

Gravitational effect on the refractive index: A hypothesis that the permittivity, ϵ_0 , and permeability, μ_0 are dragged and modified by the gravity

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Abstract: Using the analogy of an acoustic wave in the atmosphere of the Earth, the hypothesis that the ether is the permittivity and permeability of free space is proposed. If the ether is defined, the stationary state is also defined. The permittivity and permeability are dragged by the gravitational field of the Earth. Furthermore, the gravitational field determines the values of the permittivity of free space and the permeability of free space needed to satisfy time dilation by the gravitational potential. That is, the value of the permittivity and permeability of free space are modified by the gravity to isotropically decrease of the speed of light (that is, the increase of a refractive index).

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1. Introduction

The theory of special relativity proposed by Einstein [1] in 1905 was simple and intuitive. The Principle is invariant light speed such that light in a vacuum propagates with the speed c (a fixed constant) regardless of the state of motion of the light source. This expression is very simple and intuitive; however, the interpretation of the theory of special relativity is rather difficult. For example, I do not agree that the speed, c , is a fixed constant in terms of any system of inertial coordinates because I cannot imagine this situation. The expression “the speed, c , is a fixed constant in terms of any system of inertial coordinates” was derived from the Michelson-Morley experiment [2], but it is important to note that the Michelson-Morley experiment was carried out in the gravitational field of the Earth. That is, the Michelson-Morley experimental condition is not in free space [3]. The Michelson-Morley experiment observed the isotropic constancy of the speed of light, c , in a moving gravitational field.

In this discussion, the analogy of an acoustic wave in the atmosphere is used to interpret the theory of special relativity. This analogy is relevant because acoustic waves as well as electromagnetic waves satisfy the wave equation. Furthermore, an acoustic wave in the atmosphere around the Earth can provide an

intuitive interpretation of light. Concerning the derivation of the wave equation from Maxwell's equation, the speed c is defined in the reference frame where the permittivity of free space, ϵ_0 , and the permeability of free space, μ_0 , are in the stationary state. This hypothesis is derived from the analogy of an acoustic wave in a stationary atmosphere.

2. Wave equation

This discussion starts with the wave equation, which is defined as:

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0; \quad (1)$$

where E is the amplitude of the wave and c is the phase velocity of the wave. Table 1 shows the phase velocities of an electromagnetic wave and an acoustic wave. Equation 2 is the phase velocity of an electromagnetic wave when the medium of the wave is assumed to be the permittivity and permeability of free space [3, 4]. The classical concept of an ether is convenient and, at this stage, the permittivity and permeability of free space are assumed to be the ether.

Table 1 Comparison of electromagnetic and acoustic waves

| | Waves | Electromagnetic wave | Acoustic wave |
|---|----------------|---|--|
| 1 | Equation | Wave equation | Wave equation |
| 2 | Phase velocity | $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (2)$ | $c_A = \sqrt{\frac{C}{\rho}} \quad (3)$ |
| 3 | Coefficients | ϵ_0 : permittivity of free space μ_0 : permeability of free space | C : coefficient of stiffness ρ : density |
| 4 | Medium | Vacuum | Atmosphere |
| 5 | Speed | 300,000 km/s | 340 m/s |

3. Stationary state from the analogy of an acoustic wave and an electromagnetic wave

The medium of the acoustic wave, which is the air (represented by the coefficient of stiffness, C , and the density, ρ), is the reference frame of the phase velocity. The phase velocity is defined in the coordinate fixed to the atmosphere. The phase velocity of equation 3 is defined in the stationary atmosphere which is reference frame at rest. Thus, from the analogy of an acoustic wave and an electromagnetic wave, the permittivity and permeability of free space are assumed to be in the stationary state. The phase velocity of an electromagnetic wave is defined in the coordinate fixed to the permittivity and permeability of free space. By this analogy, the phase velocity of an acoustic or electromagnetic wave critically depends on the coefficients of the medium.

4. Hypotheses

Table 2 shows the hypotheses of the dragging caused by the gravitational field of the Earth.

Table 2 Hypotheses of ether-dragging

| | Hypothesis | Electromagnetic wave | Acoustic wave |
|---|--|--|--|
| 1 | Dragging by the gravitational field of the Earth | The permittivity of free space, ϵ_0 , and the permeability of free space, μ_0 . | Atmosphere: the coefficient of stiffness, C, and the density, ρ . |

In 1818, Fresnel proposed that the velocity of light in a medium can be represented as:

$$\frac{c}{n} + \left(1 - \frac{1}{n^2}\right)v; \quad (4)$$

where c is the speed of light, n is the refractive index, and v is the velocity of the medium. Equation 4 was experimentally confirmed by Fizeau in 1851. It has previously been stated that equation 4 demonstrates ether-dragging in that it defines the speed of light in water that is flowing at a velocity, v . That is, the speed of light in water, $\frac{c}{n}$, (n is the refractive index of water) is dragged by the water at a speed of $\left(1 - \frac{1}{n^2}\right)v$ (v is the velocity of the water flow). However, the Michelson-Morley experiment did not show any experimental results of ether-dragging. Fresnel's ether-dragging is a proportional dragging as shown in equation 4, which indicates that if there is an ether, the ether wind is only slightly observed; however, Fresnel's ether-dragging was experimentally denied by the Michelson-Morley experiment. Thus, the Michelson-Morley experiment has two possible answers: there is no ether (orthodox interpretation) or the ether exists and is completely dragged with the Earth (this is derived from the analogy of an acoustic wave). For the second case, the permittivity and permeability of free space are completely dragged by the gravitational field of the Earth and the effect of the ether wind is not observed. For this work, I choose the hypothesis of the complete ether-dragging by the gravitational field of the Earth.

5. Global positioning system (GPS) experiment

The time dilation by the velocity and gravitational potential was experimentally examined by the global positioning system (GPS) experiment summarized by Ashby [6]. GPS satellites orbit 20,000 km above ground level at a velocity 4 km/s. The GPS uses an Earth-centered Earth-fixed (ECEF) reference frame, or an Earth-centered inertial (ECI) coordinate system. This coordinate system is a substantially stationary frame (Appendix 1). In the GPS experiment, above 20,000 km from ground level, the ether-wind and Fresnel's ether-dragging are not observed (Appendix 2). The Michelson-Morley experiment conducted with the GPS, i.e. by direct experimental measurement rather than with the use of a Michelson interferometer, confirms the constancy of the speed of light.

The GPS experiments show that the GPS is not affected by the motion of the solar system or the cosmic microwave background (CMB) and it can be concluded that the GPS is in a stationary state. Only the velocity of the GPS satellite, $v_G = 4$ km/s, affects on the time dilation by the theory of special relativity.

In the GPS experiment, a Lorentz contraction is not observed. The sensitivity of the GPS experiment allows for the detection of a Lorentz contraction by the motion in the solar system ($v_E = 30$ km/s) or the CMB ($v_E = 700$ km/s) [7]. Sachs [8] described that the Lorentz-Fitzgerald contraction is not a physical change but a scale change. This interpretation explains why there is no Lorentz contraction detected, which is due to the fact that the observer is in a moving frame and does not detect the scale change. However, the time scale change cannot explain the time dilation in the GPS satellite by the velocity, $v_G = 4$ km/s. This is because the time dilation is not a scale change but a physical change. Therefore, the Lorentz contraction is a scale change while the time dilation is a physical change.

There is still a need for an explanation as to why the GPS works so precisely in the ECEF reference frame. One way to explain the precision of the GPS in the ECEF reference frame is to assume that the GPS satellites are in a substantial stationary state. This substantial stationary state can best be illustrated using the analogy of the atmosphere around the Earth.

6. The effect of gravity on the speed of light

It is known that the speed of light depends on the gravitational potential. In the gravitational fields, the speed of light becomes slow and time dilation occurs. In this discussion, the permittivity and permeability of free space are assumed to depend on gravity so that they are variable.

Figure 1 shows a rough illustration of the permittivity and permeability. From equation 2, the phase velocity of the electromagnetic wave is isotropic; however, it depends on the gravitational potential. The refractive index, n , is large near the ground and the light refracts to the higher refractive index region (toward the Earth). Around a black hole, the light would travel toward the black hole. This was interpreted using the refractive index n .

Table 1 shows the analogy of an acoustic wave and an electromagnetic wave. Atmospheric pressure is caused by the weight of the air above the measurement point. The phase velocity of an acoustic wave in an ideal gas does not depend on atmospheric pressure, whereas the phase velocity of an acoustic wave in the air has a slight dependence on the atmospheric pressure.

The phase velocity of an electromagnetic wave depends on the gravitation. From the equation of the phase velocity of an electromagnetic wave, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, it is predicted that the coefficient $\epsilon_0 \mu_0$ becomes larger near the ground than near the GPS satellite. By this prediction, the gravity critically relates to the phase velocity of an electromagnetic wave. The effects of the gravitational potential on the electromagnetic and acoustic waves are summarized in Table 3.

Table 3 Effects of the gravitational potential

| | Waves Effects | Electromagnetic wave | Acoustic wave |
|---|---|--|---|
| 1 | Height dependency | Gravitation | Atmospheric pressure |
| 2 | Height dependency on the phase velocity | The phase velocity depends on the gravitation. | The phase velocity of the ideal gas does not depend on the atmospheric pressure. The phase velocity of the air has a slight dependence on the atmospheric pressure. |
| 3 | Change of the phase velocity in the gravitational field | Change of the permittivity and permeability of free space by the gravitational potential | Change of the coefficient of stiffness and the density by atmospheric pressure |

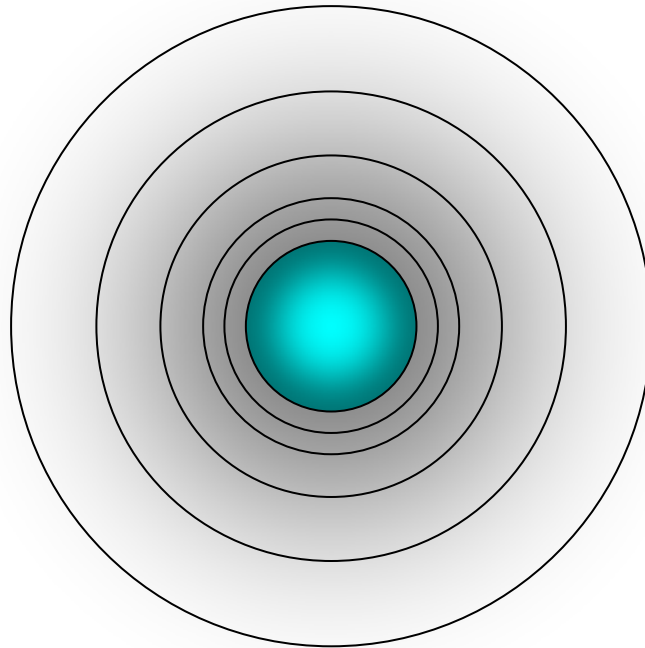


Fig. 1 Gravitational field illustrated from the analogy of the atmosphere of the Earth. The values of the permittivity of free space, ϵ_0 , and permeability of free space, μ_0 , vary depending on the height. That is, the values are changed in order to satisfy the effect of the gravitational field on time dilation.

7. Doppler shift between two moving gravitational fields

Let us adopt a hypothesis of complete frame-dragging by the gravitational field. If the gravitational field drags the frame around its space, there is no time dilation by the velocity. A clock in the gravitational fields is not affected by the motion of the frame. This is similar to a sound wave in the atmosphere. The clock on earth is only affected by the gravitational potential of the earth. The GPS satellites orbit around the earth at the velocity of 4 km/s. The clock in the GPS satellite is affected by the velocity of 4 km/s as well as the gravitational field of the earth. The GPS satellites move in the dragged frame by the gravitational field of the earth. This hypothesis is compatible with the GPS experimental results.

Figure 2 shows two gravitational fields moving at the absolute velocities v_A and v_B in free space. Observer A and Light source B are both in their own ECI coordinate frames. Thus, there is no time dilation by the velocities v_A and v_B . The speed of light, c , is constant, not only in free space but also in the gravitational fields. Thus, the assumption that “the speed of light, c , is constant regardless of the velocity of the light source and the observer” is satisfied.

The equation of the longitudinal Doppler shift critically differs from that of the orthodox one: it is similar to that of an acoustic wave. Equation 7 shows gravitational field A sees gravitational field B, where the subscript $A \rightarrow B$ denotes that A sees B. In equation 8, f_0 is the reference frequency in the stationary state. The reference frequencies in the gravitational fields are f_A and f_B . Thus, the coefficient f_B/f_A denotes the modification that gravitational field A sees gravitational field B.

$$f_{A \rightarrow B} = f_0 \frac{f_B}{f_A} \left(\frac{1 + v_A/c}{1 + v_B/c} \right). \quad (7)$$

Figure 3 shows that gravitational field B sees gravitational field A, and the frequency is represented in equation 8.

$$f_{B \rightarrow A} = f_0 \frac{f_A}{f_B} \left(\frac{1 - v_B/c}{1 - v_A/c} \right). \quad (8)$$

From equations 7) and 8, we conclude

$$f_{A \rightarrow B} \neq f_{B \rightarrow A}.$$

This is particularly different from the orthodox representation (Appendix 3), though rather similar to the acoustical one (Appendix 4).

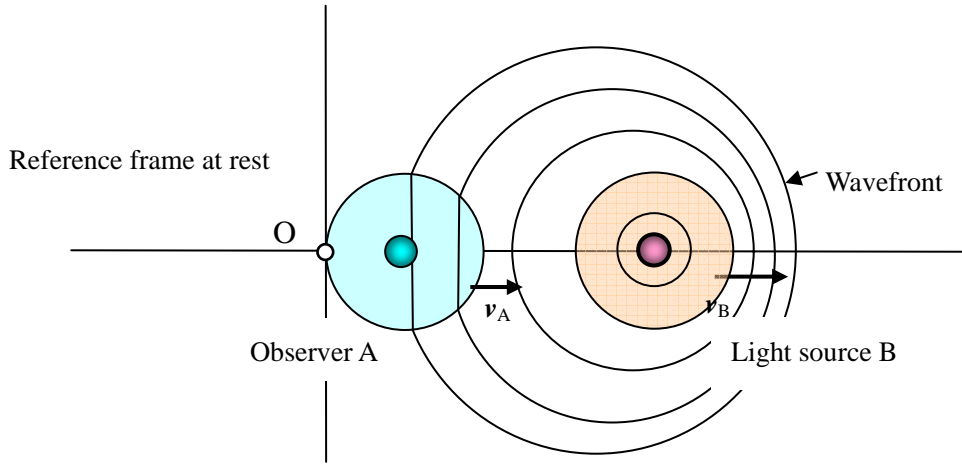


Fig. 2 Longitudinal Doppler shift between two moving gravitational fields. Wavefronts are viewed by a distant observer.

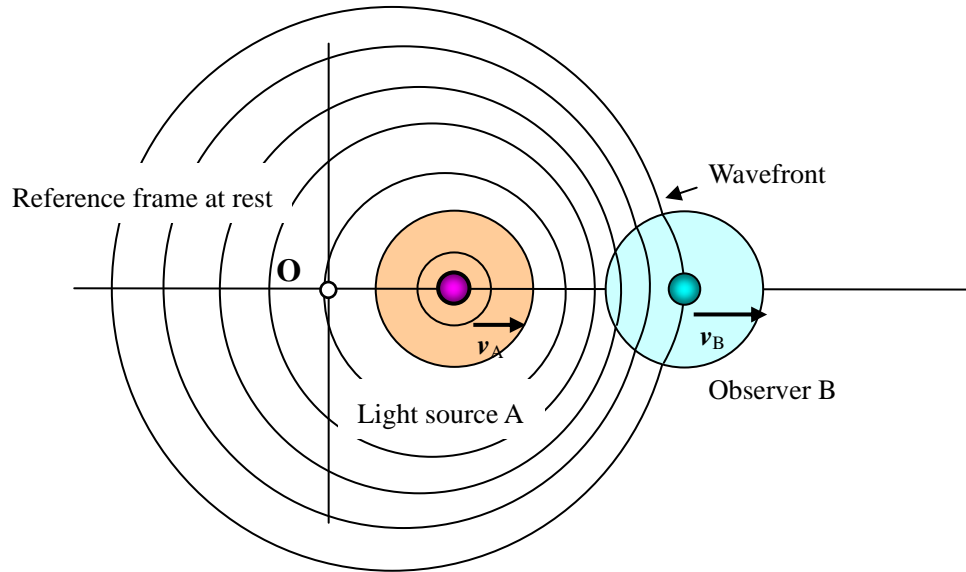


Fig. 3 Longitudinal Doppler shift between two moving gravitational fields. Observer and light source are changed from those in **Fig 2**.

8. Conclusion

The hypothesis that the ether is the permittivity and permeability of free space is proposed. If the ether is defined, the stationary state is also defined. Thus, the discussion can be carried out with the stationary state, which makes it simple and intuitive. The hypothesis of complete frame-dragging by the gravitational field is also discussed, which gives a different expression from that of the orthodox one. Furthermore, the

gravitational field varies the values of the permittivity and permeability of free space in order to satisfy the time dilation by the gravitational potential. That is, the value of the permittivity and permeability of free space become large due to gravity in order to decrease the speed of light, c . This conclusion is derived from the analogy of an acoustic wave in the atmosphere.

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Appendix 1:

In the previous report [9], two equations (A-1) and (A-2) of the reference time were discussed.

$$t'_G = \frac{t_E}{\sqrt{1 - \frac{v_E^2 + v_G^2(t)}{c^2}}} \quad , \quad (A-1)$$

$$t''_G = \frac{t_E}{\sqrt{1 - \left(\frac{v_E + v_G(t)}{c} \right)^2}} \quad . \quad (A-2)$$

Where, t_G is the reference time of the atomic clock in the GPS satellite, t_E is the reference time at the velocity $v_G=0$, that is the reference time on earth. The reference time, t_G only depends on the velocity $v_G=4$ km/s, though not on the synthetic velocity of v_G and v_E : that is, we should put $v_E=0$, where v_E is the velocity of the earth in the CMB. If the GPS experiment is carried out in free space (that is not in the gravitational field of the earth), the time dilation by the velocity v_E will obviously appear. **Figure A** shows that the orbital plane of the GPS satellite is perpendicular to the direction of the mother ship motion in the CMB. The orbit of the GPS satellite in the CMB is helical. There is constant time dilation caused by the velocities $v_G=4$ km/s and $v_E=700$ km/s as described in equation (A-1). Where, t'_G is the reference time on the helical orbit, and t''_G is that of cycloid.

Figure B shows the parallel orbital plane. The orbit of the GPS satellite in the CMB is cycloid. Therefore, the periodic deviation of the reference time described by equation (A-2) will be observed. However, the GPS experiment shows that the effect of the velocity v_E does not appear, and there is no periodic deviation detected. That is, in the gravitational field of the earth, the velocity v_E should be set to 0.

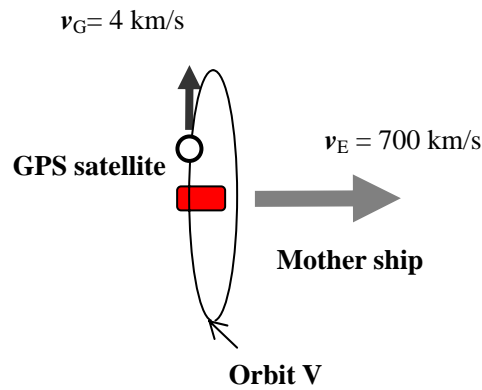


Fig. A Orbital plane of the GPS satellite is perpendicular to the direction of the mother ship motion in the CMB. The trajectory is helical. The GPS satellite and the mother ship are in free space.

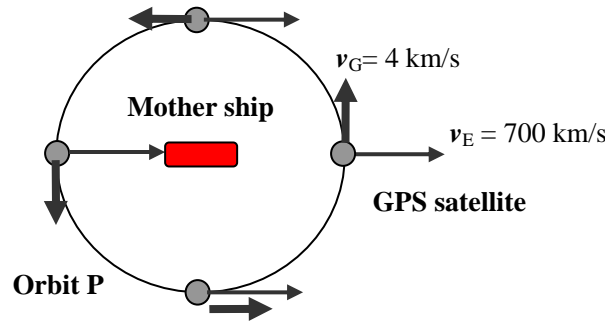


Fig. B Orbital plane of the GPS satellite is parallel to the direction of the mother ship motion in the CMB. The trajectory is cycloid.

Figure C shows the traveling path of the GPS satellite in an arbitrary free reference frame. The trajectory of orbit P in **Fig. B** appears cycloid. The clock in the satellite travels according to the red line. The satellite's motion is periodic accelerated motion. Thus the reference time of the GPS satellite around the mother ship in free space will show a periodic deviation as shown in equation (A-2). However, the reference time of the atomic clock in the GPS satellite around the earth does not show any periodic deviation. Therefore the gravitational field of the earth can be treated as a special frame, an absolute stationary frame.

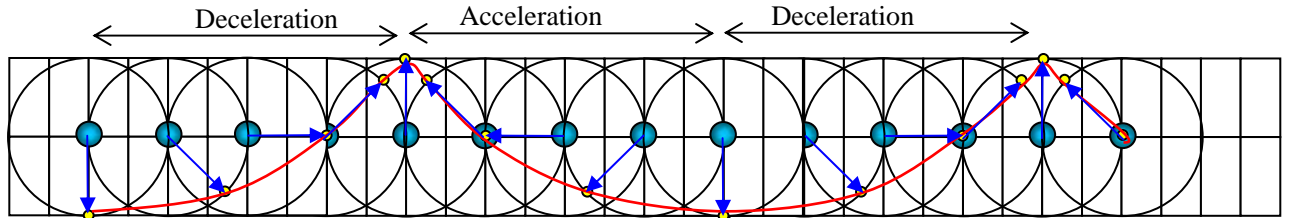


Fig. C Traveling path of the GPS satellite in an arbitrary free reference frame.

The trajectory of orbit P in **Fig. B** appears cycloid. The clock in the satellite travels according to the red line. The satellite's motion is periodic accelerated motion. Thus the reference time of the GPS satellite around the mother ship in free space will show a periodic deviation. However, the reference time of the atomic clock in the GPS satellite around the earth does not show any periodic deviation. Therefore the gravitational field of the earth can be treated as a special frame. That is the absolute stationary frame.

Appendix 2: Ether-drift and frame-dragging

Figure D shows an illustration of the classical hypothesis of the ether-drift and frame-dragging. The earth assumes to travel at the velocity of 700 km/s in the CMB toward the constellation Virgo. At this stage, the ether is tentatively considered as the permittivity ϵ_0 and permeability μ_0 . The GPS satellites are in the lower orbit of 20,000 km from the ground level. In the light blue region, the permittivity ϵ_0 and permeability μ_0

are completely dragged by the gravitational field of the earth.

The satellites in the higher orbit (in the yellow region) have a possibility to detect the ether-drift. The evidence of the ether-drift can be proven by the fact that the ECI coordinate system does not work well. Of course, these experiments have not been carried out yet.

The discussions of the ether-drift and frame-dragging were carried out more than 100 years ago. I have not carried out any calculation of the height of the frame-dragging using the theory of general relativity. At this stage, I consider that the height of the ether-drift detected is more than 20,000 km from the ground level.

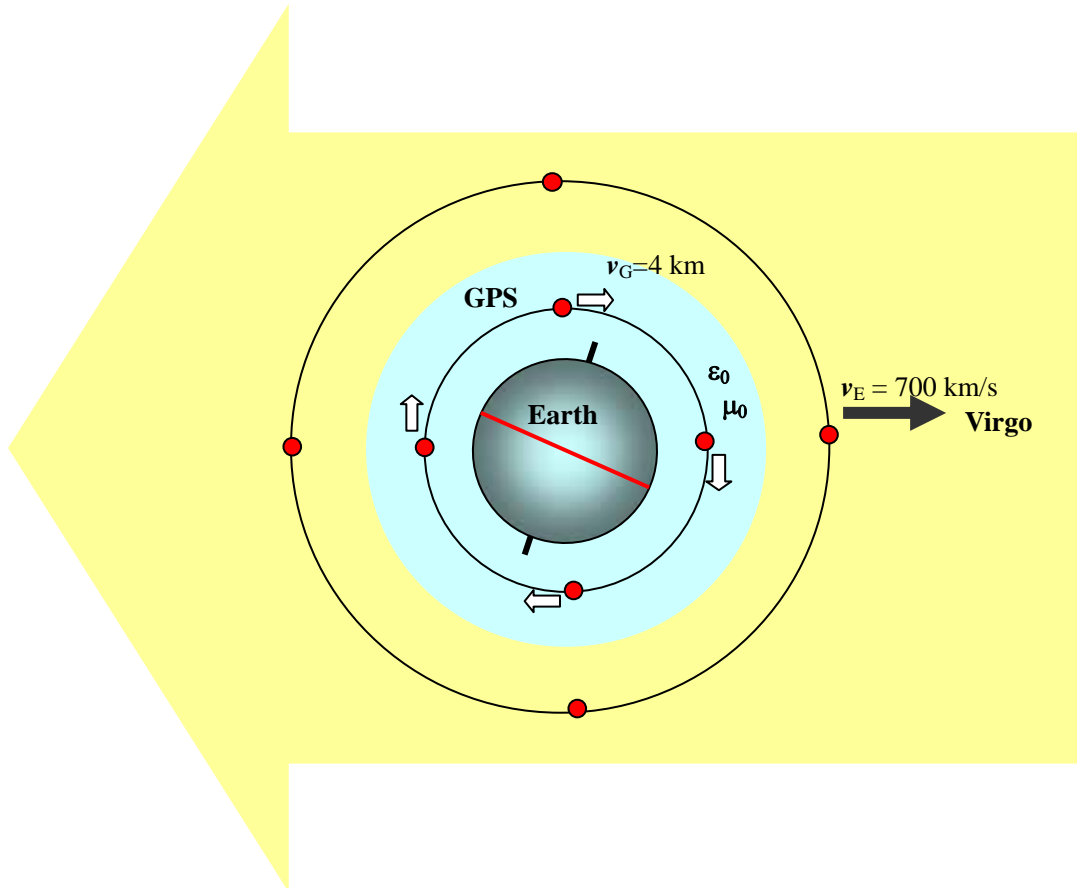


Fig. D Illustration of the complete ether-drift. The satellite on the GPS orbit does not detect the ether-drift. This is because the ether adheres to the gravitational fields of the earth (light blue region). However, the satellite that orbits on the higher level (yellow region) has a possibility to detect the ether-drift. Where $v_E = 700 \text{ km/s}$ is the velocity of the earth in the CMB. These discussions were carried out 100 years ago.

Table A shows the effects and hypothesis expected on the earth. The Sagnac and Lense-Thirring effects were observed. The speed of light in the moving medium depends on the velocity of the medium according to the relativistic velocity addition law. Thus, in the atmosphere, the speed of light is affected by the velocity of the atmosphere adhering to the earth's rotation. On the earth's equator, where the atmosphere moves with earth's rotation speed, the Sagnac effect critically occurs. On the north and south poles, the atmosphere is almost stationary, thus the Sagnac effects are not observed. The Sagnac effect in the GPS experiment was summarized by Ashby [6]. In these days, the Lense-Thirring effect [10], that is the rotational frame-dragging, was experimentally observed.

On the other hand, the discussion of the ether-drift and frame-dragging was arisen in the end of 19th century, and has disappeared in the middle of 20th century. The ether-drift and frame-dragging have already been a historic hypothesis. However, I believe that the historic hypothesis of the ether-drift and frame-dragging is worth to be experimentally checked. From the GPS experimental results (that is the GPS works precisely), at the orbits of 20,000 km from the ground level, there is no ether-drift detected, it can be interpreted by the hypothesis of the frame-dragging. The higher orbits more than 20,000 km from the ground level has a possibility to detect the ether-drift. The Global Navigation Satellite System Galileo will have the orbit of approximately 23,500 km from the ground level (orbital radii of approximately 30,000 km) [6]. Thus, we can check the effects of the orbit height.

Table A Effects and hypothesis on the earth

| | Effects and hypothesis | Comments |
|---|--------------------------------|---|
| 1 | Sagnac | The speed of light in the atmosphere depends on the velocity of the atmosphere. The atmosphere is the medium of light. |
| 2 | Lense-Thirring | Rotational frame-dragging. Earth's rotation drags the frame around the earth. |
| 3 | Ether-drift and frame-dragging | <p>Historic hypothesis: This hypothesis has been denied for more than a century: however the hypothesis of ether-drift and frame-dragging looks very attractive. The GPS experiments can test the hypothesis.</p> <p>Frame dragging, that is an inertial motion of the earth drags the frame around the earth, is very interesting. The earth motion in the CMB or the solar system drags the ether. If there is a frame-dragging, the ether-drift can be detected on higher (more than 20,000 km) orbit.</p> |

Appendix 3: The longitudinal Doppler shift frequency in free space [11]

The relativistic velocity addition in **Fig. E**, where v_A is the addition of v_S and u , is represented as,

$$v_A = \frac{v_S + u}{1 + \frac{v_S u}{c^2}}. \quad (\text{A-3})$$

Thus, the relative velocity u is derived as,

$$u = \frac{v_A - v_S}{1 - \frac{v_S v_A}{c^2}}. \quad (\text{A-4})$$

Then using equation (A-4), the longitudinal Doppler shift frequency between rockets S and A is represented as,

$$f_{S \rightarrow A}^L = f_{A \rightarrow S}^L = f_0 \sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}}, \quad (\text{A-5})$$

subscript $S \rightarrow A$ denotes that rocket S sees rocket A ($A \rightarrow S$ denotes rocket A sees rocket S), and f_0 is the reference frequency in the reference frame at rest. Equation (A-5) can be rewritten as,

$$f_{S \rightarrow A}^L = f_{A \rightarrow S}^L = f_0 \sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}} = f_0 \sqrt{\frac{1 - \frac{v_A - v_S}{c \left(1 - \frac{v_S v_A}{c^2}\right)}}{1 + \frac{v_A - v_S}{c \left(1 - \frac{v_S v_A}{c^2}\right)}}} = f_0 \sqrt{\frac{\left(1 - \frac{v_A}{c}\right) \left(1 + \frac{v_S}{c}\right)}{\left(1 - \frac{v_S}{c}\right) \left(1 + \frac{v_A}{c}\right)}}. \quad (\text{A-6})$$

Equation (A-6) shows the Doppler frequency using the absolute velocities [11]. The difference between equations 7 and (A-6) is caused by the time dilation of the reference time. When rockets A and B are in free space (or in the gravitational field of the Earth), equation (A-6) can be used.

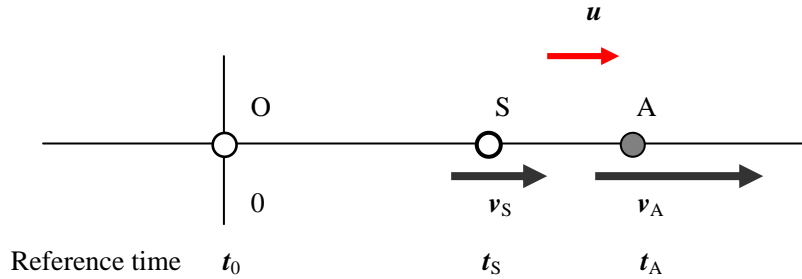


Fig. E Doppler shift of two moving rockets in free space

O: Reference frame at rest

t_0 (reference time)

A: Moving object A (rocket A)

t_A (reference time)

S: Moving object S (rocket S)

t_S (reference time)

Appendix 4: Acoustical Doppler shift

The acoustical Doppler shift of a frequency f' , represented by equation (A-7), is derived from a geometrical drawing v_A is the velocity of the observer, v_S is that of the light source, and f_0 is the frequency of the sound source.

$$f' = f_0 \left(\frac{1 \pm v_A/c}{1 \pm v_S/c} \right), \quad (\text{A-7})$$

where, + and – indicate the direction of the relative motion of the light source and the observer. The + or – sign is selected according to the relative motion between the light source and the observer. For example, the + sign of v_S is selected when the light source moves to decrease the frequency according to v_S . That is, when the light source leaves from the observer, the + sign is selected.

In the acoustical Doppler shift, f_0 is constant. In the light, modification of the Lorentz transformation or gravitational potential is required. If the Lorentz transformation is applied, equation (A-6) is obtained [11].