Further considerations on the Vector Potential.

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1. Radiation from moving charged sphere

It is customary when considering radiation effects from moving charges to assume that the charge remains constant while it is accelerating. This is reasonable since we mostly deal with moving electrons or ions. But an interesting point arises when dealing with say spheres that are moving while at the same time their charge is changing. If we take a point in space that is at a distance r large compared to the sphere radius we can assume the sphere acts like a point charge. Then the **A** field at that distant point is given by

$$\mathbf{A} = \frac{\mu_0 Q_{(t)} \mathbf{v}}{4\pi r} \qquad (1)$$

where $Q_{(t)}$ is the charge as a function of time and **v** is the velocity. It follow that **A** must also be a function of time and we can write

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\mu_0 \mathbf{v}}{4\pi r} \cdot \frac{\partial Q}{\partial t} \qquad (2)$$

Then from $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$ we see that there exists an electric field

$$\mathbf{E} = -\frac{\mu_0 \mathbf{v}}{4\pi r} \cdot \frac{\partial Q}{\partial t} \qquad (3)$$

that is a radiation field following the $1/r$ radiation law. Note that the charge is not accelerating, the velocity can be constant. It appears that this aspect of EM radiation has been overlooked, certainly it is not mentioned in texts dealing with classical electrodynamics. The vector direction of **E** is parallel to the velocity direction of Q, and its polarity determined by whether Q is changing positively or negatively. In figure 1 Q is becoming more positive and E is anti-parallel to the velocity of Q.

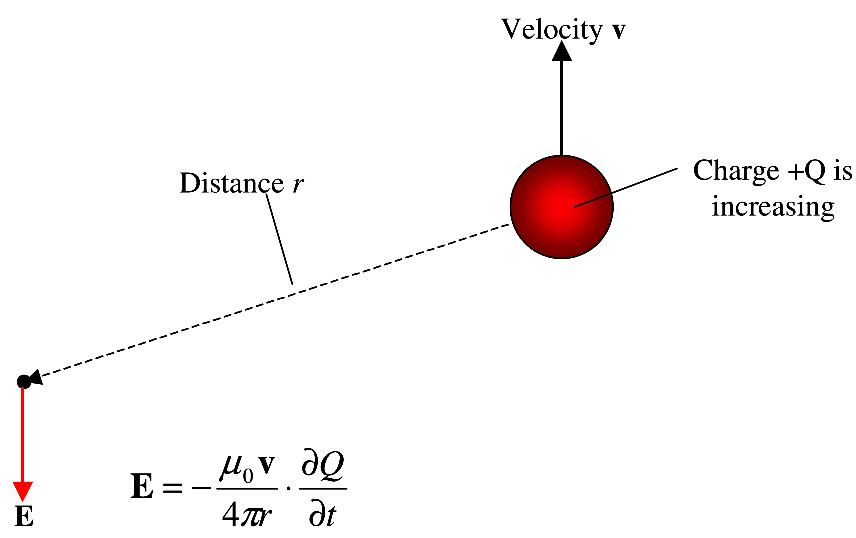

Figure 1. E field from moving sphere with changing charge.

If that Q changes value for only a short period of time Δt during which it has moved only a short distance that is small compared to r, we can assume that r remains constant during Δt then (3) yields the magnitude of an electric impulse. Let Q change by an amount ΔQ in the time Δt. Then the electric impulse $E{\cdot}\Delta t$ is given by

$$E \cdot \Delta t = -\frac{\mu_0 v}{4\pi r} \cdot \Delta Q \qquad (4)$$

Now consider a sphere that travels round a circular orbit at velocity v as shown in figure 2. It carries charge Q that alternates in polarity twice per revolution, that change of polarity being a fast switching-action taking place at diametrically opposite positions. Thus at the switching point $\Delta Q = 2Q$. The electric impulses seen at the distant point P depicted in figure 2 as given by (4) are now all unidirectional. Of course these are superimposed onto the Coulomb field from the sphere, but that alternates and it follows a different range law. It is not difficult to conceive of a system of multiple spheres where the Coulomb fields cancel but the impulse fields do not.

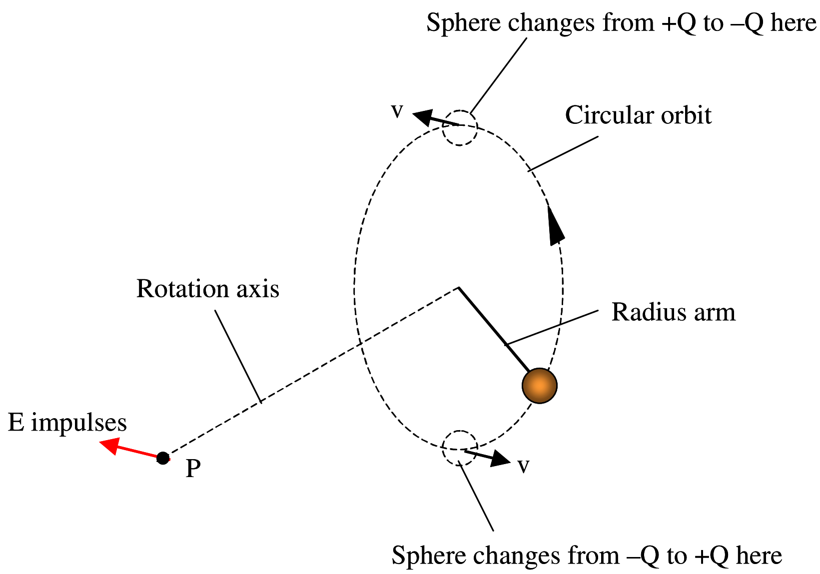

Figure 2. Switched sphere with circular orbit.

The distant point also sees an **A** field from the sphere movement through each semi-circle where the charge is constant, but this results in an **A** field vector that has constant amplitude between impulses but it rotates. If we take the two Cartesian coordinates in the plane of rotation of that vector, then differentiate those with respect to time, it could be argued that this results in a rotating **E** vector that is at right angles to the **A** vector. Were that the case, we would find that the mean DC value of the electric impulses is negated by the mean DC value of the interposing half sine waves. However there is the possibility that such a rotating **A** field does not yield an **E** field, i.e. the **A** field has to change magnitude with time in order to create an **E** field. For example it is known that electrons in fixed orbits do not radiate, yet they do produce a rotating **A** field. Another pointer to this possibility is the limitation placed on the gradient function for the **E** field arising from the scalar product $v \cdot \mathbf{A}$, where the gradient only applies to **A** and not *v*. Assuming the rotating **A** vector does not produce an electric field, then we have just the unidirectional impulses with zero field between successive impulses. This leaves us with the satisfying feature that the impulses have a mean DC value, we have discovered a method for radiating a DC electric field. This has enormous implication since it unlocks the means for extracting energy from the persistent electron spins and orbits that create ferromagnetism.

A more sensible arrangement has a sphere at each end of a rod, and means for transferring charge from one sphere to the other such as via a resonant inductor, figure 3. This doubles the impulse rate. This can then evolve into multiple spheres set around a rotating disc yielding an even greater impulse rate.

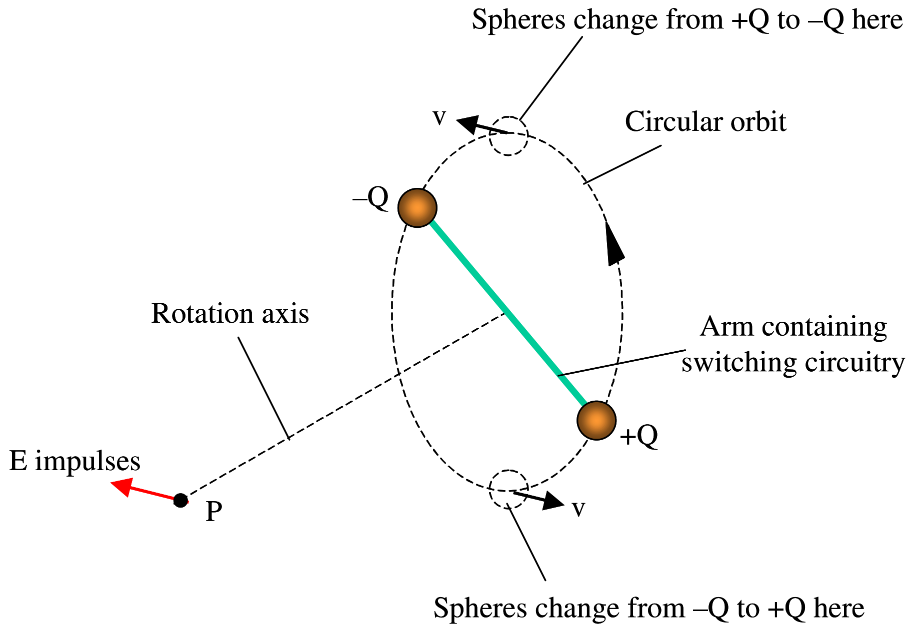

Figure 3. Two spheres

2. **Using the radiation to extract energy**

We now wish to consider how the unidirectional **E** field pulses can extract energy from a permanent magnet. We start by considering an arbitrary closed current loop driven by a constant current generator *I*. Let this loop create an **A** field at some arbitrary position where our sphere is placed, as shown in Figure 4.

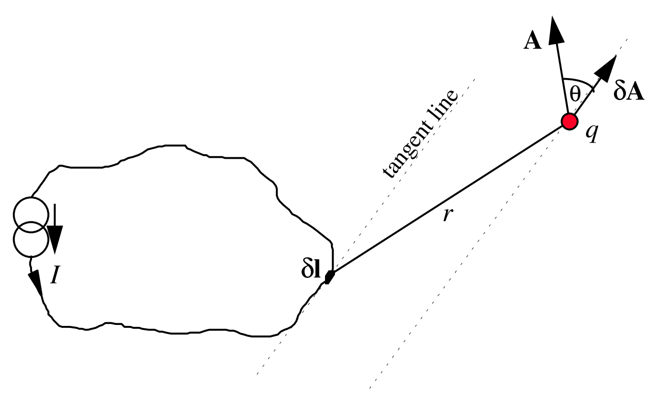

Figure 4.

Take a small element of length **δl** of the loop creating the differential field component δA at our sphere. **δA** lies parallel to **δl** and is given by

$$\delta\mathbf{A} = \frac{\mu_0 I \delta\mathbf{l}}{4\pi r} \tag{5}$$

where *r* is the distance from the current element to the field point. Note that **δA** does not necessarily lie along **A**, let the angle between **δA** and **A** be θ. Thus **δA** contributes a value δAcosθ to the total field **A**, and we can express the **A** field magnitude at this point by the integral

$$A = |\mathbf{A}| = \frac{\mu_0 I}{4\pi} \oint \frac{\cos\theta}{r} \cdot dl \tag{6}$$

where θ and r are variables of the integration. We cannot evaluate this integral for an arbitrary loop, fortunately we don't have to.

Now let our sphere's charge q be changing with time while it is moving at velocity v along the **A** field direction, see Figure 5.

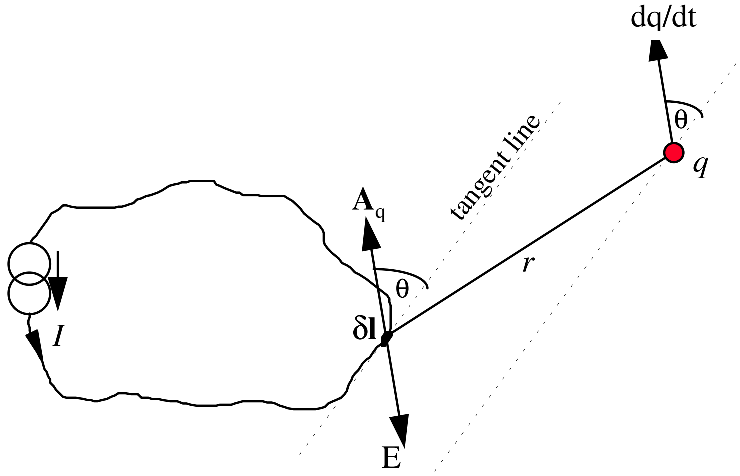

Figure 5

We wish to evaluate the voltage induced into the current loop due to the moving sphere, and we start by finding the voltage induced into the element **δl**. We find that the moving charge produces an **E** field at **δl** as given by (3). This **E** field induces a differential voltage δV along δl

$$\delta V = -\frac{\mu_0 v}{4\pi} \frac{dq}{dt} \frac{\cos\theta}{r} \tag{7}$$

giving the total voltage induced into the loop as

$$V = -\frac{\mu_0 v}{4\pi} \frac{dq}{dt} \oint \frac{\cos\theta}{r} \cdot dl \tag{8}$$

The integral part of this equation is seen to be identical to that in (6), so combining (6) with (8) eliminates the integral and yields

$$V = -\frac{vA}{I} \frac{dq}{dt} \tag{9}$$

This voltage appears across the current generator, indicating a power flow P which we can express by rearranging (9) as

$$P = VI = -vA \frac{dq}{dt} \tag{10}$$

Thus movement of the charge q along the ***A*** *field direction combined with its changing value has caused a power flow at the energy source which created the* ***A*** *field. This power flow can be negative or positive depending on the velocity direction, with or against the* ***A*** *field, and the polarity of the change in charge.*

3. **Where does this power flow to or from?**

The charge q is known to have "hidden" momentum $q\mathbf{A}$ that has been called electro-kinetic momentum. Since q is changing with time then so is its hidden momentum, and the rate-of-change of momentum results in a force $F = A\dfrac{dq}{dt}$ that is either applied to overcome the changing momentum or is delivered by the changing momentum. The force is moving at velocity v so the mechanical power being delivered to or extracted from that force is $P = vA\dfrac{dq}{dt}$ which is exactly the power flowing at the coil given by (10). The negative sign in (10) represents power flowing from the coil into its power supply, which is the case for the velocity along the **A** field direction while *dq/dt* is positive, that power being delivered mechanically to drive the sphere. The whole system obeys conservation of energy. Reversing either v or *dq/dt* creates the opposite effect, the sphere obtains a force so as to deliver power mechanically, that power being extracted from the electrical source driving the loop. If we re-place our arbitrary current loop with a permanent magnet we have the ability to extract power from the quantum forces that drive the electron spins and orbits that are responsible for the magnetism.

4. **An over-unity electrostatic motor.**

Figure 6 shows a form of electrostatic motor. Multiple spheres are placed around the periphery of an insulated disc. Two fixed spheres are placed at diametrically opposite positions with small whisker brushes that make contact with the moving ones. The fixed spheres are connected to a high voltage DC source so that at each passing sphere obtains positive or negative charge. Thus when the disc is rotated all the spheres on one side get charged oppositely to those on the other side, yielding repulsion and attraction to the fixed spheres that drives the rotation. Each time a moving sphere makes contact with a whisker its charge gets reversed and a quantity of charge is supplied from the DC source. The net DC current of those charge impulses extracts power from the DC source, and this accounts for the power delivered to the output shaft. Now place the motor in a static uniform **A** field. At the contact points the moving charge gets a force impulse that supports the rotation, and as described previously the extra power delivered to the shaft is taken from the source of that **A** field. If the **A** field is that of the Earth the motor will appear to be over-unity if the reaction on the Earth's magnetic core is ignored.

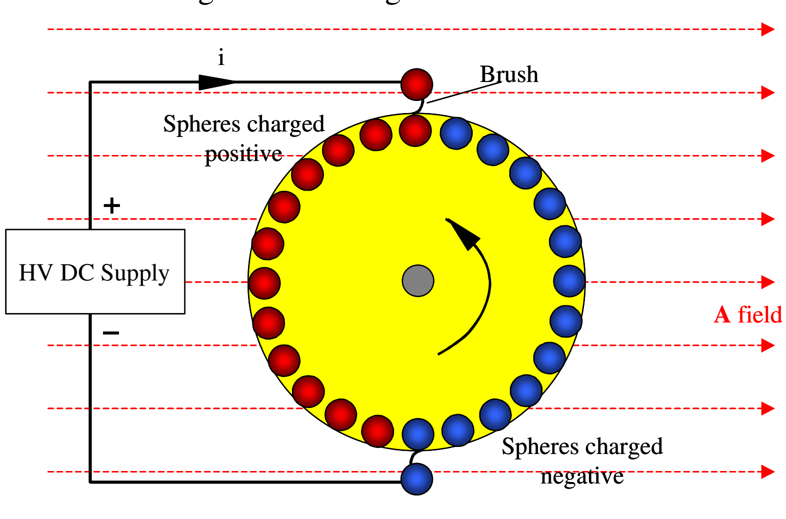

Figure 6. Electrostatic motor

A more practical version of the electrostatic motor could increase the self-capacitance of each moving body (that limits the amount of charge for a given voltage) by using mutual capacitance. This could use planar electrodes on a thin disc of high dielectric constant material that rotates close to a conducting ground plane, figure 7.

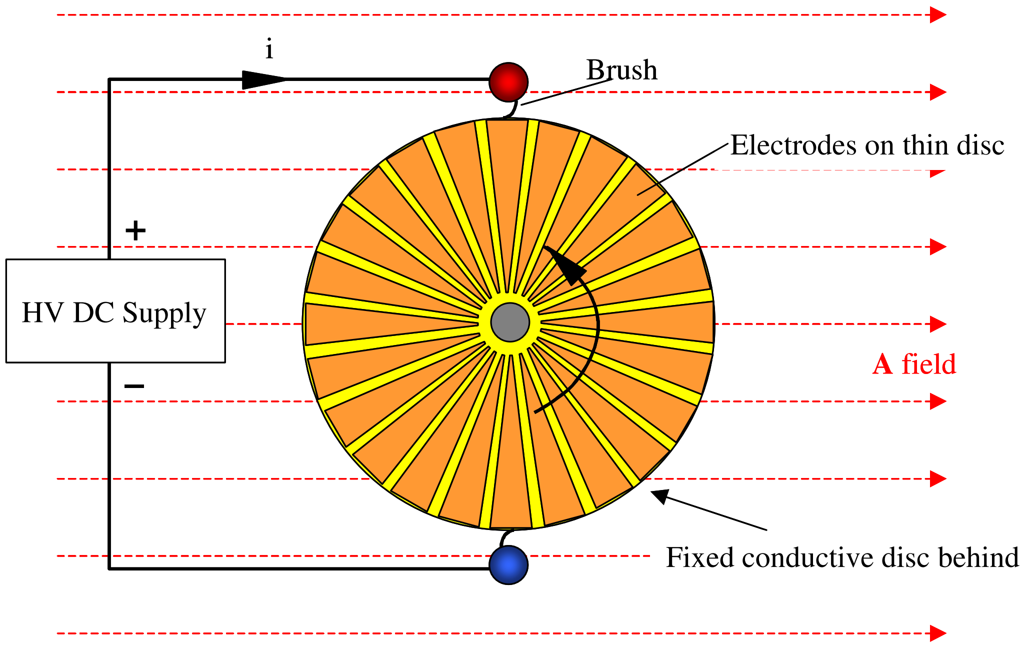

Figure 7. Motor with greater capacitance

The electrodes on the Swiss Testatika machine are of this form, perhaps that machine used this principle.