Discovery of a Lost Electrodynamic Force.

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1. Introduction

I have long been interested in the role of the magnetic vector potential and its application to our knowledge of electrodynamics. I have written several papers discussing its possible use for new forms of energy production that some people would call fringe science. Over the years I have studied many papers on the subject, more recently examining so-called hidden momentum. For those interested in doing the same a Google search for “hidden momentum” plus “magnetic vector potential” will reveal those texts of interest. One in particular [1] has presented the subject in a manner that has led me to an interesting discovery of a “new” electrodynamic force. It is not really new, it may have been known in the late 19th and early 20th centuries when the classical laws of electromagnetism were being developed. The pioneers of those early days used word such as “electrotonic” and “electrokinetic”, expressions that have since been abandoned. Not only have the words been abandoned, but also the methodology used by them to arrive at their conclusions.

In this paper I re-examine the Lorentz force written as a function of the potentials used by those early scientists to arrive at the classical electrodynamic forces in use today. In that exercise their “hidden” electrokinetic momentum and one part of their electrokinetic potential both disappear, leading to the belief that they play no part in modern theory. However that only applies to a body (such as a particle) that has constant mass and constant charge. It is possible to have a body (such as a charged sphere) where the charge is not constant, and in that situation the classical formula for electrodynamic force on the sphere has a missing term. That lost term has great significance because it opens the door to extracting energy from the source of permanent magnetism, be it the spins and orbits in ferromagnetism or the dynamo action of the Earth’s core.

2. The Lorentz force on a free charge

Rousseaux [1] states:

The Lorentz force can be rewritten as a function of the potentials:

$$\frac{d}{dt}(m\mathbf{v} + q\mathbf{A}) = -q\nabla(\phi - \mathbf{v}\cdot\mathbf{A}) \tag{1}$$

with:

$$\frac{d\mathbf{A}}{dt} = \frac{\partial\mathbf{A}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{A} \tag{2}$$

where ∇ *applies only to* $\mathbf{A}(\mathbf{r},t)$ *and not* $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$.

[Note: Bold characters represent vectors. I have used ϕ as the Coulomb potential in place of his *V*. It is not clear as to which equation the limitation on ∇ applies, but examination of the components of $(\mathbf{v}\cdot\nabla)\mathbf{A}$ shows there are no spatial gradients of velocity, hence it must apply to only the $\nabla(\mathbf{v}\cdot\mathbf{A})$ in (1).]

The LH side of (1) is a force expressed as rate-of-change of momentum where that momentum is the sum of mechanical momentum $m\mathbf{v}$ and electro-kinetic momentum $q\mathbf{A}$. The RH side is the electromagnetic force on charge q expressed as the gradient of the electro-kinetic potential of early teachings, that potential being the sum of the Coulomb potential ϕ and the scalar product $-\mathbf{v}\cdot\mathbf{A}$.

Putting (2) into (1) yields

$$m\frac{d\mathbf{v}}{dt}+q\frac{\partial\mathbf{A}}{\partial t}+q(\mathbf{v}\cdot\nabla)\mathbf{A}=-q\nabla\phi+q\nabla(\mathbf{v}\cdot\mathbf{A}) \qquad (3)$$

It has been shown that we can use the vector substitution

$$(\mathbf{v}\cdot\nabla)\mathbf{A}=-\mathbf{v}\times\mathbf{B}+\nabla_A(\mathbf{v}\cdot\mathbf{A}) \qquad (4)$$

where the subscript $_A$ denotes the limitation on ∇ above. Putting this into (3) and rearranging terms gives

$$m\frac{d\mathbf{v}}{dt}=q\left(-\nabla\phi-\frac{\partial\mathbf{A}}{\partial t}+\mathbf{v}\times\mathbf{B}\right) \qquad (5)$$

This is the classical expression for a free charge under the influence of an electric field, a time-changing vector magnetic potential and a magnetic field (respectively the three terms in the RH side). Note that the $\mathbf{v}\cdot\mathbf{A}$ scalar product and the $q\mathbf{A}$ electrokinetic momentum no longer appears in (5). Thus we have two methods for calculating force on a free charge, (1) and (5), that are equivalent. Not surprising then that most modern texts quote (5) as the defining equation and make no mention of the bothersome electro-kinetic momentum or potential that featured so much in the early development of electromagnetic theory. However note that both (1) and (5) apply to a particle of fixed mass m and fixed charge q. *If we now have a body of fixed mass, but where the charge is not constant (such as variable charge on a sphere), equations (1) and (5) tell quite different stories*. Of significance is the fact that the rate-of-change of electro-kinetic momentum represents a genuine force, now given by

$$\mathbf{F}=\frac{d}{dt}(q\mathbf{A})=\mathbf{A}\frac{dq}{dt}+q\frac{d\mathbf{A}}{dt} \qquad (6)$$

to appear in the LH side of (1). The substitutions (2) and (4) now only apply to the second term in (6), the first term remains, whereupon (5) now becomes

$$m\frac{d\mathbf{v}}{dt}=q\left(-\nabla\phi-\frac{\partial\mathbf{A}}{\partial t}+\mathbf{v}\times\mathbf{B}\right)-\mathbf{A}\frac{dq}{dt} \qquad (7)$$

The discarding of the electro-kinetic teachings of the 19th and early 20th centuries has kept hidden that fourth term in (7). If we take the case where there is no electric field present, where $\mathbf{A}$ is spatially uniform and static, then (7) becomes

$$m\frac{d\mathbf{v}}{dt}=\mathbf{A}\frac{dq}{dt} \qquad (8)$$

Clearly this can be tested experimentally. The Earth's **A** field is very strong and is reasonably uniform in the laboratory (its small non-uniformity accounts for the weak magnetic field). A lightweight body that obtains a rapid change of charge should endure a force impulse aligned with the **A** field (E-W).

3. References

[1] "On the interaction between a current density and a vector potential: Ampère force, Helmholtz tension and Larmor torque." By Germain Rousseaux. Pacs Number : 03.50.De ; 41.20.-q.