be positive or negative – which can be seen from the derived correlations – thus an energy excess or deficit may appear, but in special cases it can also have zero value.

Let us determine the theoretical limits (extremities) for n pieces of coherent waves.

Let's assume that $\vec{E}_0 = \vec{E}_{01} = \vec{E}_{02} = \dots = \vec{E}_{0n}$ and $\vec{H}_0 = \vec{H}_{01} = \vec{H}_{02} = \dots = \vec{H}_{0n}$

In this case for an extremity we can write: $\vec{E}_r = n\vec{E}_0$; $\vec{H}_r = n\vec{H}_0$

$$(\vec{S}_r)_{eff} = \frac{(n\vec{E}_0) \times (n\vec{H}_0)}{2} = n^2 \frac{\vec{E}_0 \times \vec{H}_0}{2} ; (P_{in})_{eff} = n \oint_A \frac{\vec{E}_0 \times \vec{H}_0}{2} d\vec{A}$$

$$(P_{out})_{eff} = (P_{sr})_{eff} = \oint_A (\vec{S}_r)_{eff} d\vec{A} = n^2 \oint_A \frac{\vec{E}_0 \times \vec{H}_0}{2} d\vec{A} = n \cdot n \oint_A \frac{\vec{E}_0 \times \vec{H}_0}{2} d\vec{A} = n (P_{in})_{eff}$$

The other extreme for the minimum (n is an even number) will take place, if the field intensities have alternately + and – signs. It is understood then that:

$$\vec{E}_r = (\vec{E}_0 - \vec{E}_0 + \vec{E}_0 - . + \dots) = 0 \quad \text{and} \quad \vec{H}_r = (\vec{H}_0 - \vec{H}_0 + \vec{H}_0 - . + \dots) = 0$$

So for the case of n -pieces of coherent waves we can write for the effective value of resultant power, as well as for $\mathcal{E} = tP_{eff}$ energy that:

$$0 \leq (P_{out})_{eff} = (P_{sr})_{eff} \leq n (P_{in})_{eff} \quad (1.18)$$

$$0 \leq \mathcal{E}_{out} = \mathcal{E}_s \leq n \mathcal{E}_{in} \quad (1.19)$$

1/d. Summary conclusions

If within a region of space bound by closed surface A there simultaneously exist electromagnetic waves, originating from multiple sources placed within this volume, further the total input power is radiated out, and the law of energy conservation is valid for each source separately, then:

- If the frequencies of the input signals at the sources differ from each other, then in all cases (independently of the configuration and electrical parameters) the law of energy conservation is valid, namely the energy of the resultant radiation is identical with the input energy (that has been radiated).
- However in the case when the frequencies at the sources are identical (coherent feeding), then a power difference and an energy difference will also appear – the value and sign of these will depend on the space characteristics and electrical characteristics – thus the law of energy conservation is not valid. Namely, in such cases the law of energy conservation can be satisfied only in special cases (e.g. in the case of two waves with perpendicular polarization).

However, if the total input power is not radiated out and/or the law of energy conservation is not valid generally for each of the sources separately (e.g. in the case of aperture radiators, or as a consequence of the losses in antennas), that is if:

$$(P_{out})_k = (P_s)_k = \oint_A \vec{S}_k \cdot d\vec{A} \neq (P_{in})_k$$

then in the derived equations with $(P_{out})_k = (P_s)_k$ quantities we should understand and always substitute the values of related integrals. The correlations derived from the values of ΔP and ΔP_{eff} remain valid.

As the result of the analysis according to chapter 1 we can state that in the field of simultaneously present electromagnetic waves the law of energy conservation is not valid in the general sense, since it can be satisfied only in special cases.

2. The scalar form of the power and energy of two coherent waves in far space

After the calculations under chapter 1 having theoretical characteristics, let us now derive such correlations – for the simultaneous presence of two waves – which could provide numerical results in practical cases.

In the following the powers and power densities will signify mean values in time (with the application of correlations suited for the elimination of time factor, in the case of waves with sine or cosine time functions). The notation \vec{E} in this case signifies the value of the amplitude of electric field intensity, which also includes its phase. For the sake of simplicity in the first case let the polarization of the two waves be identical.

At the points O_1 and O_2 of free space let there be two antennas, which should be fed at P_{in1} and P_{in2} with identical frequencies, and let us examine their radiant characteristics in free space in r, θ, φ spherical coordinates (fig. 2.)

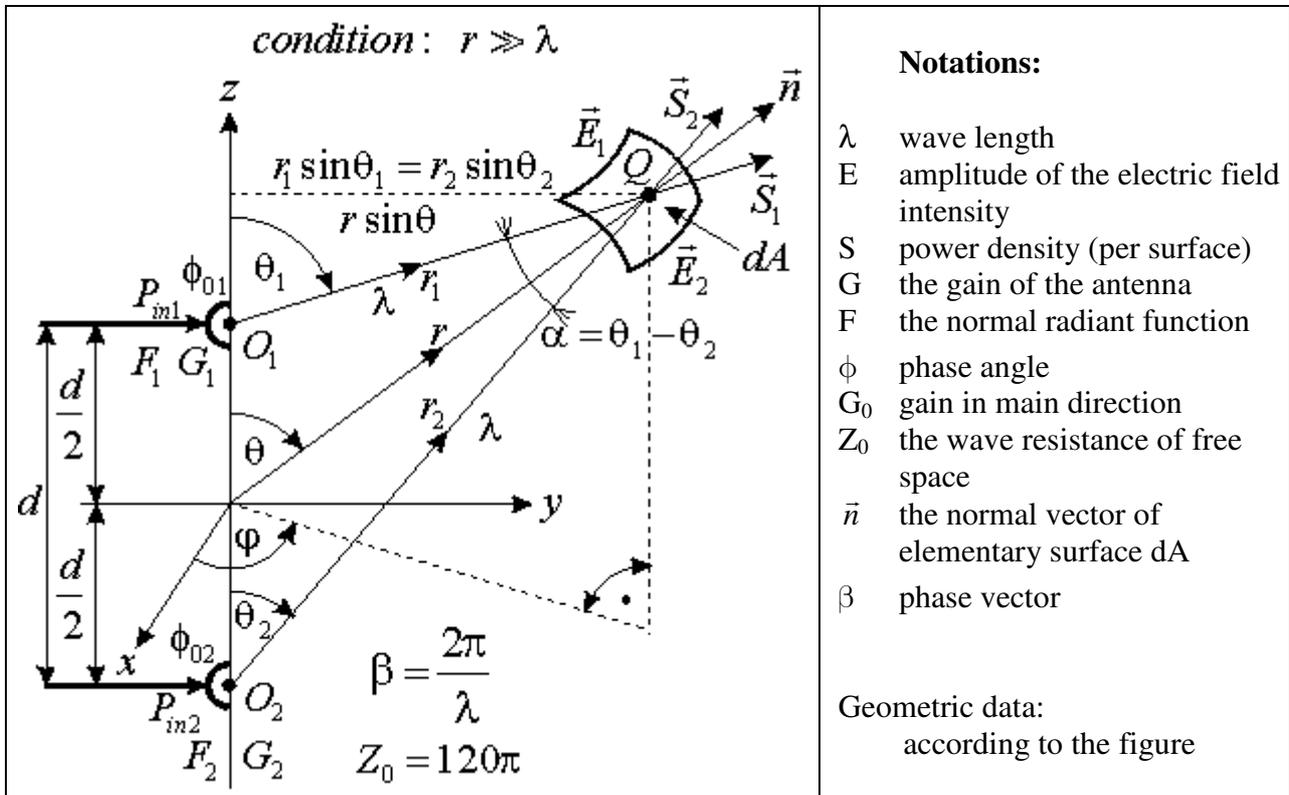


Fig. 2.

As is known from the calculations related to antennas, we can write the following correlations with general validity: $G = G_0 F^2(\theta; \varphi)$; $\vec{E} = E_0(r) F(\theta; \varphi) e^{-j\phi}$; $E = |\vec{E}|$; $S = \frac{E^2}{2Z_0} = \frac{P_{in}}{4\pi r^2} G$

since $\frac{E}{H} = Z_0 = 120\pi$

Let us first write up the powers $P_{out}=P_s$ radiated into the far-space by the antennas, and the values of field intensities at point Q , in the case with non-simultaneous radiation. In such cases it is advisable to calculate the surface integrals for spherical surfaces (wave fronts) with radiuses r_1 and r_2 .

$$P_{out1} = P_{s1} = \oint_{A1} S_1 dA_1 = \oint_{A1} \frac{P_{in1}}{4\pi r_1^2} G_1 dA_1 \quad \leftarrow dA_1 = r_1^2 \sin \theta_1 d\varphi d\theta_1$$

After substitution and rearrangement for both values it can be written that:

$$P_{out1} = P_{s1} = P_{in1} \frac{G_{01}}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta_1=0}^{\pi} F_1^2(\theta_1; \varphi) \sin \theta_1 d\varphi d\theta_1 = P_{in1} \quad (2.1)$$

$$P_{out2} = P_{s2} = P_{in2} \frac{G_{02}}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta_2=0}^{\pi} F_2^2(\theta_2; \varphi) \sin \theta_2 d\varphi d\theta_2 = P_{in2}$$

And for the electric field intensities we get:

$$S = \frac{E^2}{2Z_0} = \frac{P_{in}}{4\pi r^2} G_0 F^2 \quad \rightarrow E^2 = \frac{2Z_0}{4\pi} P_{in} \frac{G_0 F^2}{r^2} = 60 P_{in} G_0 \frac{F^2}{r^2}$$

$$E_1 = F_1 E_{01} = \frac{F_1}{r_1} \sqrt{60 P_{in1} G_{01}} \quad ; \quad E_2 = F_2 E_{02} = \frac{F_2}{r_2} \sqrt{60 P_{in2} G_{02}} \quad ; \quad \vec{E}_1 = E_1 e^{-j\phi_1} \quad ; \quad \vec{E}_2 = E_2 e^{-j\phi_2}$$

The values of phases

$$\phi_1 = (\beta r_1 + \phi_{01}) \quad ; \quad \phi_2 = (\beta r_2 + \phi_{02})$$

can be written in this form:

$$E_1 = \sqrt{60 P_{in1} G_{01}} \frac{F_1(\theta_1; \varphi)}{r_1} e^{-j(\beta r_1 + \phi_{01})} \quad \rightarrow E_1 = |\vec{E}_1| = \sqrt{60 P_{in1} G_{01}} \frac{F_1(\theta_1; \varphi)}{r_1} \quad (2.2)$$

$$E_2 = \sqrt{60 P_{in2} G_{02}} \frac{F_2(\theta_2; \varphi)}{r_2} e^{-j(\beta r_2 + \phi_{02})} \quad \rightarrow E_2 = |\vec{E}_2| = \sqrt{60 P_{in2} G_{02}} \frac{F_2(\theta_2; \varphi)}{r_2}$$

Let us now consider the case, when the antennas radiate simultaneously with identical polarization, λ wavelength, ϕ_{01} and ϕ_{02} starting phases.

If under "far space" we mean also the criterion $d \ll r$, then according to fig. 3. we can introduce further simplifications.

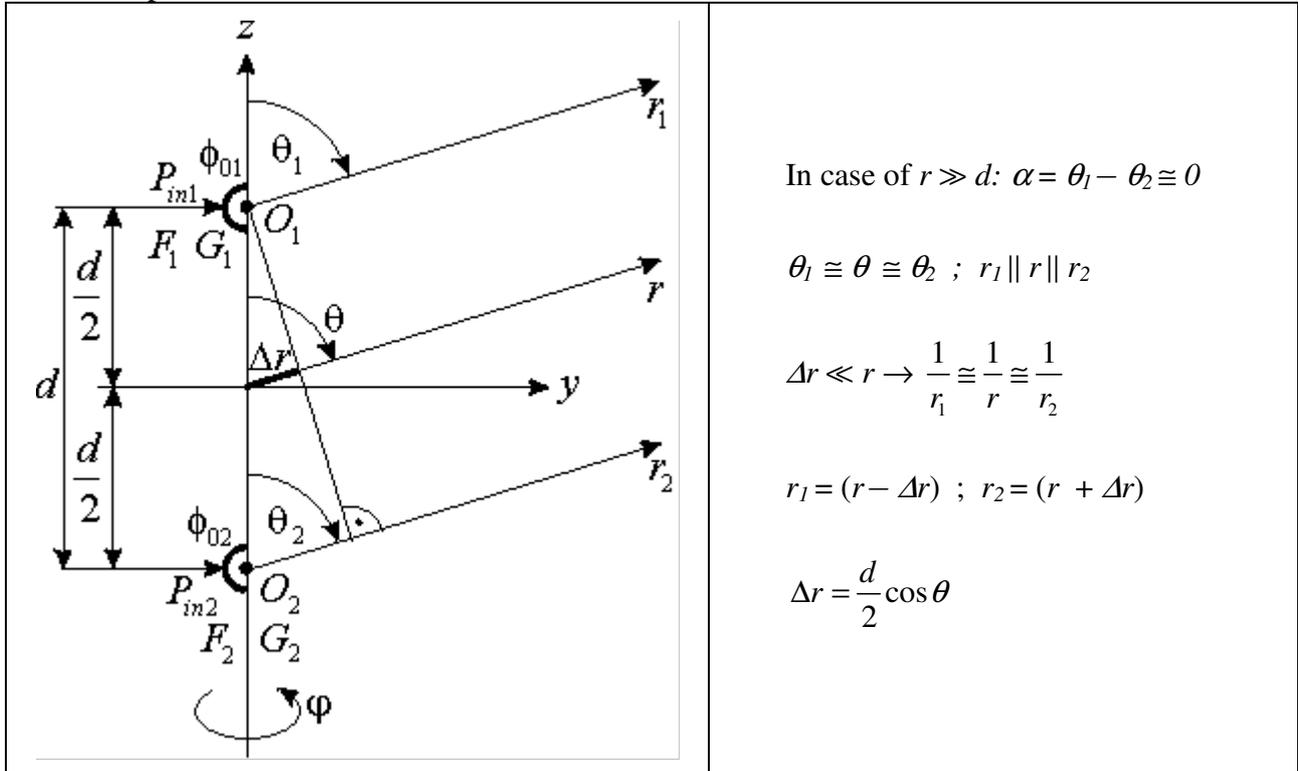


Fig. 3.

With these simplifications we can arrive to the following correlations:

$$P_{out1} = P_{s1} = P_{in1} \frac{G_{01}}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F_1^2(\theta; \varphi) \sin \theta d\varphi d\theta = P_{in1} \quad (2.3)$$

$$P_{out2} = P_{s2} = P_{in2} \frac{G_{02}}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F_2^2(\theta; \varphi) \sin \theta d\varphi d\theta = P_{in2}$$

The values of phases are:

$$\phi_1 = \beta(r - \Delta r) + \phi_{01} \quad ; \quad \phi_2 = \beta(r + \Delta r) + \phi_{02} \quad ; \quad \Delta r = \frac{d}{2} \cos \theta \quad (2.4)$$

And the electric field intensities are (since; $r_1 \cong r_2 \cong r$):

$$\vec{E}_1 = \sqrt{60P_{in1}G_{01}} \frac{F_1(\theta; \varphi)}{r} e^{-j\phi_1} \quad \rightarrow \quad E_1 = |\vec{E}_1| = \sqrt{60P_{in1}G_{01}} \frac{F_1(\theta; \varphi)}{r} \quad (2.5)$$

$$\vec{E}_2 = \sqrt{60P_{in2}G_{02}} \frac{F_2(\theta; \varphi)}{r} e^{-j\phi_2} \quad \rightarrow \quad E_2 = |\vec{E}_2| = \sqrt{60P_{in2}G_{02}} \frac{F_2(\theta; \varphi)}{r}$$

For the resultant electric field intensity at distant point Q we can write that:

$$\vec{E}_r = \vec{E}_1 + \vec{E}_2 = E_1 e^{-j\phi_1} + E_2 e^{-j\phi_2} \quad ; \quad E_r = |\vec{E}_r|$$

$$\vec{E}_r = (E_1 \cos \phi_1 + E_2 \cos \phi_2) - j(E_1 \sin \phi_1 + E_2 \sin \phi_2)$$

$$E_r^2 = |\vec{E}_r|^2 = (E_1 \cos \phi_1 + E_2 \cos \phi_2)^2 + (E_1 \sin \phi_1 + E_2 \sin \phi_2)^2$$

After squaring and rearrangement we get:

$$E_r^2 = E_1^2 + E_2^2 + 2E_1E_2(\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2) = E_1^2 + E_2^2 + 2E_1E_2 \cos(\phi_1 - \phi_2)$$

Using the expressions:

$$\Delta \phi = \phi_1 - \phi_2 = \beta r - \beta \Delta r + \phi_{01} - \beta r - \beta \Delta r - \phi_{02} = (\phi_{01} - \phi_{02}) - 2\beta \Delta r \quad (2.6)$$

We get:

$$E_r^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \Delta \phi$$

and with this, the resultant power density S_r is:

$$S_r = \frac{E_r^2}{2Z_0} = \frac{E_1^2}{2Z_0} + \frac{E_2^2}{2Z_0} + \frac{E_1E_2}{Z_0} \cos \Delta \phi = S_1 + S_2 + \frac{E_1E_2}{Z_0} \cos \Delta \phi = S_1 + S_2 + \Delta S$$

For the resultant power output of the radiation now we can write that:

$$P_{outr} = P_{sr} = \oint_A S_r dA = \oint_A S_1 dA + \oint_A S_2 dA + \oint_A \frac{E_1E_2}{Z_0} \cos \Delta \phi dA$$

As it is evident from correlations 1.1 and 1.2 (when the law of energy conservation is valid for the single sources of radiation, and the total input power is radiated out):

$$P_{in1} = \oint_A S_1 dA \quad ; \quad P_{in2} = \oint_A S_2 dA$$

further

$$\Delta P = \oint_A \Delta S dA \quad \text{where} \quad \Delta S = \frac{E_1E_2}{Z_0} \cos \Delta \phi$$

$$P_{outr} = P_{sr} = P_{in1} + P_{in2} + \frac{1}{Z_0} \oint_A E_1E_2 \cos \Delta \phi dA = P_{in1} + P_{in2} + \Delta P \quad (2.7)$$

Thus the mean power difference is:

$$\Delta P = \frac{1}{Z_0} \oint_A E_1E_2 \cos \Delta \phi dA \quad \leftarrow \quad dA = r^2 \sin \theta d\varphi d\theta$$

Substituting the E_1 and E_2 values given by 2.5, further $Z_0=120\pi$ and $\Delta r = \frac{d}{2} \cos \theta$ values we get:

$$\Delta P = \sqrt{P_{in1} P_{in2}} \frac{\sqrt{G_{01} G_{02}}}{2\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F_1(\theta; \varphi) F_2(\theta; \varphi) \cos \Delta \phi \sin \theta d\varphi d\theta \quad (2.8)$$

$$\text{where: } \Delta \phi = (\phi_{01} - \phi_{02}) - 2\pi \frac{d}{\lambda} \cos \theta$$

According to our aim, with equation 2.8 we have arrived at a correlation, which contains only scalar quantities, thus by its computerized evaluation we can get numerical values for the mean power-difference (or by multiplying with time, the energy-difference) produced in far space, as the resultant of the electromagnetic waves with identical wavelengths and polarization radiated from two antennas.

It is worthwhile to give the value of ΔP for the case, when the two antennas are identical and oriented into the same direction, that is when:

$$F_1(\theta; \varphi) = F_2(\theta; \varphi) = F(\theta; \varphi) \quad \text{and} \quad G_{01} = G_{02} = G_0$$

$$\Delta P = \sqrt{P_{in1} P_{in2}} \frac{G_0}{2\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F^2(\theta; \varphi) \cos \Delta \phi \sin \theta d\varphi d\theta \quad (2.9)$$

$$\Delta \phi = (\phi_{01} - \phi_{02}) - 2\pi \frac{d}{\lambda} \cos \theta$$

With the help of the correlation given in 2.9 we can calculate the limits (extremities) of the value of ΔP for the case of two waves.

When $P_{out1} = P_{out2} = \frac{P_{in}}{2}$ and $d \rightarrow 0$:

$$\Delta P_{ext} = P_{in} \frac{G_0}{4\pi} \cos(\phi_{01} - \phi_{02}) \int_{\varphi=0}^{2\pi} F^2(\theta; \varphi) \sin \theta d\varphi d\theta$$

But as follows from 2.3:

$$\frac{G_0}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F^2(\theta; \varphi) \sin \theta d\varphi d\theta = 1 \quad \text{thus} \quad \Delta P_{ext} = P_{in} \cos(\phi_{01} - \phi_{02})$$

If the two antennas are fed with identical phases $\phi_{01} = \phi_{02}$, then $(\Delta P_{ext})_{max} = P_{in}$; but with counter-phase $(\Delta P_{ext})_{min} = -P_{in}$.

Thus in the case of two antennas we can write in perfect consonance with the correlations 1.18 and 1.19 (here $n=2$) that:

$$P_{in} - P_{in} = 0 \leq (P_{out} = P_{sr} = P_{in} + \Delta P) \leq P_{in} + P_{in} = 2P_{in} \quad (2.10)$$

Finally let us determine the power- and energy correlations with general validity for the case of two waves, if the polarization of the two waves are not identical, that is when the planes of polarization enclose angle γ (fig. 4.)

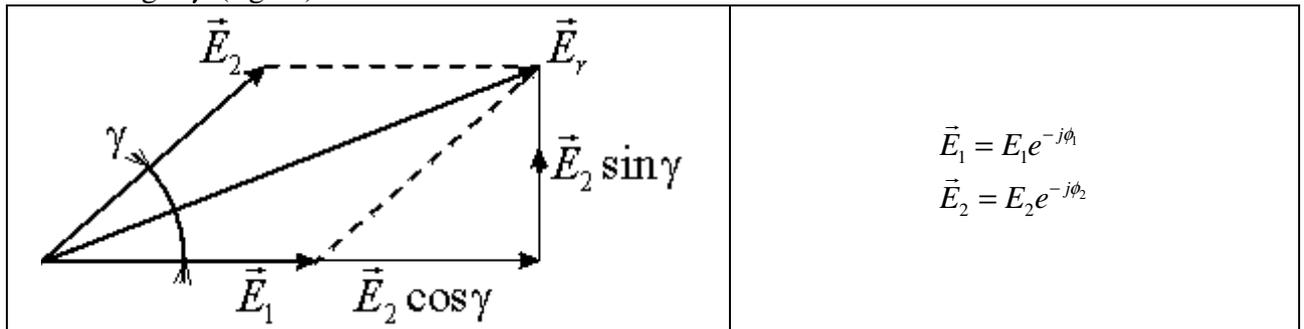


Fig. 4.

In such case, for the resultant field intensity it can be written that:

$$E_r^2 = |\vec{E}_r|^2 = |\vec{E}_1 + \vec{E}_2 \cos \gamma|^2 + |E_2 \sin \gamma|^2$$

$$E_r^2 = |E_1 e^{-j\phi_1} + E_2 \cos \gamma e^{-j\phi_2}|^2 + |E_2 \sin \gamma e^{-j\phi_2}|^2$$

$$E_r^2 = [E_1 \cos \phi_1 + E_2 \cos \gamma \cos \phi_2]^2 + [E_1 \sin \phi_1 + E_2 \cos \gamma \sin \phi_2]^2 + E_2^2 \sin^2 \gamma$$

After squaring, rearranging, and applying the related trigonometric correlations, we arrive at the following result:

$$E_r^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos \gamma \cos(\phi_1 - \phi_2) = E_1^2 + E_2^2 + 2E_1 E_2 \cos \gamma \cos \Delta\phi$$

In analogous manner with the demonstration of 2.6 and 2.7 correlations, the following end result is derived:

$$P_{out} = P_{sr} = P_{in1} + P_{in2} + \frac{\cos \gamma}{Z_0} \oint_A E_1 E_2 \cos \Delta\phi dA = P_{in1} + P_{in2} + \Delta P$$

$$\Delta P = \sqrt{P_{in1} P_{in2}} \frac{\sqrt{G_{01} G_{02}}}{2\pi} \cos \gamma \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F_1(\theta; \phi) F_2(\theta; \phi) \cos \Delta\phi \sin \theta d\phi d\theta \quad (2.11)$$

where: $\Delta\phi = (\phi_{01} - \phi_{02}) - 2\pi \frac{d}{\lambda} \cos \theta$

It can be seen that with identical polarization $\gamma = 0^\circ$ the $|\Delta P|$ is maximal, and with perpendicular polarization $\gamma = 90^\circ \rightarrow \Delta P = 0$.

The correlations derived above are valid for the case, when the law of energy conservation is valid for the two single sources separately and the total input powers are radiated out, that is when $P_{out1} = P_{s1} = P_{in1}$ and $P_{out2} = P_{s2} = P_{in2}$.

If the law of energy conservation is not valid for each single source separately in general sense and/or the input powers are not radiated out (e.g. with aperture radiating sources, or because of the losses in the antennas), that is when $P_{out1} = P_{s1} \neq P_{in1}$ and $P_{out2} = P_{s2} \neq P_{in2}$ then instead of the expressions 2.11 we should use and understand the following correlations, which have general validity:

$$P_{out} = P_{sr} = P_{out1} + P_{out2} + \Delta P$$

$$P_{out1} = P_{in1} \frac{G_{01}}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F_1^2(\theta; \phi) \sin \theta d\phi d\theta \quad ; \quad P_{out2} = P_{in2} \frac{G_{02}}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F_2^2(\theta; \phi) \sin \theta d\phi d\theta$$

$$\Delta P = \sqrt{P_{in1} P_{in2}} \frac{\sqrt{G_{01} G_{02}}}{2\pi} \cos \gamma \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F_1(\theta; \phi) F_2(\theta; \phi) \cos \Delta\phi \sin \theta d\phi d\theta \quad (2.12)$$

$\Delta\phi = (\phi_{01} - \phi_{02}) - 2\pi \frac{d}{\lambda} \cos \theta$

3. The general calculation of the power and energy of coherent waves with scalar quantities

In the case of more than two simultaneous radiations according to the preceding, knowing the parameters of the single radiating sources, we should determine the resultant field intensity E_r , and having this, we calculate the following generally valid integral:

$$P_{out} = P_{sr} = \oint_A \frac{E_r^2}{2Z_0} dA = \frac{1}{2Z_0} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} E_r^2 r^2 \sin \theta d\varphi d\theta \quad (3.1)$$

For the calculations performed with scalar quantities it is justified to mention that for the solution of problems according to our purpose, in the case of $d \ll r$ the following formula (derived from 3.1) also has general validity, whether the question is about the resultant of radiation originating from only one single source, or from several sources.

$$P_{out} = P_{sr} = P_{in} \frac{G_{0r}}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F_r^2(\theta; \varphi) \sin \theta d\varphi d\theta = P_{in} \frac{1}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} G_r(\theta; \varphi) \sin \theta d\varphi d\theta$$

$$\xi = \frac{P_{out}}{P_{in}} = \frac{P_{sr}}{P_{in}} = \frac{G_{0r}}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F_r^2(\theta; \varphi) \sin \theta d\varphi d\theta \quad (3.2)$$

From this we can already see, that when $\xi=1$, then the law of energy conservation is valid, but if $\xi \neq 1$ then it is not valid.

The correlation given with 3.2 is valid not only for the radiation in far space, but also for the quasi far space, if G_{0r} and $F_r(\theta; \varphi)$ are related to that.

Sometimes it happens in the practice that we want to determine the value of ξ only for a partial surface; in that case with the correlation 3.2 the integration boundaries should be selected according to the desired surface.

Summarizing: the correlations derived in the previous- (2.) and this chapter describe the power- and energy conditions – with scalar quantities in a form suitable for numerical evaluation – in cases with coherent systems (with identical λ) when $d \ll r$, which is in perfect harmony with the results obtained in the first chapter and with the conclusions derived from them. Therefore their repetition is unnecessary.

4. The Maxwell equations and the violation of the law of energy conservation in the case of electromagnetic waves

Regarding the violation of the law of energy conservation, the results and conclusions obtained in previous chapters by all means justify the examination whether they are in consonance with Maxwell's equations. If we get the same results from the Maxwell equations, then we can be sure that the law of energy conservation has no general validity, thus it can not be accepted as a postulate.

For the examinations let us write up the Maxwell equations in one of its usual forms as a reminder.

$$\begin{aligned}
 \text{I.} \quad & \text{rot } \vec{H} = \vec{i} + \frac{\partial \vec{D}}{\partial t} \\
 \text{II.} \quad & \text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\
 \text{III.} \quad & \text{div } \vec{B} = 0 \\
 \text{IV.} \quad & \text{div } \vec{D} = \rho \\
 \text{V.} \quad & \vec{D} = \varepsilon \vec{E} \ ; \ \vec{B} = \mu \vec{H} \ ; \ \vec{i} = \sigma(\vec{E} + \vec{E}_e) \\
 \text{VI.} \quad & w = \frac{1}{2} \varepsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2
 \end{aligned}
 \tag{4.1}$$

Usually we include the Lorentz law about the force acting upon a charge that moves in electromagnetic field and the equations expressing the conservation of charge; but in the present case they will be unnecessary. The notations used in the equations: \vec{E} electric-, \vec{H} magnetic-, \vec{E}_e extraneous field intensity vectors, \vec{D} displacement-, \vec{B} magnetic induction vectors, ρ the electric charge density, σ specific conductivity, \vec{i} current density vector, ε dielectric constant, μ magnetic permeability, w energy density. The ε and μ are the material constants of the space. These quantities are the functions of the coordinate of place and time (four-dimensional space), thus they signify the momentary values present at the different points in the geometrical space. (The σ , ε , μ material constants are generally constant quantities). The equations according to 4.1 represent such a system, by which the characteristics of an electromagnetic field can be calculated at any later moment in time, when given the starting conditions. We will deal with the many facets of these equations with the analysis of their content, and with the variations of its solutions satisfying the given conditions only in connection with the present examination. Thus we will work mainly with the equation VI. describing the energy density of the space, naturally taking into account also the rest of the equations.

As first, in the next section we will examine the determination and interpretation of the radiant power of the electromagnetic wave.

4/a. The Poynting vector and the radiant power

Let the electromagnetic field be known according to the vector functions \vec{E} and \vec{H} , which are originating from the source of radiation (e.g. antenna) as the result of quantities \vec{i} and \vec{E}_e . Let's determine how much radiant power P_s is departing from the volume V bound by the surface A , which includes the source.

As is known, with the Poynting vector $\vec{S} = (\vec{E} \times \vec{H})$ and the Gauss theorem we can write:

$$P_s = \oint_A (\vec{E} \times \vec{H}) d\vec{A} = \int_V \text{div}(\vec{E} \times \vec{H}) dV$$

Let's write up the value of $\text{div}(\vec{E} \times \vec{H})$ taking into consideration the Maxwell equations:

$$\text{div}(\vec{E} \times \vec{H}) = -\vec{E} \text{rot} \vec{H} + \vec{H} \text{rot} \vec{E}$$

Substituting the I., II. and V. Maxwell equations we get:

$$\text{div}(\vec{E} \times \vec{H}) = -\vec{E} \left(\vec{i} + \frac{\partial \vec{D}}{\partial t} \right) - \vec{H} \frac{\partial \vec{B}}{\partial t} = - \left(\epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \mu \vec{H} \frac{\partial \vec{H}}{\partial t} \right) - \vec{E} \vec{i}$$

Since $\vec{E} = \frac{\vec{i}}{\sigma} - \vec{E}_e$ further $\frac{\partial}{\partial t} \frac{1}{2} \epsilon \vec{E}^2 = \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t}$ and $\frac{\partial}{\partial t} \frac{1}{2} \mu \vec{H}^2 = \mu \vec{H} \frac{\partial \vec{H}}{\partial t}$

We get:

$$\text{div}(\vec{E} \times \vec{H}) = -\frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 \right) - \frac{\vec{i}^2}{\sigma} + \vec{E}_e \vec{i} = -\frac{\partial w}{\partial t} - \frac{\vec{i}^2}{\sigma} + \vec{E}_e \vec{i} \quad (4.2)$$

Where w is the energy density per unit volume according to equation VI.

Let's recognize that the equation 4.2 signifies an identity, thus we can calculate the wanted radiant power P_s in two ways by knowing the vectors \vec{E} and \vec{H} :

$$P_s = \oint_A (\vec{E} \times \vec{H}) d\vec{A} = \int_V \text{div}(\vec{E} \times \vec{H}) dV$$

or in possession of the quantities of $\vec{E}, \vec{H}, \vec{i}, \vec{E}_e$:

$$P_s = -\frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 \right) dV - \int_V \frac{\vec{i}^2}{\sigma} dV + \int_V \vec{E}_e \vec{i} dV = -\int_V \frac{\partial w}{\partial t} dV - \int_V \frac{\vec{i}^2}{\sigma} dV + \int_V \vec{E}_e \vec{i} dV$$

This can be demonstrated with the following equation:

$$\oint_A (\vec{E} \times \vec{H}) d\vec{A} = P_s = -\int_V \frac{\partial w}{\partial t} dV - \int_V \frac{\vec{i}^2}{\sigma} dV + \int_V \vec{E}_e \vec{i} dV \quad (4.3)$$

Thus we can choose freely whether to calculate the radiant power from 4.3 with the expressions standing on its left side or right side.

We can write the derived result in differential form as well, if p_s represents the radiant power density per unit volume, then:

$$\int_V p_s dV = P_s = \oint_A (\vec{E} \times \vec{H}) d\vec{A} = \int_V \text{div}(\vec{E} \times \vec{H}) dV$$

$$\text{div}(\vec{E} \times \vec{H}) = p_s = -\frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 \right) - \frac{\vec{i}^2}{\sigma} + \vec{E}_e \vec{i} \quad (4.4)$$

As we know, in the equation VI. given by 4.1 the first part represents the power density per unit volume w_E of the electric force field, and the second part w_H that of the magnetic force field; thus the equations 4.4 can be interpreted in the following way in the case of electromagnetic waves.

According to the right side of the equation the radiant power density per unit volume p_s is originating from the $\vec{E}_e \cdot \vec{i}$ input power density, and from the rate of decreasing the w_E and w_H (electric and magnetic) power densities in time (as they are transformed into radiation); after subtracting the \vec{i}^2/σ power density corresponding with the Joule-heat.

According to the left side of the equation we can also calculate the same p_s radiant power density also by forming the divergence of the Poynting vector $\vec{S} = (\vec{E} \times \vec{H})$.

The examinations as per our aims relate to the electromagnetic waves, but still it is of use to touch upon the following points. As we know, the Maxwell equations have general validity, thus they apply not only to the fast changing, but also to the slow changing and static electromagnetic fields. So we can ask the question, what is the meaning of the equations according to 4.4 in the case of electrostatic fields, when \vec{E} and \vec{H} are constant values in time? In such cases the right side of the equation will reduce to $\left(-\frac{\vec{i}^2}{\sigma} + \vec{E}_e \vec{i}\right)$ since the electric and magnetic power densities w_E and w_H are constant values in time, thus their time derivatives are zero. We know from practice that in static fields there is no radiation, thus $p_s=0$. From this follows that in this case $\vec{E}_e \vec{i} = \vec{i}^2/\sigma$, namely the input power will dissipate into Joule-heat. That is right, but what is happening in this case with the left side of the 4.4 equation, since the $\text{div}(\vec{E} \times \vec{H})$ seemingly appears to be unchanged. This is really only an appearance, since with static fields the related Maxwell equations are the followings.

$$\frac{\partial \vec{D}}{\partial t} = 0 \quad \text{and} \quad \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{thus:}$$

$$\text{rot } \vec{H} = \vec{i} \quad ; \quad \text{rot } \vec{E} = 0 \quad ; \quad \vec{E} = \frac{\vec{i}}{\sigma} - \vec{E}_e$$

with these it can be written that:

$$\text{div}(\vec{E} \times \vec{H}) = -\vec{E} \text{ rot } \vec{H} + \vec{H} \text{ rot } \vec{E} = -\vec{E} \vec{i} + \vec{H} \cdot 0 = -\vec{E} \vec{i}$$

$$\text{div}(\vec{E} \times \vec{H}) = -\left(\frac{\vec{i}}{\sigma} - \vec{E}_e\right) \vec{i} = -\frac{\vec{i}^2}{\sigma} + \vec{E}_e \vec{i} = p_s = 0$$

So we get here also that: $\vec{E}_e \vec{i} = \frac{\vec{i}^2}{\sigma}$

Then we can see that we have arrived at the former result, thus for the two sides of the equation the identity stands here too. We can conclude that with static fields (in consonance with the practice) there is no radiation, but there is radiation in the electromagnetic fields changing in time (whether slowly or rapidly), and the power density of the radiation per unit volume is calculated with equation 4.4, or the total momentary power P_s with the correlation 4.3.

Returning to the electromagnetic waves it is suitable to mention the following.

When the waves are propagating in free space, then in place of material constants ϵ and μ , the dielectric and magnetic permeability of the vacuum $\epsilon_0 = 8.859 \cdot 10^{-12} \text{ As/Vm}$ and $\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$ will be used. Further, as is known, in such case for the absolute value of the field intensities we get

$$|\vec{E}| = Z_0 |\vec{H}| \quad \text{where} \quad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \quad \text{is the wave resistance of the free space.}$$

Taking into account further that $\vec{E}^2 = \vec{E}\vec{E} = |\vec{E}|^2 = E^2$ and $\vec{H}^2 = \vec{H}\vec{H} = |\vec{H}|^2 = H^2$ and $\vec{i}^2 = \vec{i}\vec{i} = |\vec{i}|^2 = i^2$, then with energy density w we get that the electric w_E and magnetic w_H power densities are equal in value:

$$w_E = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 Z_0^2 H^2 = \frac{1}{2} \epsilon_0 \frac{\mu_0}{\epsilon_0} H^2 = \frac{1}{2} \mu_0 H^2 = w_H$$

So in this case:

$$w_0 = (w_E + w_H) = 2w_E = 2w_H = \epsilon_0 E^2 = \mu_0 H^2 \quad (4.5)$$

In case of free space the formulas 4.3 and 4.4 will get the following form:

$$\oint_A (\vec{E} \times \vec{H}) d\vec{A} = P_s = - \int_V \frac{\partial w_0}{\partial t} dV - \int_V \frac{i^2}{\sigma} dV + \int_V \vec{E}_e \vec{i} dV \quad (4.6)$$

$$\text{div}(\vec{E} \times \vec{H}) = p_s = - \frac{\partial w_0}{\partial t} - \frac{i^2}{\sigma} + \vec{E}_e \vec{i} \quad (4.7)$$

Where according to 4.5: $\frac{\partial w_0}{\partial t} = \epsilon_0 \frac{\partial E^2}{\partial t} = \mu_0 \frac{\partial H^2}{\partial t} = 2\epsilon_0 E \frac{\partial E}{\partial t} = 2\mu_0 H \frac{\partial H}{\partial t}$

As a reminder, it is justified to mention that the previously determined P_s and p_s quantities are momentary values. In the practice however, usually the knowledge of their time mean value – that is, their effective value – is necessary. The effective (mean) values for waves with time functions of sine or cosine can be derived by the calculation of the mean integral taken for the time period τ , so:

$$(P_s)_{eff} = \frac{1}{\tau} \int_{t=0}^{\tau} P_s dt \quad ; \quad (p_s)_{eff} = \frac{1}{\tau} \int_{t=0}^{\tau} p_s dt$$

We have seen such calculations in chapter 1, now we will not deal with this here in detail (to avoid repetition). Knowing the effective power and power density obtained this way, we can derive the values of energies by integrating them further by time. If the amplitudes of the waves with time functions of sine and cosine are not modulated, then instead of integrating by time, the value of the energy can be obtained by the multiplication of the effective power with time.

In the next chapter we will examine the case, when there are two waves (radiation) in the space of radiation simultaneously using the Maxwell equations.

4/b. The Maxwell equations for the simultaneous presence of two waves

Let be given two radiating sources (1st and 2nd antenna in the following) within the volume V bound by surface A with \vec{i}_1 and \vec{i}_2 current densities, \vec{E}_{e1} and \vec{E}_{e2} extraneous (external) field intensities associated to them separately (according to the indexes). In this case the vectors \vec{i}_1 and \vec{E}_{e1} of the 1st antenna will create \vec{E}_1 and \vec{H}_1 field intensities within the space of radiation, while the \vec{i}_2 and \vec{E}_{e2} at the 2nd antenna create the \vec{E}_2 and \vec{H}_2 quantities (waves).

If the two antennas radiate simultaneously, then the waves are also present in the field of radiation at the same time, which will create the phenomena of interference as the result of their superposition. We can state that the Maxwell equations are valid also for the resultant fields generated by the superposition of different fields, since:

$$\text{rot}(\vec{H}_1 + \vec{H}_2) = \text{rot} \vec{H}_1 + \text{rot} \vec{H}_2 = \left(\vec{i}_1 + \frac{\partial \vec{D}_1}{\partial t} \right) + \left(\vec{i}_2 + \frac{\partial \vec{D}_2}{\partial t} \right) = \vec{i}_1 + \vec{i}_2 + \frac{\partial \vec{D}_1}{\partial t} + \frac{\partial \vec{D}_2}{\partial t}$$

where: $\vec{H}_r = (\vec{H}_1 + \vec{H}_2)$; $\vec{i}_r = (\vec{i}_1 + \vec{i}_2)$; $\frac{\partial \vec{D}_r}{\partial t} = \frac{\partial}{\partial t}(\vec{D}_1 + \vec{D}_2)$

Thus we get:

$$\text{rot} \vec{H}_r = \vec{i}_r + \frac{\partial \vec{D}_r}{\partial t}$$

Similarly:

$$\text{rot} \vec{E}_r = -\frac{\partial \vec{B}_r}{\partial t} ; \vec{E}_r = (\vec{E}_1 + \vec{E}_2) ; \vec{B}_r = (\vec{B}_1 + \vec{B}_2)$$

In the same way we get:

$$\text{div}(\vec{B}_1 + \vec{B}_2) = \text{div} \vec{B}_1 + \text{div} \vec{B}_2 = \text{div} \vec{B}_r = 0 ; \vec{B}_r = (\vec{B}_1 + \vec{B}_2)$$

$$\text{div}(\vec{D}_1 + \vec{D}_2) = \text{div} \vec{D}_1 + \text{div} \vec{D}_2 = \text{div} \vec{D}_r = \rho_r ; \vec{D}_r = (\vec{D}_1 + \vec{D}_2) ; \rho_r = (\rho_1 + \rho_2)$$

The \vec{H} , \vec{E} , ρ , \vec{B} , \vec{D} , \vec{i} are the quantities known from earlier, and the index r signifies the resultant of these quantities.

After all these, for the case of two simultaneous radiations (in this case it is sufficient to deal only with the correlation 4.4) we can write that:

$$\begin{aligned} \text{div}(\vec{E}_1 \times \vec{H}_1) &= p_{s1} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}_1^2 + \frac{1}{2} \mu \vec{H}_1^2 \right) - \frac{\vec{i}_1^2}{\sigma} + \vec{E}_{e1} \vec{i}_1 \\ \text{div}(\vec{E}_2 \times \vec{H}_2) &= p_{s2} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}_2^2 + \frac{1}{2} \mu \vec{H}_2^2 \right) - \frac{\vec{i}_2^2}{\sigma} + \vec{E}_{e2} \vec{i}_2 \\ \text{div} \vec{E}_r &= p_{sr} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}_r^2 + \frac{1}{2} \mu \vec{H}_r^2 \right) - \frac{\vec{i}_r^2}{\sigma} + \vec{E}_{er} \vec{i}_r \end{aligned} \quad (4.8)$$

By substituting the followings into the correlation 4.8 and developing it we get:

$$\vec{H}_r = (\vec{H}_1 + \vec{H}_2) ; \vec{E}_r = (\vec{E}_1 + \vec{E}_2) ; \vec{i}_r = (\vec{i}_1 + \vec{i}_2) ; \vec{E}_{er} = (\vec{E}_{e1} + \vec{E}_{e2})$$

$$\text{div}[(\vec{E}_1 + \vec{E}_2) \times (\vec{H}_1 + \vec{H}_2)] = p_{sr} = -\frac{\partial}{\partial t} \left[\frac{1}{2} \epsilon (\vec{E}_1 + \vec{E}_2)^2 + \frac{1}{2} \mu (\vec{H}_1 + \vec{H}_2)^2 \right] - \frac{(\vec{i}_1 + \vec{i}_2)^2}{\sigma} + (\vec{E}_{e1} + \vec{E}_{e2})(\vec{i}_1 + \vec{i}_2)$$

$$\begin{aligned} \operatorname{div}\left[\left(\vec{E}_1+\vec{E}_2\right) \times\left(\vec{H}_1+\vec{H}_2\right)\right] &= \operatorname{div}\left[\left(\vec{E}_1 \times \vec{H}_1\right)+\left(\vec{E}_2 \times \vec{H}_2\right)+\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right]= \\ &= \operatorname{div}\left(\vec{E}_1 \times \vec{H}_1\right)+\operatorname{div}\left(\vec{E}_2 \times \vec{H}_2\right)+\operatorname{div}\left[\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right] \\ &= \frac{\left(\vec{i}_1+\vec{i}_2\right)^2}{\sigma}=\frac{\vec{i}_1^2}{\sigma}+\frac{\vec{i}_2^2}{\sigma}+2 \frac{\vec{i}_1 \vec{i}_2}{\sigma}=\frac{\vec{i}_1^2}{\sigma}+\frac{\vec{i}_2^2}{\sigma}+0 \end{aligned}$$

Namely $\vec{i}_1 \vec{i}_2$ is zero, because each of the current densities are different from zero only in its own antenna, thus where one is existing the other one is zero and vice versa, therefore we get $\vec{i}_1 \vec{i}_2=0$

$$\left(\vec{E}_{e1}+\vec{E}_{e2}\right)\left(\vec{i}_1+\vec{i}_2\right)=\vec{E}_{e1} \vec{i}_1+\vec{E}_{e2} \vec{i}_2+\vec{E}_{e1} \vec{i}_2+\vec{E}_{e2} \vec{i}_1=\vec{E}_{e1} \vec{i}_1+\vec{E}_{e2} \vec{i}_2+0+0$$

Owing to similar reasons: $\vec{E}_{e1} \vec{i}_2=0 ; \vec{E}_{e2} \vec{i}_1=0$

$$\left(\vec{E}_1+\vec{E}_2\right)^2=\vec{E}_1^2+\vec{E}_2^2+2 \vec{E}_1 \vec{E}_2 ; \left(\vec{H}_1+\vec{H}_2\right)^2=\vec{H}_1^2+\vec{H}_2^2+2 \vec{H}_1 \vec{H}_2$$

Now for the correlation 4.8 we can write that:

$$\begin{aligned} \operatorname{div}\left(\vec{E}_1 \times \vec{H}_2\right)+\operatorname{div}\left(\vec{E}_2 \times \vec{H}_1\right)+\operatorname{div}\left[\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right] &= p_{sr}= \\ =-\frac{\partial}{\partial t}\left[\frac{1}{2} \varepsilon \vec{E}_1^2+\frac{1}{2} \mu \vec{H}_1^2\right]-\frac{\partial}{\partial t}\left[\frac{1}{2} \varepsilon \vec{E}_2^2+\frac{1}{2} \mu \vec{H}_2^2\right]-\frac{\partial}{\partial t}\left[\varepsilon \vec{E}_1 \vec{E}_2+\mu \vec{H}_1 \vec{H}_2\right]-\frac{\vec{i}_1^2}{\sigma}-\frac{\vec{i}_2^2}{\sigma} &+\vec{E}_{e1} \vec{i}_1+\vec{E}_{e2} \vec{i}_2 \end{aligned}$$

Importing the notations p_{s1} and p_{s2} :

$$p_{s1}+p_{s2}+\operatorname{div}\left[\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right]=p_{sr}=p_{s1}+p_{s2}-\frac{\partial}{\partial t}\left[\varepsilon \vec{E}_1 \vec{E}_2+\mu \vec{H}_1 \vec{H}_2\right] \quad (4.9)$$

We can see that $p_{sr}=p_{s1}+p_{s2}+\Delta p_s$, where

$$\operatorname{div}\left[\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right]=\Delta p_s=-\frac{\partial}{\partial t}\left[\varepsilon \vec{E}_1 \vec{E}_2+\mu \vec{H}_1 \vec{H}_2\right] \quad (4.10)$$

Thus with momentary power densities p_{s1} and p_{s2} each represented by one of the two waves, following from the interference of two waves, one more Δp_s additional power density difference appears in the resultant power density. This Δp_s quantity can be calculated according to correlation 4.10 also in two ways (either using the expression on the left side, or that on the right side of the equation).

Let us write up further the total momentary resultant power P_{sr} of the space in the case of two simultaneous waves, based on the equations 4.9 and 4.10:

$$\int_V p_{s1} dV+\int_V p_{s2} dV+\int_V \Delta p_s dV=P_{sr}=P_{s1}+P_{s2}+\Delta P_s$$

Where ΔP_s is the momentary power difference:

$$\int_V \operatorname{div}\left[\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right] dV=\oint_A\left[\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)\right] d\vec{A}=\Delta P_s=-\frac{\partial}{\partial t} \int_V\left(\varepsilon \vec{E}_1 \vec{E}_2+\mu \vec{H}_1 \vec{H}_2\right) dV$$

or with the notation $\Delta \vec{S}=\left(\vec{E}_1 \times \vec{H}_2\right)+\left(\vec{E}_2 \times \vec{H}_1\right)$

$$\int_V \operatorname{div} \Delta \vec{S} dV=\oint_A \Delta \vec{S} d\vec{A}=\Delta P_s=-\frac{\partial}{\partial t} \int_V\left(\varepsilon \vec{E}_1 \vec{E}_2+\mu \vec{H}_1 \vec{H}_2\right) dV \quad (4.11)$$

For the effective value (time mean value) of the power difference we can write that:

$$\frac{1}{\tau} \int_{t=0}^{\tau} \int_V \operatorname{div} \Delta \vec{S} dV dt=\frac{1}{\tau} \int_{t=0}^{\tau} \oint_A \Delta \vec{S} d\vec{A} dt=\left(\Delta P_s\right)_{eff}=-\frac{1}{\tau} \int_{t=0}^{\tau} \frac{\partial}{\partial t} \int_V\left(\varepsilon \vec{E}_1 \vec{E}_2+\mu \vec{H}_1 \vec{H}_2\right) dV dt \quad (4.12)$$

As we know, for the correlations derived for free space, instead of ε we can write ε_0 , and instead of μ similarly μ_0 , taking also into consideration that $|\vec{E}|=Z_0|\vec{H}|$.

For the sake of completeness it is enough to mention that we can get the energies corresponding to the powers after integrating them by time, or in the case of waves without amplitude modulation, simply by multiplying with time.

Summarizing the results obtained in this chapter we can state that the possibility of the violation of energy conservation instead of contradicting the Maxwell equations, directly follows therefrom. Thus we can consider it to be a proven fact, that the law of energy conservation can not be accepted as a postulate, since under certain conditions (which have been already analyzed in details in chapters 1, 2, and 3) a power- and energy difference can appear, that depending on circumstances can be zero, excess, or deficit as well.

From the derived results it also appears that this power- or energy difference in the free space originates exclusively from the interference of the waves. The losses (joule-heat) of the sources influence the magnitude of these quantities only indirectly, namely so that the wave generating ability of the input powers will decrease according to the losses. The same can also be said for the case when the parameters of the space ϵ and μ are not lossless.

It can be mentioned in advance that the law of energy conservation can be violated not only with the interference of two or more waves, but even in the case of one single source, in spite of the fact that Joule heat does not appear and the free space is also lossless. We will see such a case later e.g. with aperture radiators.

It is the characteristic of the many sidedness and wealth of meaning of the Maxwell equations that even such (at first sight) unbelievable results follow from them, which are valid not only for the spherical waves, as the consequence of the general validity of the equations.

4/c. The general form of Maxwell equations in the case of multiple sources and waves

The examinations completed so far, and the field theory concerning vectors provide a ground for writing up the Maxwell equations in their general form, applicable also for several sources and waves as follows. The index (r,n) utilized by the expressions signify the resultant of the simultaneously existing n -pieces of physical quantities. The σ , ε , μ material constants can be considered as quantities independent from n .

$$\begin{aligned}
 \text{I.} \quad & \text{rot } \vec{H}_{r,n} = \left(\vec{i}_{r,n} + \frac{\partial \vec{D}_{r,n}}{\partial t} \right) \\
 \text{II.} \quad & \text{rot } \vec{E}_{r,n} = -\frac{\partial \vec{B}_{r,n}}{\partial t} \\
 \text{III.} \quad & \text{div } \vec{B}_{r,n} = 0 \\
 \text{IV.} \quad & \text{div } \vec{D}_{r,n} = \rho_{r,n} \\
 \text{V.} \quad & \vec{D}_{r,n} = \varepsilon \vec{E}_{r,n} \quad ; \quad \vec{B}_{r,n} = \mu \vec{H}_{r,n} \quad ; \quad \vec{i}_{r,n} = \sigma (\vec{E}_{r,n} + \vec{E}_{re,n}) \\
 \text{VI.} \quad & \text{div} (\vec{E}_{r,n} \times \vec{H}_{r,n}) = p_{s,n} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon \vec{E}_{r,n}^2 + \frac{1}{2} \mu \vec{H}_{r,n}^2 \right) - \frac{\vec{i}_{r,n}^2}{\sigma} + \vec{E}_{re,n} \vec{i}_{r,n} \\
 \text{VII.} \quad & \vec{E}_{r,n} = \sum_{k=1}^n \vec{E}_k \quad ; \quad \vec{H}_{r,n} = \sum_{k=1}^n \vec{H}_k \quad ; \quad \vec{i}_{r,n} = \sum_{k=1}^n \vec{i}_k \quad ; \quad \vec{E}_{re,n} = \sum_{k=1}^n \vec{E}_{e,k} \quad ; \quad \rho_{r,n} = \sum_{k=1}^n \rho_k
 \end{aligned} \tag{4.13}$$

For the sake of completeness let us write up the equations expressing the conservation of charge, and also the law about the force $\vec{F}_{r,n}$ acting upon the charge q , moving in electromagnetic field with speed \vec{v} (Lorentz-law).

$$\begin{aligned}
 \text{div } \vec{i}_{r,n} + \frac{\partial \rho_{r,n}}{\partial t} &= 0 \\
 \vec{F}_{r,n} &= q \vec{E}_{r,n} + q (\vec{v} \times \vec{B}_{r,n})
 \end{aligned} \tag{4.14}$$

This generally valid form of Maxwell equations according to the examinations and verifications completed so far prove the possibility, that with the forming of favorable structures of electromagnetic waves, and the arrangement of the field structures into appropriate structures, it is possible to create excess power and excess energy above the powers and energies fed into the system. This fact proves at the same time that the law of energy conservation has no general validity.

As the finalization of the related theoretical examinations referring back to the correlations 1.18 and 1.19, the general form of the law of energy conservation for n -pieces of sources will take the following shape, where \mathcal{E} signifies the energy:

$$\begin{aligned}
 0 \leq (P_{s,r})_{\text{eff}} &\leq n \sum_{k=1}^n (P_{in,k})_{\text{eff}} \\
 0 \leq \mathcal{E}_{r,n} &\leq n \sum_{k=1}^n \mathcal{E}_{in,k}
 \end{aligned} \tag{4.15}$$

Further the efficiency of radiation (energy factor) ξ_n for n sources is:

$$\begin{aligned}
 \xi_n &= \frac{(P_{s,r})_{\text{eff}}}{\sum_{k=1}^n (P_{in,k})_{\text{eff}}} = \frac{\mathcal{E}_{r,n}}{\sum_{k=1}^n \mathcal{E}_{in,k}} \\
 0 \leq \xi_n &\leq n
 \end{aligned} \tag{4.16}$$

5. The general discussion of straight antennas with scalar quantities

In the previous chapters according to our aim settings we have dealt mainly with theoretical calculations and provings. In the followings we will deal with the practical meanings of the results originating from these examinations in quantitative form.

We already have the generally valid formula 3.2 and the correlation 2.12 (for the case of two waves) for the quantitative representation of the power- and energy conditions in far space using scalar quantities. With the computerized evaluation of these correlations (since in the great majority of practical cases these evaluations are not possible in explicit form) for different antenna configurations (radiation sources) usually we can obtain numerical results. The radiant characteristics of the antennas in the literature are known almost exclusively only for the far space, but in certain cases it might be necessary also to know the results for the quasi far space. Therefore it seems to be useful to examine in this chapter the radiant characteristics of straight antennas (appearing often in practice) with scalar quantities, valid also for quasi far space. The arrangement necessary for the examinations is shown in fig. 5. where the meanings of the letters used for the notation of quantities can be interpreted according to the figure, or identical with the notations used so far. The calculations relate to the ideal case, assuming that the straight antenna of length l and its surrounding free space is lossless.

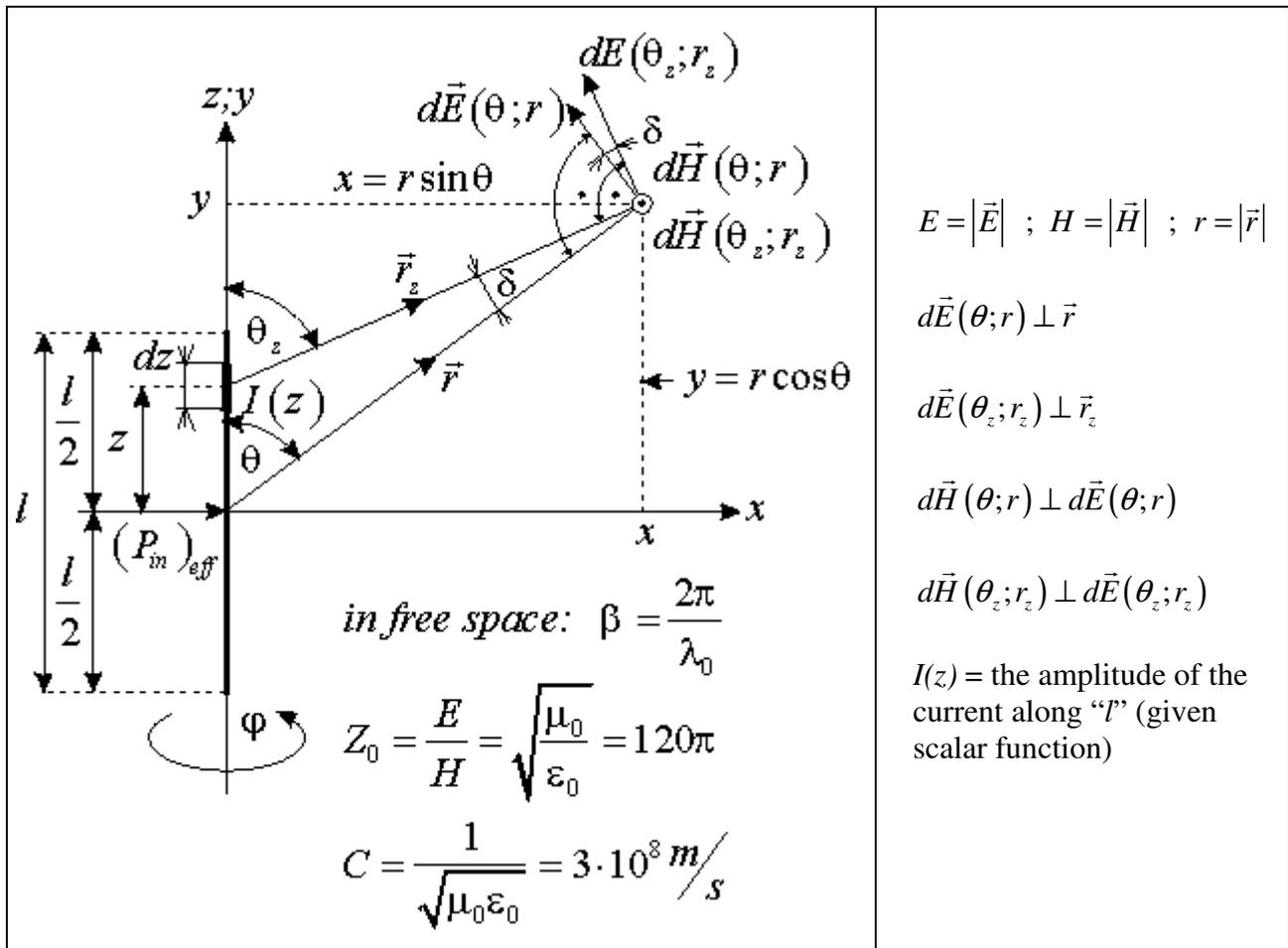


Fig. 5.

The electric quantities are changing according to sine or cosine time functions, but the notations signify only the amplitudes depending on the space coordinates. Therefore the dependence on time and φ are not marked with these, since we are dealing with a rotationally symmetrical case. The effect of the dependence on time will be taken into account at the appropriate places (when determining the effective values).

Our task is to determine the absolute values of the resultant amplitudes perpendicular to the $E(\theta; r)$ and $H(\theta; r)$ appearing at the point at space coordinates r, θ as the effect of the current determined by $I(z)$ amplitude function, for the antenna of length l fed with $(P_{in})_{eff}$ power. Then knowing these, we should determine the effective value of the Poynting vector's absolute value corresponding to this case. Further the total effective power radiated through the closed surface A (including the setup) as the function of r distance. The related literature determines these quantities (as already mentioned) usually only for the far space, when they assume the $l \ll r$ approximation, that is when they assume that $\vec{r}_z \parallel \vec{r}$. Let us attempt to determine the wanted quantities without these approximations, taking into the account the δ angle.

It is known from the solution of Maxwell equations for the $(I(z)dz)$ elementary current (Hertz dipole) that:

$$dE(\theta_z; r_z) = \frac{60\pi}{\lambda_0 r_z} I(z) \sin \theta_z dz \quad \rightarrow \quad d\vec{E}(\theta_z; r_z) = dE(\theta_z; r_z) e^{-j\beta r_z}$$

In our case the value of the differential electric field intensity perpendicular to the \vec{r} , by the application of the above is:

$$d\vec{E}(\theta; r) = d\vec{E}(\theta_z; r_z) \cos \delta = \frac{60\pi}{\lambda_0 r_z} I(z) \sin \theta_z \cos \delta e^{-j\beta r_z} \quad (5.1)$$

For the 5.1 we have to determine the $r_z \sin \theta_z \cos \delta$ quantities as the functions of r and θ .

With the notations of fig. 5. for the values of r_z and $\sin \theta_z$ we can write that:

$$\begin{aligned} r_z^2 &= x^2 + (y-z)^2 = r^2 \sin^2 \theta + r^2 \cos^2 \theta - 2zr \cos \theta + z^2 \\ r_z &= (r^2 - 2zr \cos \theta + z^2)^{\frac{1}{2}} \quad ; \quad \sin \theta_z = \frac{x}{r_z} = \frac{r \sin \theta}{r_z} \end{aligned} \quad (5.2)$$

We get the value of $\cos \delta$ according to the fig. 6.

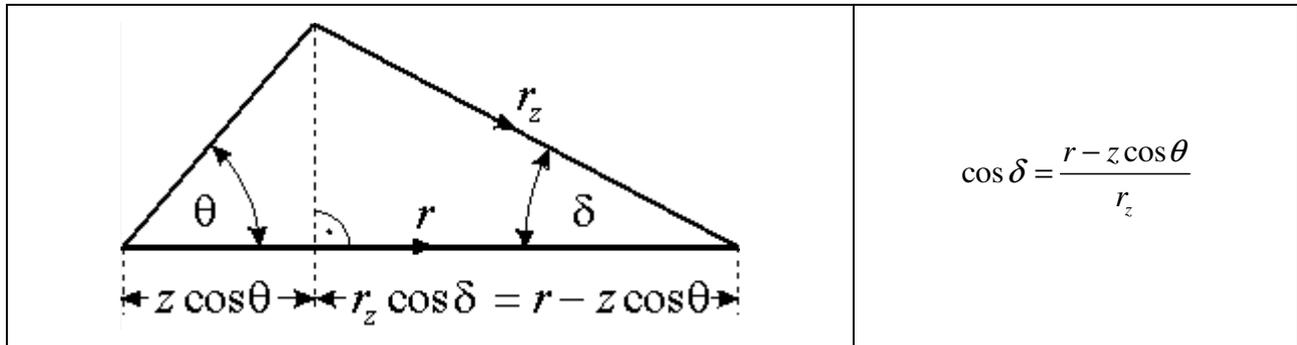


Fig. 6.

By substituting these into 5.1 and integrating by z for the whole length l of the antenna we get:

$$\begin{aligned} \vec{E}(\theta; r) &= \frac{60\pi}{\lambda_0} \int_{-l/2}^{+l/2} I(z) \frac{(r - z \cos \theta) r \sin \theta}{r_z^3} e^{-j\frac{2\pi}{\lambda_0} r_z} dz \\ E(\theta; r) &= |\vec{E}(\theta; r)| = \frac{60\pi}{\lambda_0} \left| \int_{-l/2}^{+l/2} I(z) \frac{(r - z \cos \theta) r \sin \theta}{r_z^3} e^{-j\frac{2\pi}{\lambda_0} r_z} dz \right| \end{aligned} \quad (5.3)$$

where:

$$r_z = (r^2 - 2zr \cos \theta + z^2)^{\frac{1}{2}}$$

We can see in equation 5.3 that in this case of $r \gg z$, $r_z \cong r$ and $r - z \cos \theta \cong r$, thus we get the expression of the straight antennas valid for the far space.

By describing the exponential factor in equation 5.3 with the Euler correlation we get two integrals denoted as:

$$I_1 = \int_{-\frac{1}{2}}^{+\frac{1}{2}} I(z) \frac{(r - z \cos \theta) r \sin \theta}{r_z^3} \cos \left(\frac{2\pi}{\lambda_0} r_z \right) dz$$

$$I_2 = \int_{-\frac{1}{2}}^{+\frac{1}{2}} I(z) \frac{(r - z \cos \theta) r \sin \theta}{r_z^3} \sin \left(\frac{2\pi}{\lambda_0} r_z \right) dz$$
(5.4)

With these we can write that:

$$E(\theta; r) = \frac{60\pi}{\lambda_0} |I_1 - jI_2| = \frac{60\pi}{\lambda_0} \sqrt{I_1^2 + I_2^2}$$
(5.5)

With the equation 5.5 (by substituting the values r_z , I_1 , I_2) we can calculate the absolute values of the electric field intensity $E(\theta; r)$ in question, as the function of coordinates $(\theta; r)$. Naturally the calculation can not be done in explicit form, but it can be completed without difficulty with computer.

Knowing the $E(\theta; r)$ it is simple to get the absolute value of the magnetic field intensity.

$$H(\theta; r) = \frac{E(\theta; r)}{Z_0} = \frac{E(\theta; r)}{120\pi}$$
(5.6)

The absolute value of the Poynting vector's effective value belonging to the $(\theta; r)$ coordinates is:

$$S_{eff}(\theta; r) = \frac{E^2(\theta; r)}{2Z_0} = \frac{E^2(\theta; r)}{240\pi}$$
(5.7)

By having the $S_{eff}(\theta; r)$ we can calculate the total radiant power $P_{s,eff}(r)$, radiated through the spherical surface A with radius r as follows:

$$P_{s,eff}(r) = \oint_A S_{eff}(\theta; r) dA$$

Since the straight antenna is a circle radiator in φ , thus:

$$dA = 2\pi x r d\theta = 2\pi r^2 \sin \theta d\theta$$

$$P_{s,eff}(r) = 2\pi r^2 \int_{\theta=0}^{\pi} S_{eff}(\theta; r) \sin \theta d\theta = \frac{r^2}{120} \int_{\theta=0}^{\pi} E^2(\theta; r) \sin \theta d\theta$$
(5.8)

We can realize that the derived results are applicable not only for one single antenna with length l , but also can be adapted for the case of several straight antennas (placed above each other on a common axis), if we know the $I(z)$ amplitude functions and the lengths of the antennas, when the limits of the integration will be as the lengths of the specific antennas according to the configuration. Naturally the above calculations can be done – besides straight antennas – also for other antennas, but due to their bulkiness we will not deal with them here.

Returning to the results obtained for the straight antennas – as a finalization we can also write up the expression of the radiant efficiency (energy balance) $\xi(r)$ related to the examined antenna as the function of distance r ; the value of which is valid not only for the far space, but also for the quasi far space, for waves without amplitude modulation.

$$\xi(r) = \left(\frac{P_s(r)}{P_{in}} \right)_{eff} = \frac{\mathcal{E}_s}{\mathcal{E}_{in}} = \frac{r^2}{120 P_{in,eff}} \int_{\theta=0}^{\pi} E^2(\theta; r) \sin \theta d\theta$$
(5.9)

where $E(\theta; r)$ can be calculated with the correlations 5.3; 5.4; and 5.5.

6. The radiant efficiency of elemental antennas

During the theoretical calculations and demonstrations we have met several times with the limitation as condition that the derived result is valid only if the law of energy conservation is valid for the source (or sources) of radiation, and if the total input power is radiated out. We have also shown how these results will be modified if this condition is not satisfied. Naturally, this does not mean that the violation of the law of energy conservation will take place at the sources of radiation due to skin losses and matching errors. Since the Joule heat losses and the matching losses can be taken into account intelligibly. In practice a case may arise where the antenna does not radiate the total input power that could be radiated. It is commonly understood that this may be due to heat loss or matching error. However, in certain cases the antenna will not radiate the total input power even though the matching is perfect and the radiator is constructed from superconductor material. We meet with such a case in aperture radiators.

Regarding the later considerations it is expedient to examine some elementary antennas (radiating sources) from the point of view whether the law of energy conservation is valid for them or not, since the antennas in practice are made up (will be formed) by such elementary antennas.

In the following we will examine the radiation efficiency of some elementary antennas of this kind, that is their energy balance apart from the above mentioned losses. With our knowledge so far we can do these examinations in the simplest way by applying the equation 3.2 (in the case of free space), where $G(\theta; \varphi)$ is the antenna gain and $F(\theta; \varphi)$ is the normalized radiant characteristic.

6/a. The isotropic antenna

Although the isotropic antenna does not exist in actual practice (it is a fictional construct), it is advisable to determine its energy balance, since it serves in many cases as a base of comparison. Based on fig. 7. we can write the followings:

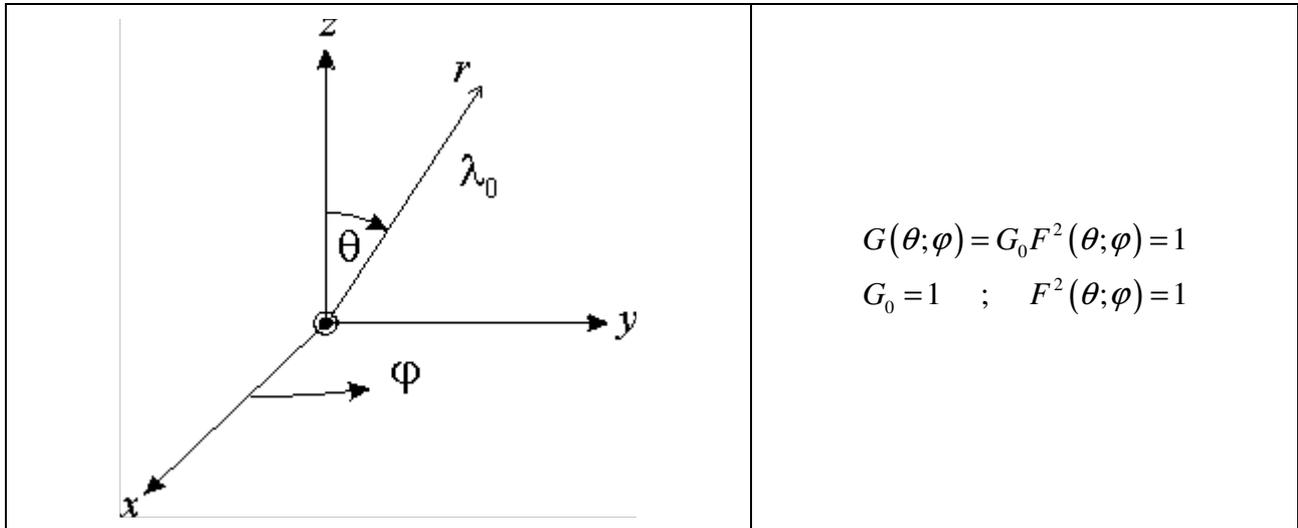


Fig. 7.

$$\xi = \frac{G_0}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F^2(\theta; \varphi) \sin \theta d\varphi d\theta = \frac{1}{2} \int_{\theta=0}^{\pi} \sin \theta d\theta = \frac{1}{2} 2 = 1$$

Thus in the case of isotropic antenna the energy conservation is valid, since we have got $\xi=1$.

6/b. The Hertz dipole

As we know, the Hertz dipole, also referred to as using elementary current ($I dl$) is one of the most important radiators of the practical and theoretical calculations, therefore the examination of its radiant energy correlations is justified, which can be done according to the fig. 8.

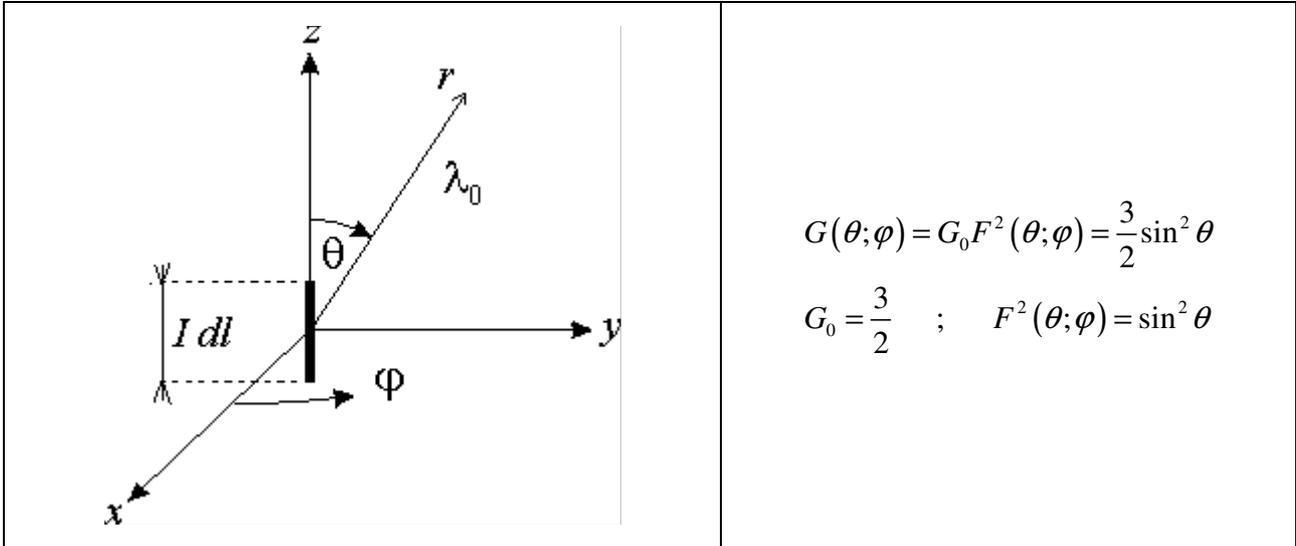


Fig. 8.

$$\xi = \frac{G_0}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F^2(\theta; \varphi) \sin \theta d\varphi d\theta = \frac{3}{2} \frac{1}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin^3 \theta d\varphi d\theta = \frac{3}{4} \int_{\theta=0}^{\pi} \sin^3 \theta d\theta$$

After integration we get:

$$\xi = \frac{3}{4} \left(\frac{\cos^3 \theta}{3} - \cos \theta \right) \Big|_{\theta=0}^{\pi} = \frac{3}{4} \left[\left(-\frac{1}{3} + 1 \right) - \left(\frac{1}{3} - 1 \right) \right] = \frac{3}{4} \frac{4}{3} = 1$$

Accordingly, the law of energy conservation is valid also for the case of Hertz dipole, since we have got $\xi=1$ here too.

This result is important, because the Hertz dipole is an antenna which exists in reality, and the straight antennas of length l (or also other antennas) are built up from such radiators, corresponding to the elementary currents.

6/c. The elementary aperture

The elementary aperture (with infinitesimal dimensions) is also one of the important determinative elements of the antenna theory and practice. Namely its radiant characteristics constitute the basis of the aperture analysis. Therefore it is also justified to examine the radiant- and energy correlations for this elementary radiator. These examinations can be done according to fig. 9. where the aperture with surface of infinitesimal size dA is placed into the starting point of the coordinate system, and the amplitude of the electric field intensity E_0 is constant on its surface.

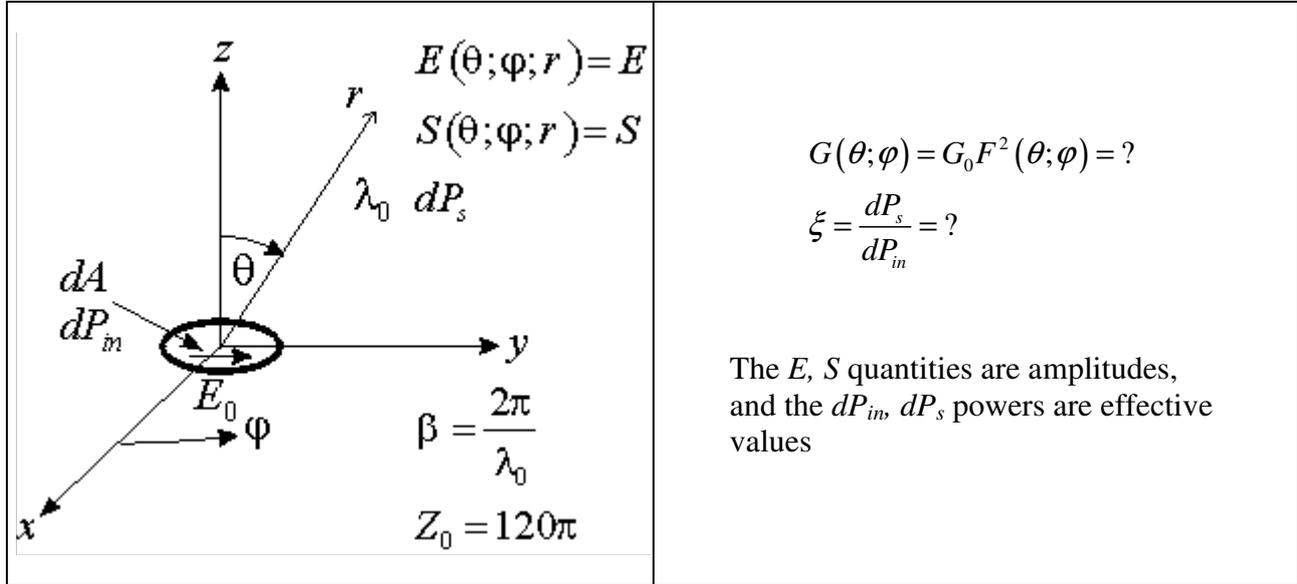


Fig. 9.

For the calculation of the radiant efficiency ξ or energy factor in question, first it is necessary to determine the function G of the gain of the elementary surface, together with the radiant characteristics F .

As is known, for the gain we can generally write that:

$$G = \frac{\frac{E^2}{2Z_0}}{\frac{P_{in}}{4\pi r^2}} = \frac{4\pi r^2 E^2}{2Z_0 P_{in}} = 2\pi \frac{r^2 E^2}{Z_0 P_{in}}$$

In present case $P_{in} = \frac{E_0^2 dA}{2Z_0}$ and the absolute value of the electric field intensity E according to

Huygens-Kirchhoff method is:

$$E = \frac{\beta}{4\pi r} (1 + \cos\theta) E_0 dA = \frac{E_0}{\lambda_0 r} \frac{1 + \cos\theta}{2} dA = \frac{E_0 dA}{\lambda_0 r} \cos^2\left(\frac{\theta}{2}\right)$$

Substituting these back into the expression for G we get:

$$G = 2\pi r^2 \frac{E_0^2 (dA)^2}{\lambda_0^2 r^2} \frac{2Z_0}{Z_0 E_0^2 (dA)} \cos^4\left(\frac{\theta}{2}\right) = \frac{4\pi}{\lambda_0^2} dA \cos^4\left(\frac{\theta}{2}\right) = G_0 F^2$$

It can be seen that neither G nor F depends on φ , thus:

$$G(\theta) = G_0 F^2(\theta) ; G_0 = \frac{4\pi}{\lambda_0^2} dA ; F^2(\theta) = \cos^4\left(\frac{\theta}{2}\right) = \left(\frac{1 + \cos\theta}{2}\right)^2$$

Now we can calculate the value of ξ as follows.

$$\xi = \frac{G_0}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F^2(\theta) \sin \theta d\varphi d\theta = \frac{G_0}{2} \int_{\theta=0}^{\pi} F^2(\theta) \sin \theta d\theta$$

$$\xi = 2\pi \frac{dA}{\lambda_0^2} \int_{\theta=0}^{\pi} \left(\frac{1 + \cos \theta}{2} \right)^2 \sin \theta d\theta = 2\pi \frac{dA}{\lambda_0^2} I$$

Where integral I is:

$$I = \int_{\theta=0}^{\pi} \left(\frac{1}{4} + \frac{\cos \theta}{2} + \frac{\cos^2 \theta}{4} \right) \sin \theta d\theta = I_1 + I_2 + I_3$$

The values of each integral in order are:

$$I_1 = \frac{1}{4} \int_{\theta=0}^{\pi} \sin \theta d\theta = -\frac{1}{4} \cos \theta \Big|_0^{\pi} = \frac{1}{2}$$

$$I_2 = \frac{1}{2} \int_{\theta=0}^{\pi} \cos \theta \sin \theta d\theta = \frac{1}{2} \frac{\sin^2 \theta}{2} \Big|_0^{\pi} = 0$$

$$I_3 = \frac{1}{4} \int_{\theta=0}^{\pi} \cos^2 \theta \sin \theta d\theta = -\frac{1}{4} \frac{\cos^3 \theta}{3} \Big|_0^{\pi} = \frac{1}{4} \frac{2}{3} = \frac{1}{6}$$

So we can already write up the requested end result:

$$\xi = 2\pi \frac{dA}{\lambda_0^2} (I_1 + I_2 + I_3) = 2\pi \frac{dA}{\lambda_0^2} \frac{2}{3} = \frac{1}{3} \frac{4\pi}{\lambda_0^2} dA = \frac{G_0}{3} = \frac{4\pi}{3} \frac{dA}{\lambda_0^2}$$

The result is surprising, since $\frac{dA}{\lambda_0^2} \ll 1$, therefore $\xi \ll 1$, thus in the case of the elementary aperture the law of energy conservation is significantly violated (a big loss takes place). Therefore the law of energy conservation loses its validity in the case of elementary apertures. As we will see later, this effect comes into play (unfavorably) even with the apertures of big sizes.

6/d. The half wave dipole

Although the half wave dipole does not belong to the category of elementary antennas in the strict sense, since it is an important radiator type in the practice – as well as in theory – it is justified to examine its power- and energy correlations. The examination similar to the previous case can be completed in a simple way according to the fig. 10.

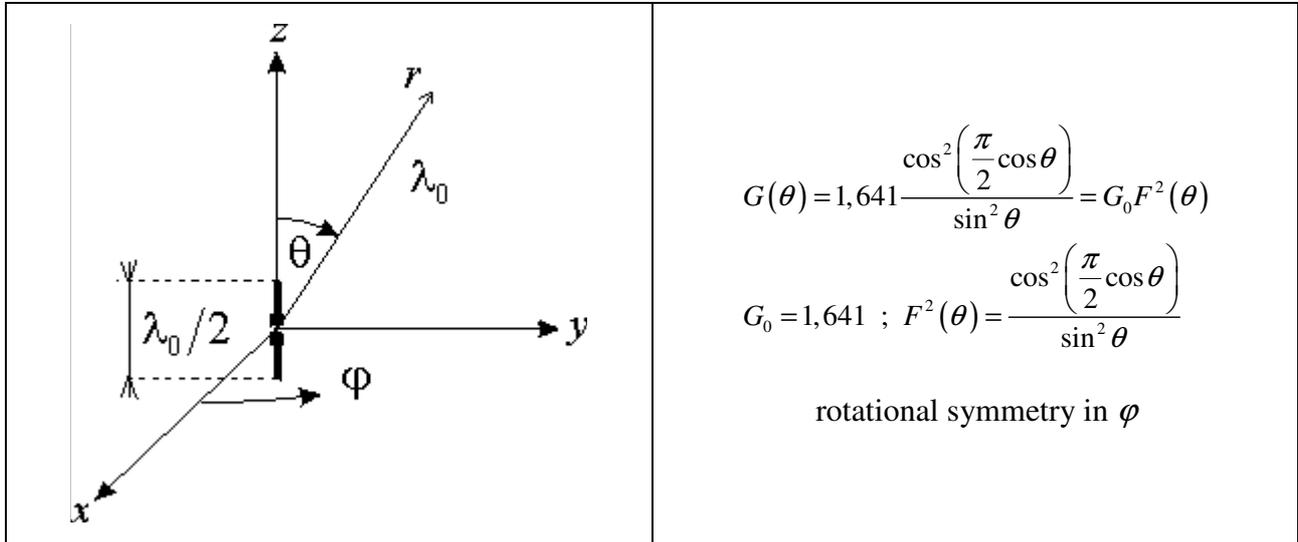


Fig. 10.

For the value ξ in question we can write that:

$$\xi = \frac{G_0}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} F^2(\theta) \sin \theta d\varphi d\theta = \frac{G_0}{2} \int_{\theta=0}^{\pi} \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} d\theta$$

$$\xi = G_0 \int_{\theta=0}^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} d\theta = G_0 I_0$$

The value of the integral I_0 can be determined in the simplest way by computerized evaluation, and we get a value of $I_0 \cong 0,6094$. Thus the value of ξ in question is:

$$\xi = G_0 I_0 = 1,641 \cdot 0,6094 = 1,000025 \cong 1$$

On the basis of this end result we can declare that the law of energy conservation is valid for the half wave dipole.

It can be mentioned in advance that for the full wave dipole of length λ_0 fed at its middle (which – concerning energetics – is equivalent with two half wave dipoles placed directly above each other) the law of energy conservation is no longer valid.

7. The energy coefficients of performance calculated for several antenna configurations

After the fairly extensive mathematical analysis, in this chapter we will show the dependence of the derived radiant efficiency ξ (energy coefficient) as the function of the geometrical data of the antenna configuration, for some simple practical antenna configurations in the form of diagrams. The diagrams represent the results of computerized calculations (with the utilization of the correlations derived in the theoretical section) related to lossless radiation sources and free space.

The performed numerical calculations were done with the expressions valid for the far space of radiation, showing the related correlations and the parameters of the antenna configuration with each of the figures. The utilized antenna characteristics and the gain function valid for far space can be found in the cited literature, or they can be derived by the methods described there. The power connected to the input of the antennas originate from the same generator (their frequency is identical), thus the calculations are valid for coherent systems.

It is unambiguously evident from the diagrams derived for the antenna configurations that the law of energy conservation, in certain cases how and in what measure will be violated, thus their detailed evaluation is unnecessary, so we will make only a few remarks and characteristic conclusions.

The notations used in the diagrams according to the practice so far: ξ - the energy coefficient, d - distance between the elements, x - the size expressed in wave lengths, n - the number of identical antennas, ϕ and ψ - the values of phase angles, P - the effective power, E_0 - the electric field intensity, E_s and E_{in} - signifies energy here, where the meaning of indexes are obvious.

Naturally, the energy coefficient ξ can be determined also for the case of any type-, any number- and any configuration of antennas (using the described formulas), if the parameters of the single antennas and their geometric configurations are given.

Two pieces of isotropic radiators

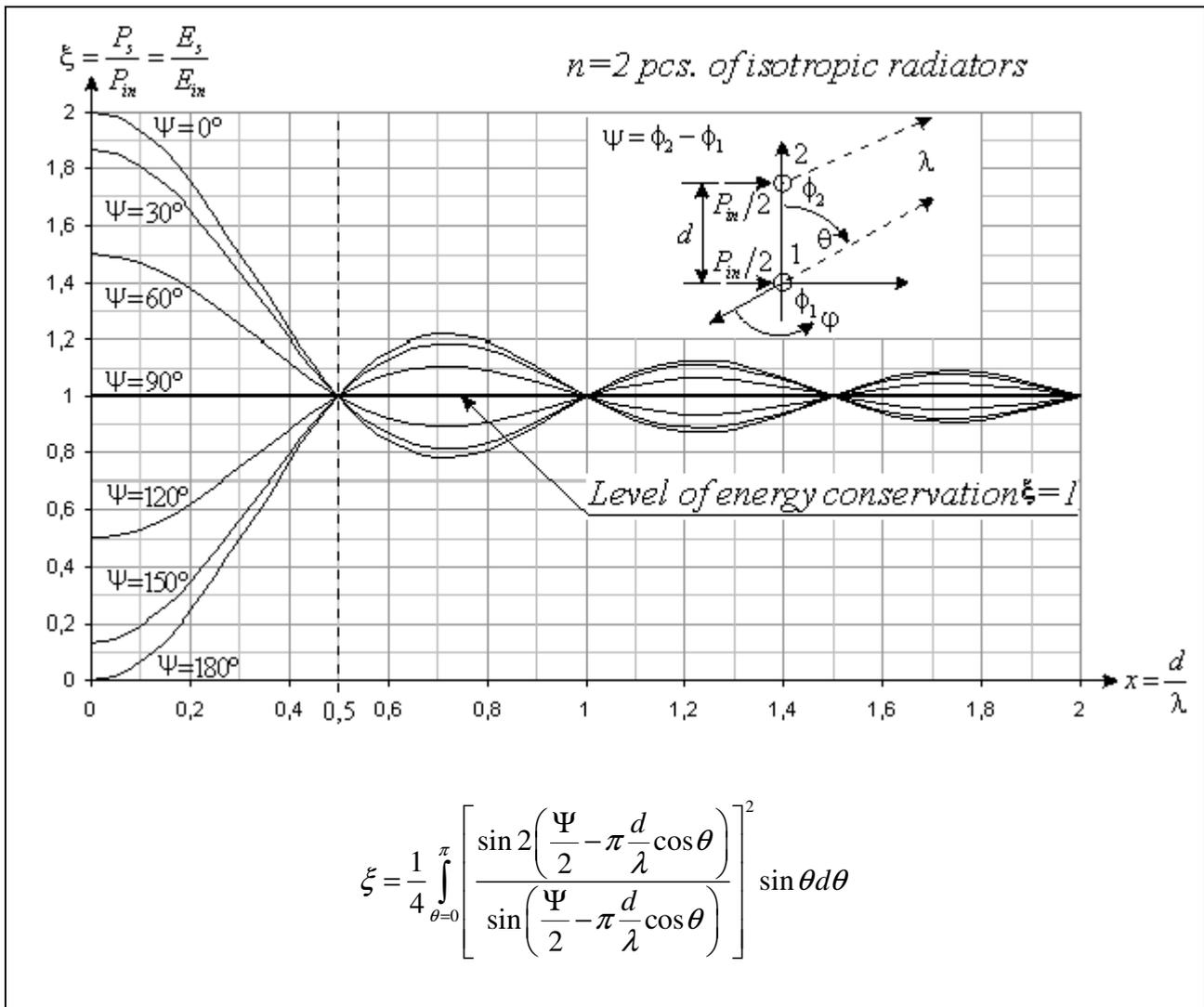


Fig. 11.

We can see from the above diagram to what measure- and with what sign the law of energy conservation is violated, depending on the phase conditions. The deviation from $\xi=1$ is diminishing with the increase of distance x and finally converges to the $\xi=1$ value, corresponding to the level of energy conservation.

Two pieces of Hertz dipoles

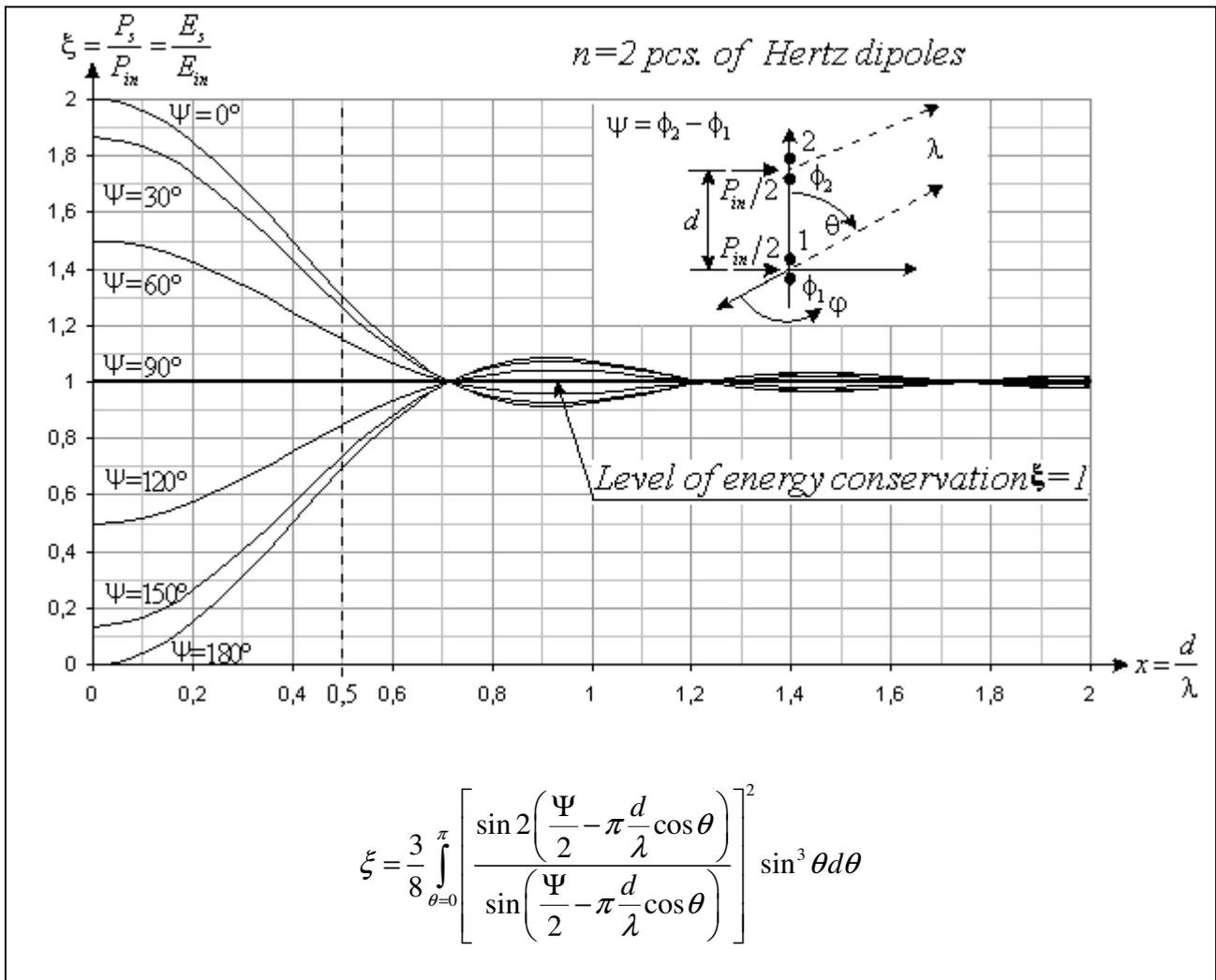


Fig. 12.

The characteristics of these curves is similar to the previous case with the difference, that the nodes along the x ($\xi=1$) are at different places, and for big values of x it converges faster to $\xi=1$.

Two pieces of half wave dipoles

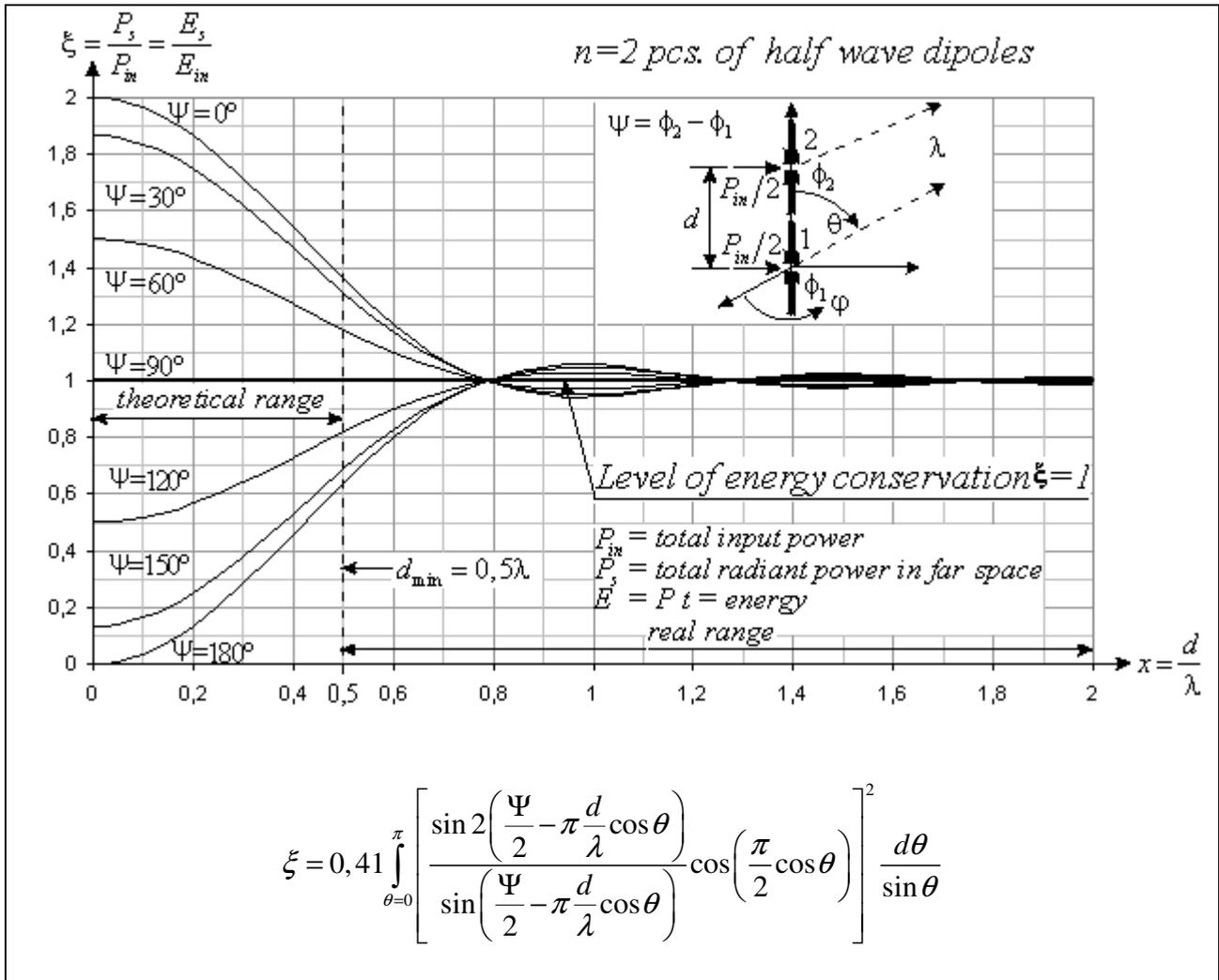


Fig. 13.

Here we can say the same as with fig.12. regarding the characteristics of the curves.

It is worth mentioning that when feeding the antennas with identical phase and power and placing them directly above each other, a significant ~36% excess of power and energy appears. As we have already mentioned, the 2 pieces of half wave dipoles radiating with identical phase is equivalent to 1 piece of whole wave dipole fed at the middle; thus the 36% of excess power is valid also for the whole wave dipole (fed at the middle).

Three pieces of half wave dipoles

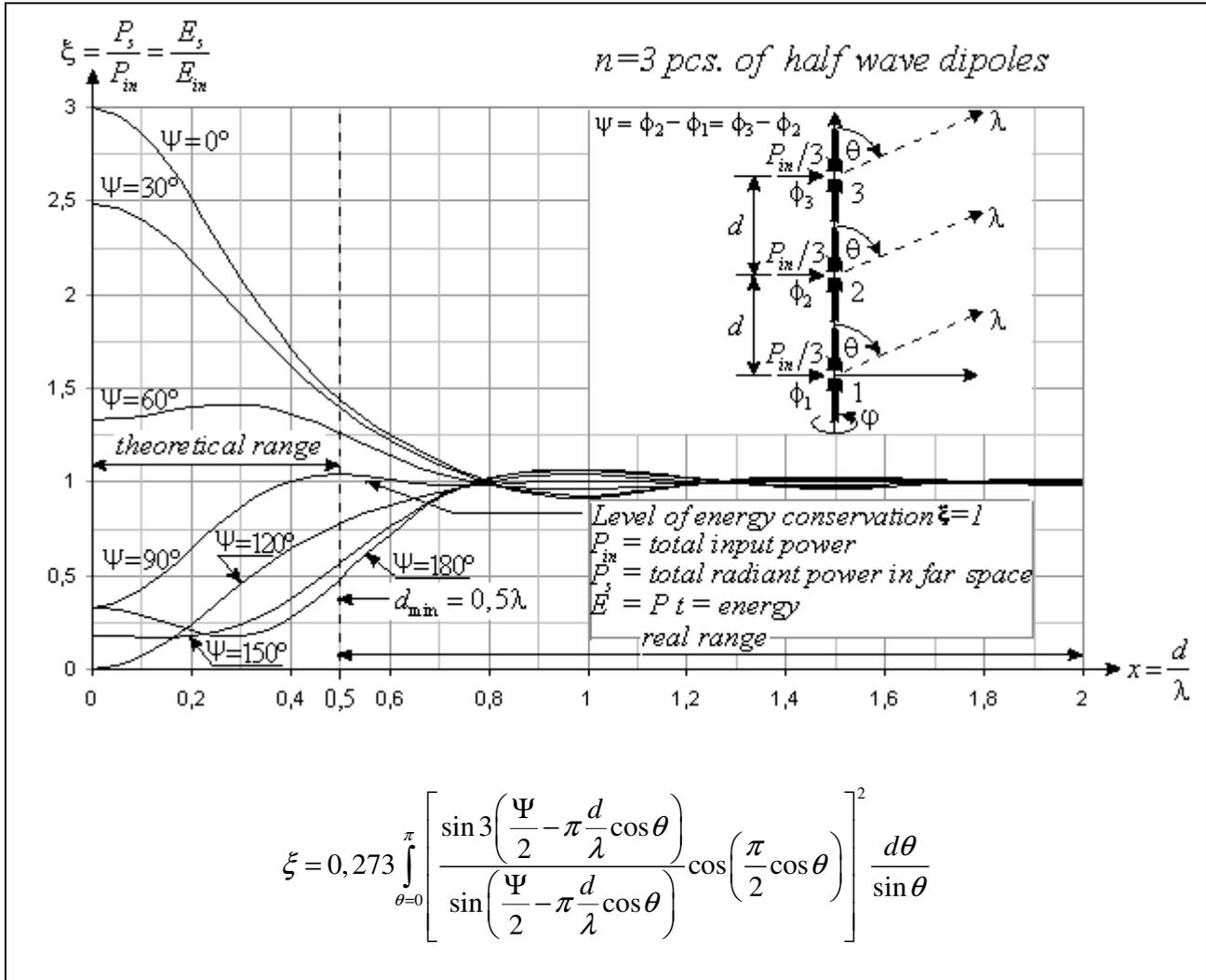


Fig. 14.

The shape of the curves in this case is dependent on the phase, but its characteristics regarding the convergence are similar to the previous ones. It can be seen further, that in the case of antennas placed directly above each other, fed with identical phase, the excess power is ~48%.

Several pieces of half wave dipoles

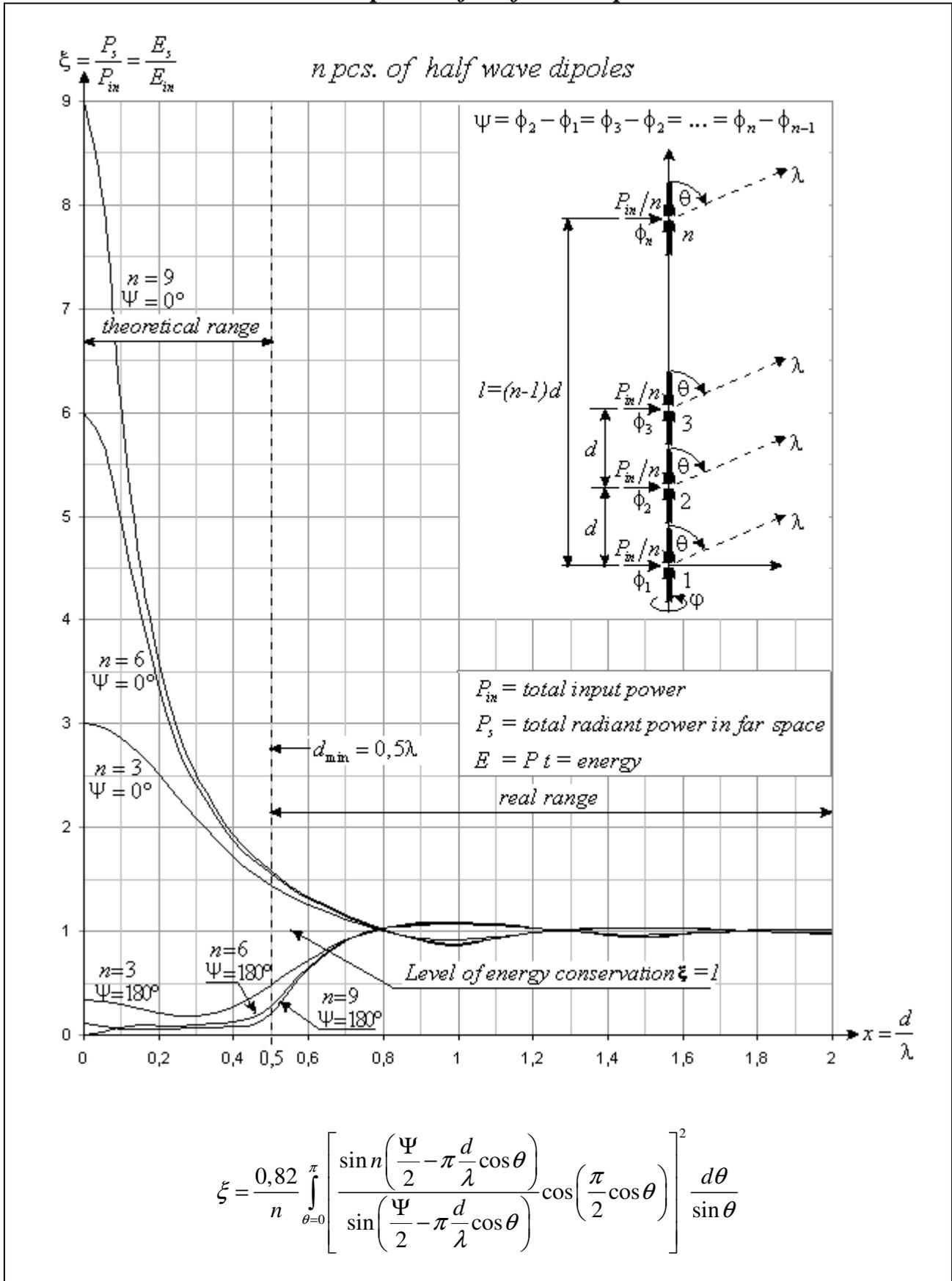


Fig. 15.

The factors in front of the integrals with the figures 11. – 15. are calculated as $G_0/(2n)$, where G_0 is the gain of the individual antenna in main direction, and n is the number of antenna elements.

Different aperture radiators

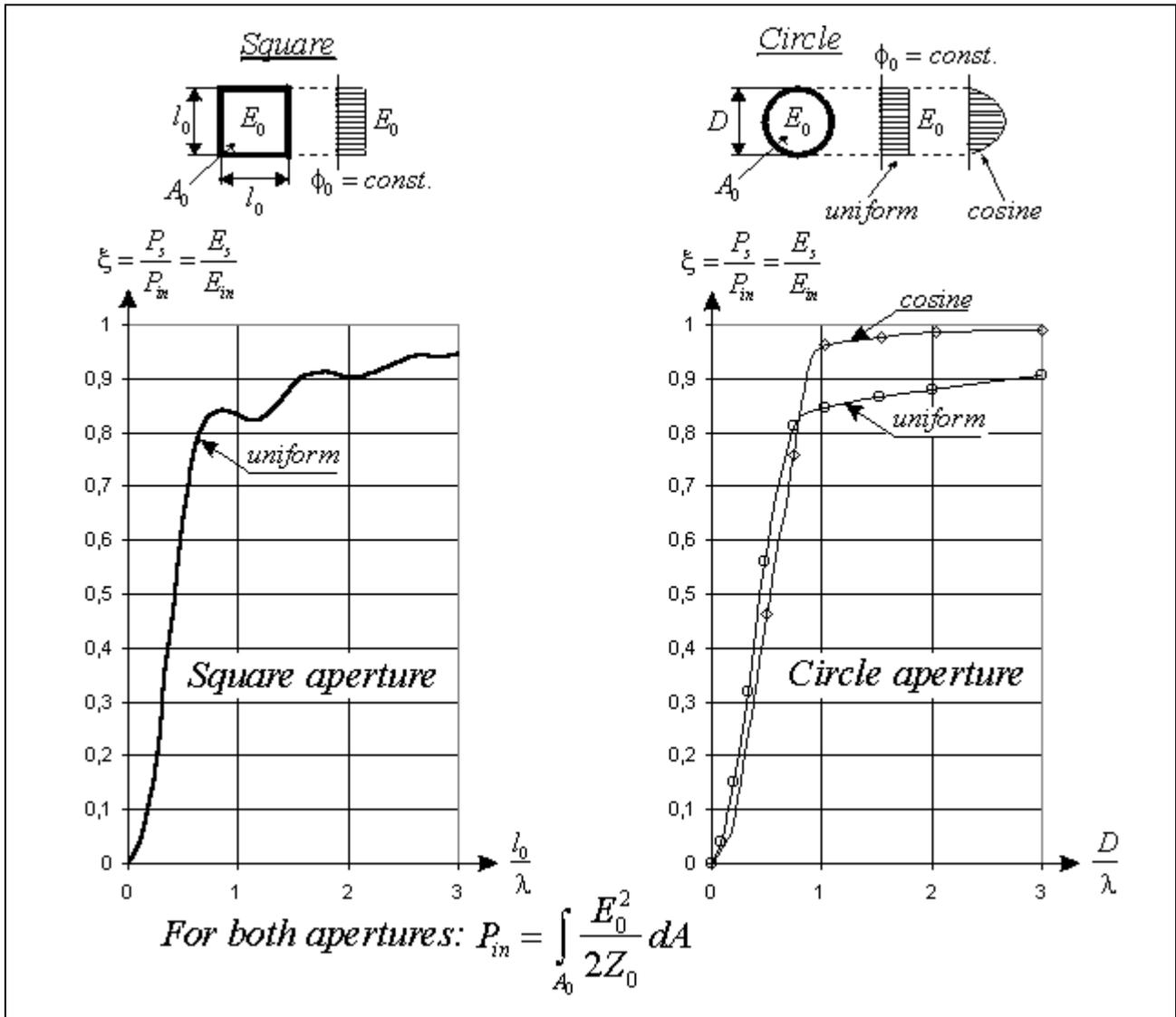


Fig. 16.

These curves about different aperture radiators have been made using the generally valid correlation 3.2 and the approximate radiant functions as found in the related literature.

The results for the aperture radiators are surprising. Namely in this case there will be always energy deficit, which although diminishes with the increase of the aperture dimensions, but it never exceeds the value of $\xi = 1$. This phenomena is connected with our discussions under $6/c$, as we have got $\xi \ll 1$ for the elementary apertures. Thus in spite of the fact that these elementary apertures develop beneficial effects, we can get $\xi = 1$ value only for apertures with infinitely big size (namely the law of energy conservation can be satisfied only in that case).

It is remarkable that among the examined shapes of constant phase apertures, the one with circular shape and cosine type of amplitude distribution has the least energy loss. In the case of apertures with quadratic shape we can also observe a fluctuation – depending on its size – which has a diminishing tendency with the increase of the dimensions.

8. Conclusions and arising questions

The analysis and calculation results according to the previous chapters unambiguously prove that the law of energy conservation is not generally valid for the resultant field of coherent spherical electromagnetic waves (radiation) with identical frequency, since an excess- or deficiency of energy can also appear, compared to the total radiated energy, the magnitude of which depends on the parameters and geometrical position of the radiating sources.

In the case of incoherent radiation (with different frequencies) the law of energy conservation will be satisfied also only, if it is valid for each radiation source separately.

These declarations are valid not exclusively for spherical waves, but also for cylindrical waves and moreover for waves in general.

By summarizing it conclusively we can declare as a fact, that the law of energy conservation is not generally valid – but only in special cases – for the energy propagation in space (as radiation) in the form of waves, concerning the resultant energy of the waves. Here under waves we should understand them to be not exclusively electromagnetic waves.

– *Further questions arising in connection with the violation of the law of energy conservation*

When the necessary conditions are satisfied, where does the excess of energy come from, compared to the energy fed into the system; and in the case of energy deficiency, into what does the energy gets transformed into (or where does it disappear to)?

Since the discovery of the antiparticles it is a known fact, that when a particle is united with its pair of antiparticle, then its rest mass disappears and their mass-energy (in quantum form) will be transformed into electromagnetic radiant energy. The question is whether this process is reversible, and if (under certain conditions) yes, then whether the excess of radiant energy can result in excess of particles and/or antiparticles. Namely whether an excess of mass or mass-energy can be created? Or whether the mentioned energy deficiency can be compensated by the rise of mass energy?

Based on the analogy of matter and antimatter, can there be energy and anti-energy? If anti-energy does exist, then how can it be interpreted?

As a conclusion we can only say briefly, that the violability of the law of energy conservation – besides its practical applicability – can hopefully represent a more generalized interpretation of the classical nuclear-physics, as well as of our knowledge about the universe, especially by answering the above questions.

Budapest
28 January 1998

János Vajda

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Remark:

János Vajda has invented a free energy device based on these principles and a patent application has been filed (Title: APPARATUS FOR GENERATING AND UTILIZING SURPLUS ENERGY BY MEANS OF ELECTROMAGNETIC WAVES, Number: P9601424, Application filed:05/28/1996). If anyone wishes to invest into the full development and utilization of that invention please contact the author.