A Possible Means for Extracting Energy from the Earth's Magnetic Vector Potential

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1. Introduction

The magnetic field of the Earth (here taken to be truly spherical) can be considered to emanate from a magnetic dipole of magnitude m=8.24×10^{22} Am^{2} at its centre [1]. Although the magnetic field at the surface is rather weak, the same cannot be said for the magnetic scalar and vector potentials. The huge magnetic scalar potential is considered in my paper [9] where the possibility exists to use it for gaining electro-magnetic thrust, and possibly also overunity systems. This present paper looks at the magnetic vector potential.

The formula for the magnetic vector potential **A** at the Earth's surface is

$$\mathbf{A} = \frac{\mu_0 m \cos\theta}{4\pi r^2} \qquad (1)$$

where r is the radius (6.37×10^{6} m) and θ is the latitude. In this paper we use bold script to denote a vector, and the vector **A** everywhere points towards the west. At the equator **A**=203 Weber/m while at the poles **A**=0. Note that 203Weber/m is a huge value unlikely to be seen in a laboratory (to get an appreciation of this you would find that value of **A** field at the cylindrical surface of a huge fully-magnetized piece of iron that is 400m in diameter and much greater than 400m long!). Surprisingly, few people are aware of this aspect of the Earth's magnetism and little attention has been given to the possibility of employing this as a useful resource. Perhaps the reason for this is current scientific dogma that the magnetic **B** and **H** fields are the primary fields responsible for magnetic effects, the **A** field being merely a mathematical artefact used for calculation purposes. This is not the only paper to challenge that position and consider possible means for extracting energy via any **A** field, but is the first to offer means for using the Earth's **A** field. For simplicity the **A** field is considered to be spatially uniform in the laboratory, its actual non-uniformity (that produces the Earth's relatively small magnetic **B** field values) is ignored.

2. The role of the A field in classical voltage induction.

Classical electrodynamics recognizes two forms of voltage induction into a conductor associated with magnetic fields, transformer induction and motional induction.

- Transformer induction occurs when both the source of the magnetic field and the conductor are stationary, but the magnetic field changes with time. In typical transformers the magnetic field is confined within a core, while the conductor is outside the core where there is no magnetic field. This absence of any magnetic field *at the conductor* demonstrates that such fields cannot be considered primary when dealing with induction effects. The induction is explained by the presence of the magnetic vector potential **A** outside the core. **A** is also changing with time giving rise to an electric field **E** where

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \qquad (2)$$

  and it is this **E** field that actually causes the induction, that drives the mobile conduction electrons in the conductor. Because the **A** vectors (and therefore also **E** vectors) form concentric loops around the core, where for any loop the instantaneous closed path integral is equal to the instantaneous magnetic flux Φ in the core, it has become common practice to denote the induction as a voltage for a single turn which is equal to $\frac{\mathrm{d}\Phi}{\mathrm{d}t}$ (volts/turn), thus hiding the real cause for the induction, the **A** field. Note that (2) is a partial derivative that contains only time increments ∂t, there are no spatial increments.

- Motional induction occurs when there is relative motion between a conductor and the source of the magnetic field. This is expressed as an induction **E** field created by vector multiplication of the velocity vector **v** with the field vector **B**,
$$\mathbf{E} = \mathbf{v} \times \mathbf{B} \tag{3}$$
This **E** field drives the conduction electrons to produce *current* in the conductor. Most people are familiar with the case when the three vectors are all at right angles to each other as expressed by Fleming's right hand rule for induced *current*, but that teaching omits to point out that the current along the conductor is the result of an induced electric field (3). This form of induction is also called flux-cutting where the conductor is considered to "cut" the lines of magnetic force. It is unfortunate that this perception hides the role of the vector potential **A** field in this induction, see next section.

- There is much debate about a third form of induction that can be derived from (2) if instead of a partial derivative it becomes a full derivative to take account of variations in **A** as "seen" by an electron moving through an **A** field. This time-variation can come from (a) variations in electron velocity magnitude, (b) variations in electron velocity direction, (c) spatial variations in **A** magnitude, (d) spatial variations in **A** direction and (e) any combinations of these. Several authors [2] [3] have shown that this full approach yields the motional induction (3) plus another term
$\mathbf{E} = -\nabla_A(\mathbf{v}\cdot\mathbf{A})$ also written as $E = -\mathbf{grad}_A(\mathbf{v}\cdot\mathbf{A})$ (4)
where the gradient is applied only to spatial variations in **A**, i.e. any spatial variations in velocity are suppressed (hence the subscript $_A$). Here **v•A** is the scalar product of the two vectors, which is simply the product of the electron velocity value v with the component of **A** along the velocity direction. Note that **v•A** has dimensions of volts, hence it represents an electric potential that varies throughout space and, just like the well-known Coulomb potential, the spatial gradient (4) is an electric field. The fact that motional induction (3) can be derived directly from the full time-derivative of the **A** field emphasises the role of that field in *all* forms of induction: the "flux-cutting" perspective is simply a useful visual tool and not necessarily the primary aspect of that induction.

3. Why use suppressed terms?

The full Cartesian components of $\mathbf{E} = \nabla(\mathbf{v}\cdot\mathbf{A})$ (i.e. without velocity derivatives suppressed) are:

$$\begin{aligned}
E_x &= \left[v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_y}{\partial x} + v_z \frac{\partial A_z}{\partial x} + A_x \frac{\partial v_x}{\partial x} + A_y \frac{\partial v_y}{\partial x} + A_z \frac{\partial v_z}{\partial x} \right] \\
E_y &= \left[v_x \frac{\partial A_x}{\partial y} + v_y \frac{\partial A_y}{\partial y} + v_z \frac{\partial A_z}{\partial y} + A_x \frac{\partial v_x}{\partial y} + A_y \frac{\partial v_y}{\partial y} + A_z \frac{\partial v_z}{\partial y} \right] \\
E_z &= \left[v_x \frac{\partial A_x}{\partial z} + v_y \frac{\partial A_y}{\partial z} + v_z \frac{\partial A_z}{\partial z} + A_x \frac{\partial v_x}{\partial z} + A_y \frac{\partial v_y}{\partial z} + A_z \frac{\partial v_z}{\partial z} \right]
\end{aligned} \tag{5}$$

Note the first three terms in each parenthesis deal with spatial variations in the **A** field whilst the remaining terms deal with spatial changing velocity, and these latter terms are suppressed in (4). For an almost uniform **A** field (here to be along the *x* direction) such as that of the Earth as seen in the restricted dimensions of the laboratory, the spatial variations in **A** are all zero as are A_y and A_z, then we can simplify (5) to

$$E_x = \left[A_x \frac{\partial v_x}{\partial x} \right], \quad E_y = \left[A_x \frac{\partial v_x}{\partial y} \right], \quad E_z = \left[A_x \frac{\partial v_x}{\partial z} \right] \tag{6}$$

Only terms with velocity derivatives survive and these are the very terms that are suppressed in (4). It appears that the only reason for that suppression is that the full derivative of the **A** field is taken as

$$\frac{D\mathbf{A}}{Dt} = \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{A} \tag{7}$$

then (2), (3) and (4) follow from the vector identity

$$\mathbf{v} \cdot \nabla \mathbf{A} = \mathbf{v} \times \mathbf{B} - \nabla_A (\mathbf{v} \cdot \mathbf{A}) \tag{8}$$

It seems wrong to formulate EM theory to satisfy vector math identities especially when it leads to the absurd situation where a moving electron, undergoing a 180 degree change of direction within an **A** field, moves from a positive potential to a negative one (or vice versa) without the presence of an electric force-field. If the potential **v·A** has any validity then that movement must be through an effective **E** field, not as given by (4) having suppressed terms, but as given by (5), or in the case of a uniform **A** field, by (6). For the remainder of this paper we will assume that this is the case.

4. What evidence is there?

Sommerfeld [4] points out that in 1903 Schwarzschild introduced his "electrokinetic potential" $L = (\phi - \mathbf{v} \cdot \mathbf{A})$ where ϕ is the Coulomb potential. Wu & Yang [8] give a historical overview of the vector potential showing that Larmor's 1895 version of the kinetic energy of a moving electron included the energy *e*(**v·A**). So it is over 120 years since **v·A** was recognized as a potential. Schwarzschild's electrokinetic potential *L* is really a *potential difference* (note the minus sign) which when multiplied by the charge density forms a relavistic invariant, which was important in the development of his Principle of Least Action. The mathematical intricacies of that development are of little interest to engineers, they are interested in electric and magnetic forces that can do work. The **E** field (5) is just that, hence it would be more sensible if the electrokinetic potential capable of doing work in a fixed laboratory frame were the *sum* $(\phi + \mathbf{v} \cdot \mathbf{A})$ and not the difference. Thus, ignoring for the moment the *electric* (Coulomb) potential ϕ, an electron moving at velocity **v** through an **A** field can be considered to have an *electro-kinetic* potential **v.A**, or an *electro-kinetic energy e*(**v.A**). This is in addition to its accepted kinetic energy associated with its mass. Note this is a maximum when **v** is parallel to **A**, when the electron travels along the **A** field, and has a value *evA*.

In 1959 Aharanov & Bohm discovered that electrons moving through a static but spatially non-uniform **A** field were effected by that field. Their experiment involved an electron beam that split to travel along different paths then recombined to produce an interference pattern. The **A** field created a differential phase effect between the two beams, which became known as the Aharanov-Bohm effect. Current dogma assumes this effect is just a quantum interference effect having no significance in regard to energy extraction. However Spavieri & Rodriguez [4] have shown that this effect may have a classical origin whereby an additional force related to the static **A** field may be present on a moving electron. It is quoted as a longitudinal (along the velocity direction) force and involves the longitudinal component of the gradient of the scalar product of the vector velocity **v** and magnetic vector potential field **A** as given by

$$\mathbf{F}_{\mathrm{L}} = -e\nabla(\mathbf{v}\cdot\mathbf{A}) \qquad (9)$$

This is simply the electric field (4), without any suppressed terms, applied to the electron charge. This force creates an em phase lag that exactly accounts for the Aharonov-Bohm effect, so the Aharonov-Bohm measurements could be considered as evidence that these longitudinal forces, and the **v•A** potentials, really do exist. However in view of the fact that for any closed path (9) integrates to zero its use in energy production has not been considered feasible.

Marinov [5] mentions a cathode ray experiment by Solunin Kostin where a toroidal coil was placed around the neck of a cathode ray tube. When the electron beam was given some deflection by the usual CRT means it was found possible to alter the amount of deflection by varying the DC current in the toroidal coil. Thus the electron beam velocity was altered by that current. A toroidal coil produces only an **A** field at its centre, hence here again we have a demonstration of a static **A** field applying longitudinal force to moving electrons and changing their velocity.

Carpenter [6] and Murgatroyd [7] both discuss E.G. Cullwicks "anomaly in electromagnetics" where an electric charge is fired through a toroidal coil carrying constant current where only an **A** field exists along the axis. According to current theories, as the charge moves along the axis it experiences no magnetic or induced force whatever, whereas it exerts a magnetic force of repulsion on the toroid. Where is the reaction to that force? In order to resolve this dilemma they both use (Schwartzchild's) electro-kinetic potential to calculate the changing momentum of the charge resulting in the charge enduring longitudinal forces as it moves along the **A** field, and show that overall momentum is preserved.

It can be concluded that there is a reasonable body of evidence that longitudinal forces induced by movement through an **A** field really do exist, but they have been considered to be of no importance because they come from the gradient of a scalar potential where for any closed path they integrate to zero. That null is a mathematical certainty, it cannot be challenged, but the next section highlights the difference between a closed path and a closed circuit.

5. Closed paths and closed circuits.

When dealing with closed path integrals through a scalar field there is a subtle difference between that for the Coulomb potential ϕ and that for the scalar product $(\mathbf{v}\cdot\mathbf{A})$. For a given source, the potentials ϕ will have set values at any point in space, those values will not depend on any characteristic of the device used to sense them. It therefore makes sense to consider the presence of a "field" of those scalar values throughout space. It is the math for this scalar "field" that cannot be challenged. Such a "field" of scalar values is not true for $(\mathbf{v}\cdot\mathbf{A})$ since its value at any point depends on the velocity of the observer. If the observer is not moving there is no potential.

In our considerations the observer is an electron. In both cases, if the electron follows a closed *path* the net force on it integrates to zero, we have a closed *circuit* that cannot extract electrical energy from the potentials. This identification of a mathematical *closed path* with an electrical *closed circuit* is the invisible stumbling block that we have unwittingly stumbled over. *A closed electrical circuit does not necessarily require electrons to traverse the full closed path*. For example if part of that path involves Maxwell's displacement current we can have current flowing around a closed circuit but the electron flow is discontinuous. In the case of the scalar potential $(\mathbf{v}\cdot\mathbf{A})$ we have the ability to use a discontinuous path where the moving electron gains potential from the **A** field along one part of its path, then transfers some of that energy via electromagnetic means to another electron that can follow a different path that includes the load. *That different path need not negate the potential gained from the **A** field in the first place.*

Consider electrons at their maximum EK potential, i.e. they are travelling along the **A** field. To extract energy from a moving electron via its EK potential it is important that the electron does not lose that potential before it reaches the transfer point. That loss of potential occurs if (a) the electron loses velocity when it reaches a capacitor plate to then become some quasi-static charge, or (b) the electron moves onto a take-off path at right angles, where in both cases $(\mathbf{v}\cdot\mathbf{A})$ drops to zero. Overcoming this problem requires careful consideration of the circuit geometry and leads to the use of unusual and seemingly strange three-terminal capacitors, where two terminals connect to opposite ends of the same electrode so that current can be maintained along that electrode. The surfeit of electrons that represent the charge on that capacitor plate are not stationary, they retain their velocity hence also their $(\mathbf{v}\cdot\mathbf{A})$ potential. The third terminal of course connects to the other electrode where, because of the unusual circuit geometry, change of velocity is not a consideration.

6. More on $(\mathbf{v}\cdot\mathbf{A})$ and $\nabla(\mathbf{v}\cdot\mathbf{A})$

We are interested in drift velocity of electrons along a stationary filamentary wire, see section 7 for more details. The spatial variations in electron velocity can come from classical linear acceleration (or deceleration) along the wire or from constant velocity around a curved wire. For a filamentary conductor the electron drift velocity direction is along the wire, hence $\mathbf{v_d}\cdot\mathbf{A}$ is simply the component of **A** as measured along the wire multiplied by the electron drift velocity (to emphasise we are dealing with drift velocity we have added the subscript $_\mathbf{d}$). As its name implies this is a scalar that has the units of volts, it is a scalar electric potential. Equation (5) or (6) yields the effective **E** field along the wire (whatever contour it follows) and $\mathbf{v_d}\cdot\mathbf{A}$ at any point along the wire is the induced potential. It follows that the voltage induced across a finite length of conductor of uniform cross section is given by the difference in potential $\mathbf{v_d}\cdot\mathbf{A}$ evaluated at the far end from that evaluated at the beginning, and this is independent of the route followed by the wire. It also follows that for a closed loop, where start and end points are the same, the induced voltage is zero, hence there is zero induced current into the loop. However we can deliberately drive current around a closed loop to obtain variations in the EK potential $\mathbf{v_d}\cdot\mathbf{A}$ at different points around the loop. This applies even when **A** is uniform leading to the unusual situation where the drift velocities or currents around the loop may not be uniform. This seemingly is a violation of Kirchoff's Law!

Variation in current and potential along a wire is a well-known phenomenon in standing waves where the effect is produced by em waves propagating in opposite directions. There the potential maxima coincide with current (hence drift velocity) minima, for 100% standing waves the drift velocity is zero at the potential maxima. So we have quasi-static surface electrons at the negative potential maxima that explain the seeming violation of Kirchoff's Law. These can be bled off into a load via connections to the wire. As stated previously we cannot do this in our loop, we must maintain the electron velocity at the potential maxima. The Coulomb field from the surfeit of moving electrons is present and if we deliberately cause the EK potential to be alternating energy can be extracted via Maxwell's displacement current. By capacitively coupling to the wire as shown in figure 1, AC current can flow into the load without any alteration to the velocity direction of the electrons in the wire. That extraction of power can result in change of the electron velocity magnitudes in the wire, but this is simply using the induced field as the source of energy whereby the load reacts on that source. In effect the presence of the **A** field supplies force to push the moving electrons along the curved trajectory while the presence of the load supplies some reaction to push.

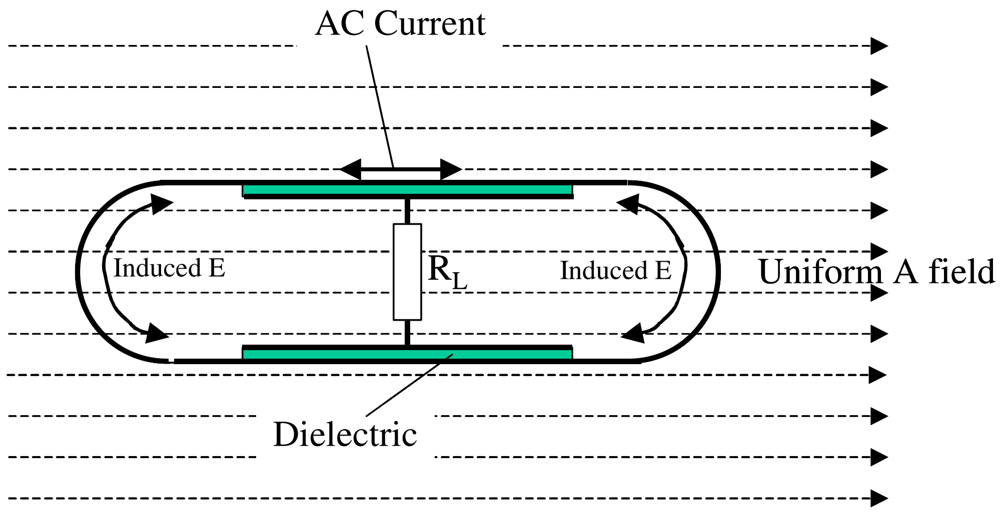

Figure 4. Closed loop experiment.

Here we have a closed loop of current within the **A** field, and the electro-kinetic potentials induced create charge on the straight sections that couple via dielectric to other electrodes. If the current in the loop is alternating then we should see an alternating voltage across the resistor. *Note that we cannot simply connect capacitors to the top and bottom of the loop* because in those capacitors the charge on the plates becomes stationary, and that implies deceleration which negates the induced E field. *We must have current flowing along one electrode of each capacitor*. If we wish to use classical techniques for obtaining high value capacity then we need specially constructed capacitors to allow this situation to occur.

Figure 2 shows the situation at the peak of one half cycle of current with of course reversed polarity for the other half cycle.

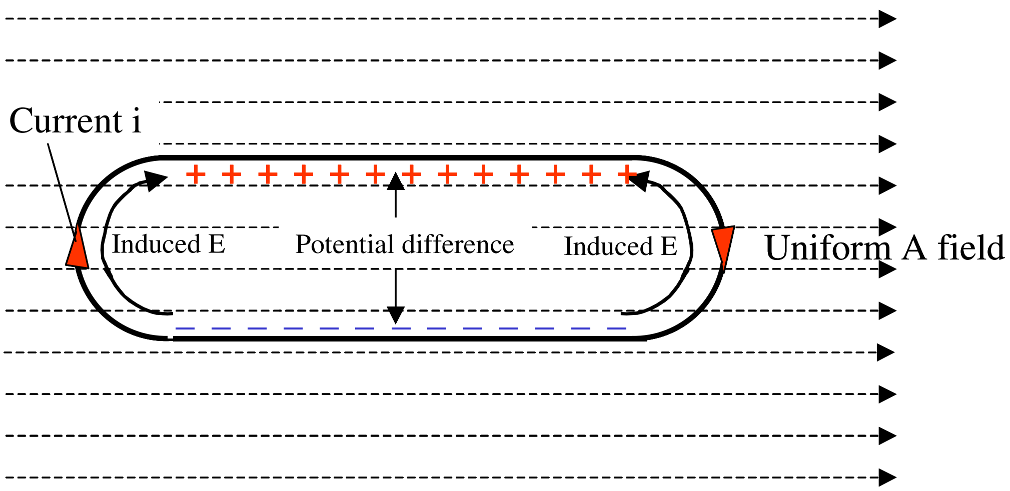

Figure 2. One half cycle peak

A practical realization of this experiment is shown in figure 3. The input has to drive current in the loop that will appear as a short circuit to the source, but we should see the voltage induced from the Earth's **A** field. The straight sections must lie on the E-W axis and reversing the whole system should create a 180° phase reversal of that induced voltage.

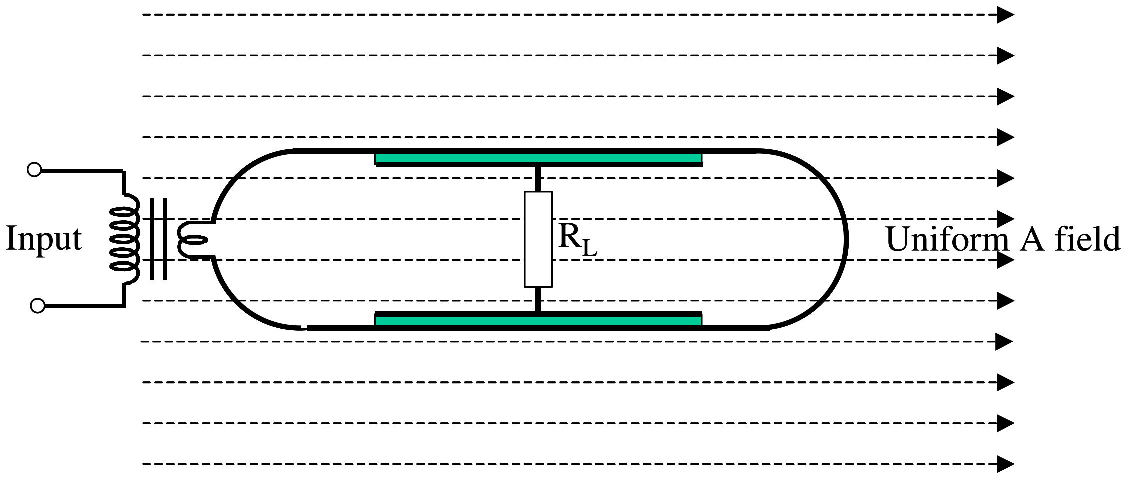

Figure 3. Practical experiment

If this experiment gives a positive result it will be the first conclusive evidence that energy can be extracted via the vector magnetic potential. Also it provides the first ever instrument for measuring that vector potential. And it offers potential as a means of creating an electronic compass.

7. Drift velocity considerations

The volumetric number density N_D of atoms in a material is given by

$$N_D = \frac{N_A \rho}{W_A} \text{ per cm}^3 \tag{10}$$

where N_A is Avagrado's number, ρ is the density and W_A the atomic weight. For copper ρ is 8.92 g/cm^3, W_A is 63.54 and Avagrado's number is 6.025×10^{23}. Thus 1 cm^3 of copper contains 8.46×10^{22} atoms. With one conduction electron per atom that same number applies to the electron density. For a wire of diameter d mm carrying a current i the drift velocity v_d is given by

$$v_d = \frac{4000i}{\pi e N_D d^2} \text{ mm/s} \tag{11}$$

where e is the electron charge 1.602×10^{-19} Coulombs.
Thus a copper wire 1mm diameter carrying 1 amp has a drift velocity of 9.39×10^{-2} mm/s, or 9.39×10^{-5} m/s in SI units. Using that SI value against typical laboratory vector potential differences we get trivial induced potentials of only fractions of a micro-volt. Such tiny voltage is unlikely to be of any use. However in the Earth's equatorial vector field varying tangentially from +200 to –200 Weber/m a U shaped wire would have an induced potential of 37.6 milli-volts which is much more respectable. Since that voltage is for a current of 1 amp the induction can be represented by a negative resistor of magnitude 37.6 milliohms but note this value applies to our 1mm diameter wire. In view of the simplicity of denoting the induction by a resistor value, enabling equivalent circuits to be created and solved, a useful formula for the induced resistance R_{ind} in U shaped copper wires of diameter d mm undergoing a 180 degree change from parallel to anti-parallel within a uniform vector potential field is

$$R_{ind} = \pm \frac{9 \cdot 4 \times 10^{-5} A}{d^2} \text{ Ohms} \tag{12}$$

where R_{ind} can be positive or negative depending on the initial parallel current direction relative to the **A** field. Note that this value may not apply to HF alternating current where skin effect will increase the drift velocity (for a given current) and therefore increase the value of R_{ind}. Figure 4 shows the equivalent circuit for the proposed experiment.

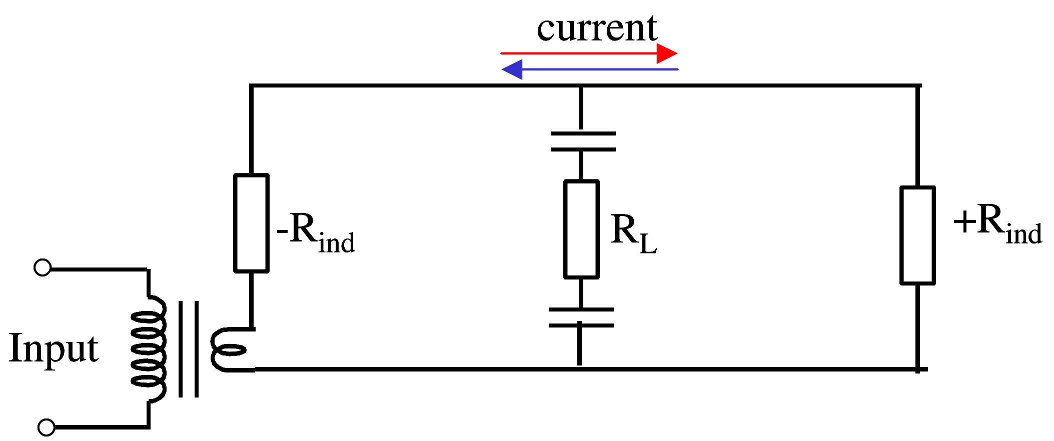

Figure 4. Equivalent Circuit

(Note that in this circuit we show capacitors as components connected by wires to the loop, and therefore current flows along these wires into the load R_L. However in the real circuit no electrons actually leave the loop, the current flow there is Maxwell's displacement current.)

The driver sees induced $+R_{ind}$ and $-R_{ind}$ in series hence sees only a short circuit that does not consume any power (we have ignored actual wire resistance here). Solving this circuit gives some indication of how the system can obtain power in its load R_L. If energy is gained from the **A** field then we should see OU with regard to the input power and the power dissipated in R_L.

8. Conclusions

For typical drift velocity of electrons in a copper wire their Electro-Kinetic (EK) potential in the Earth's magnetic vector potential **A** field is not trivial. This paper suggests how that EK potential could be measured. If the experiment proves successful it could provide the first ever instrument for measuring a static magnetic vector potential. It would also offer possibilities as a magnetic compass since the Earth's **A** field points E-W. There would also be the possibility of extracting energy from the Earth's field.

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