A Possible Means for Extracting Energy from the Earth's Magnetic Vector Potential

1. Introduction

The Earth's magnetic field can be considered to emanate from a magnetic dipole of magnitude m=8.24×10^{22} Am^2 at its centre. The formula for the magnetic vector potential **A** at the Earth's surface is

$$\mathbf{A} = \frac{\mu_0 m \cos\theta}{4\pi r^2} \qquad (1)$$

where r is the radius (6.37×10^6 m) and θ is the latitude. In this paper we use bold script to denote a vector, and the vector **A** everywhere points towards the west. At the equator **A**=203 weber/m while at the poles **A**=0. This paper considers possible means for extracting energy via that **A** field.

2. Voltage induction from a changing A field.

Classical electrodynamics recognizes two forms of voltage induction into a conductor associated with magnetic fields.

Transformer induction occurs when both the source of the magnetic field and the conductor are stationary, but the magnetic field changes with time. In typical transformers the magnetic field is confined within a core, while the conductor is outside the core, the induction then being explained by the presence of the magnetic vector potential **A** outside the core. **A** is also changing with time giving rise to an electric field **E** where

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \qquad (2)$$

and it is this **E** field that actually causes the induction, that drives the mobile conduction electrons in the conductor. Because the **A** vectors form concentric loops around the core, where for any loop the instantaneous closed path integral is equal to the instantaneous flux Φ in the core, it has become common practice to denote the induction as a voltage for a single turn which is equal to $\frac{\mathrm{d}\Phi}{\mathrm{d}t}$ (volts/turn) thus hiding the real cause for the induction, the **A** field. Note that (2) is a partial derivative that contains only time increments ∂t, there are no spatial increments.

Motional induction occurs when there is relative motion between a conductor and the source of the magnetic field. This is expressed as an induction **E** field created by vector multiplication of the velocity vector **v** with the field vector **B**,

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} \qquad (3)$$

Most people are familiar with the case when the three vectors are all at right angles to each other as expressed by Fleming's right hand rule. Note it is again the **E** field that drives the conduction electrons.

There is much debate about a third form of induction that can be derived from (2) if instead of a partial derivative it becomes a full derivative to take account of time variations in **A** as "seen" by an electron moving through an **A** field. This time variation can come from (a) variations in electron velocity magnitude, (b) variations in electron velocity direction, (c) spatial variations in **A** magnitude, (d) spatial variations in **A** direction and (e) any combinations of these. Several authors [1] [2] have shown that this full approach yields the motional induction (3) plus another term

$$\mathbf{E} = -\nabla(\mathbf{v}\cdot\mathbf{A}) = -\mathbf{grad}_A(\mathbf{v}\cdot\mathbf{A}) \qquad (4)$$

where the gradient is applied only to spatial variations in **A**, i.e. any spatial variations in velocity are suppressed (hence the subscript **A**). Here **v**•**A** is the scalar product of the two vectors, which is simply the product of the electron velocity **v** with the component of **A** along the velocity direction. For a filamentary conductor (e.g. a wire) the velocity direction is along the wire, hence $\mathbf{v_d}\cdot\mathbf{A}$ is simply the component of **A** as measured along the wire multiplied by the electron drift velocity (to emphasise we are dealing with drift velocity we have add the subscript 'd'). As its name implies this is a scalar that has the units of volts, it is a scalar potential. Equation (4) yields the effective **E** field along the wire (whatever contour it follows) and $\mathbf{v_d}\cdot\mathbf{A}$ at any point along the wire is the induced potential. It follows that the voltage induced across a finite length of conductor of uniform cross section is given by the difference in potential $\mathbf{v_d}\cdot\mathbf{A}$ evaluated at the end from that evaluated at the beginning, and this is independent of the route followed by the wire.

Sommerfeld [3] points out that in 1903 Schwarzschild introduced his "electrokinetic potential" $L = (\phi - \mathbf{v}\cdot\mathbf{A})$, so it is over 100 years since the scalar product **v**•**A** was recognized as a potential. Schwarzschild's electrokinetic potential *L* is really a *potential difference* (note the minus sign) which when multiplied by the charge density forms a relavistic invariant, which was important in the development of his Principle of Least Action. The mathematical intricacies of that development is of little interest to engineers, they are interested in electric and magnetic forces that can do work. The **E** field (4) is just that, hence it would be more sensible if the electrokinetic potential capable of doing work in a fixed laboratory frame were the *sum* $(\phi + \mathbf{v}\cdot\mathbf{A})$ and not the difference. Thus, ignoring the *electric* (Coulomb) potential ϕ, an electron moving at velocity **v** through an **A** field can be considered to have an *electro-kinetic* potential **v.A**, or an *electro-kinetic energy* $e(\mathbf{v}.\mathbf{A})$. This is in addition to its accepted kinetic energy associated with its mass. Note this is a maximum when **v** is parallel to **A**, when the electron travels along the **A** field, and has a value *evA*. To emphasis this point we will use EKE for this electro-kinetic energy.

Spavieri & Rodriguez [4] have shown that the Aharonov-Bohm effect may have a classical origin whereby an additional force related to the static **A** field may be present on a moving electron. It is quoted as a longitudinal (along the velocity direction) force and involves the longitudinal component of the gradient of the scalar product of the vector velocity **v** and magnetic vector potential field **A** as given by $\mathbf{F_L} = -q\nabla(\mathbf{v}\cdot\mathbf{A})$ where *q* is the electron charge. This force creates an em phase lag effect that exactly accounts for the Aharonov-Bohm effect.

The Cartesian components of $\nabla(\mathbf{v}\cdot\mathbf{A})$ are:

$$\begin{aligned}
E_x &= \left[v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_y}{\partial x} + v_z \frac{\partial A_z}{\partial x} + A_x \frac{\partial v_x}{\partial x} + A_y \frac{\partial v_y}{\partial x} + A_z \frac{\partial v_z}{\partial x} \right] \\
E_y &= \left[v_x \frac{\partial A_x}{\partial y} + v_y \frac{\partial A_y}{\partial y} + v_z \frac{\partial A_z}{\partial y} + A_x \frac{\partial v_x}{\partial y} + A_y \frac{\partial v_y}{\partial y} + A_z \frac{\partial v_z}{\partial y} \right] \qquad (5) \\
E_z &= \left[v_x \frac{\partial A_x}{\partial z} + v_y \frac{\partial A_y}{\partial z} + v_z \frac{\partial A_z}{\partial z} + A_x \frac{\partial v_x}{\partial z} + A_y \frac{\partial v_y}{\partial z} + A_z \frac{\partial v_z}{\partial z} \right]
\end{aligned}$$

Note the first three terms in each parenthesis deal with spatial variations in the **A** field whilst the remaining terms deal with spatial changing velocity, and these are the terms that are suppressed in (4). For an almost uniform **A** field (here to be along the *x* direction) such as that of the Earth as seen in the restricted dimensions of the laboratory, the spatial variations in **A** are all zero as are A_y and A_z, then we can simplify (5) to

$$E_x = \left[A_x \frac{\partial v_x}{\partial x} \right], \quad E_y = \left[A_x \frac{\partial v_x}{\partial y} \right], \quad E_z = \left[A_x \frac{\partial v_x}{\partial z} \right] \tag{6}$$

Only terms with velocity derivatives survive and these are the very terms that are suppressed in (4). It appears that the only reason for that suppression is that the full derivative of the **A** field is taken as

$$\frac{D\mathbf{A}}{Dt} = \mathbf{v} \cdot \nabla \mathbf{A} \tag{7}$$

and then (3) and (5) follow from the vector identity

$$\mathbf{v} \cdot \nabla \mathbf{A} = \mathbf{v} \times \mathbf{B} + \nabla_A (\mathbf{v} \cdot \mathbf{A}) \tag{8}$$

It seems wrong to formulate EM theory to satisfy vector math identities especially when it leads to the absurd situation where a moving electron, undergoing a 180 degree change of direction within an **A** field, moves from a positive electro-kinetic potential to a negative one (or vice versa) without the presence of an electric field. If the Schwarzschild electro-kinetic potential has any validity then that movement must be through an effective **E** field as given by (5), or in the case of a uniform **A** field, by (6).

We are interested in drift velocity of electrons along a filamentary wire, see Annex A for more details. The spatial variations in velocity can come from classical linear acceleration (or deceleration) within a straight wire along a fixed axis, or from constant velocity around a curved wire.

3. Using FEMM to determine v ·A

For those who find (5) daunting and have used FEMM to solve 2D magnetic problems there is a useful trick for obtaining **v ·A** as follows.

Figure 1 shows the x-y plane **H** field around two parallel conductors with their axes along the z direction, i.e. at right angles to the paper. One conductor carries current into the paper while the other carries current in the opposite direction. The blue line is a contour of interest along which we wish to evaluate the tangential component of **H**. FEMM does this for you with the result shown in figure 2. Now the math relating the field **H** to the current in the conductors is identical to the math relating the **A** field to the flux within a core. Thus we can replace the 1 amp current with a 1 Weber flux, then figure 1 becomes a plot of the **A** field and figure 2 becomes a plot of the tangential component of **A** along the contour. Note that a 1 Weber flux is a huge value unlikely to occur in practice. For the 1 cm^2 cross section core modelled here 1 Weber represents a flux density **B** field of 10^4 Tesla. A more realistic value is 1 Tesla so figure 3 shows the tangential component of the A field for this value (i.e. a core flux of 10^{-4} Weber).

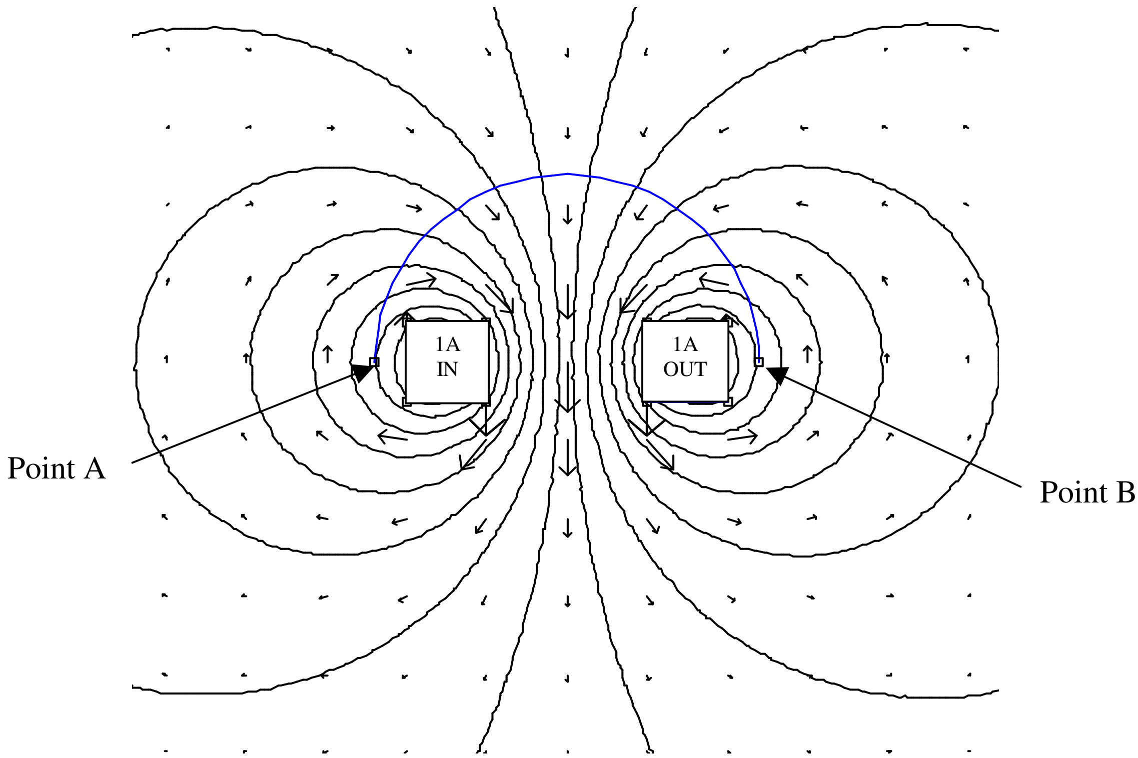

Figure1. FEMM plot of H field

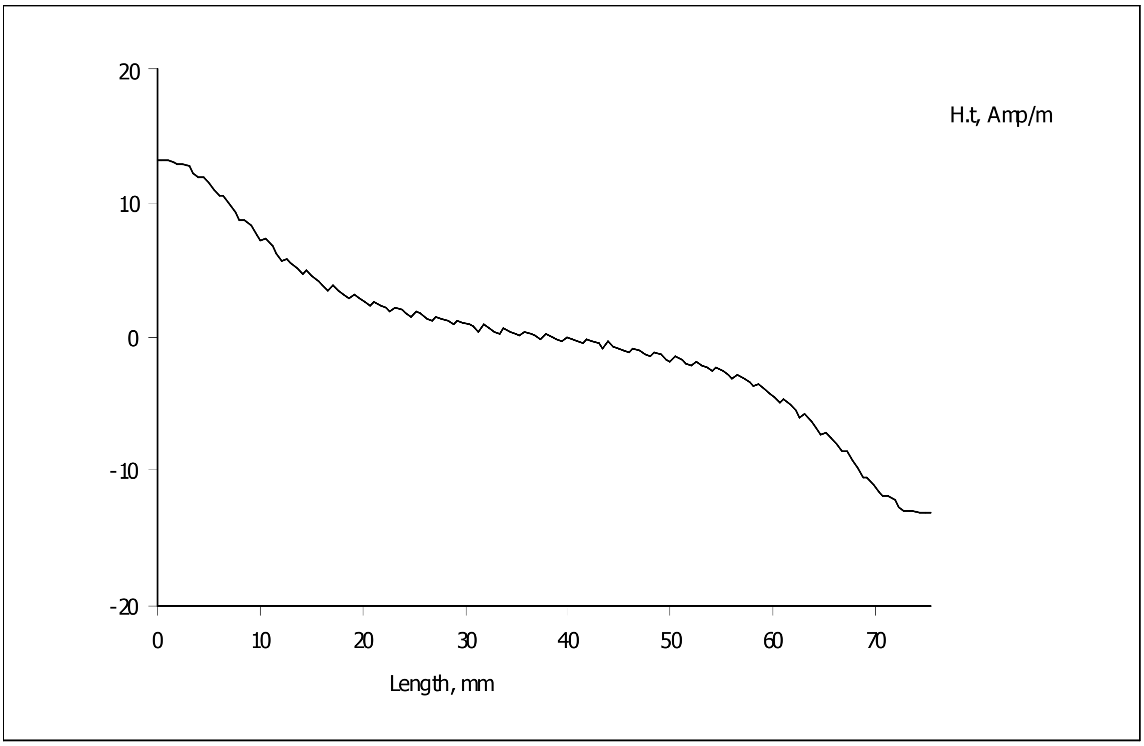

Figure 2. Tangential H component along contour

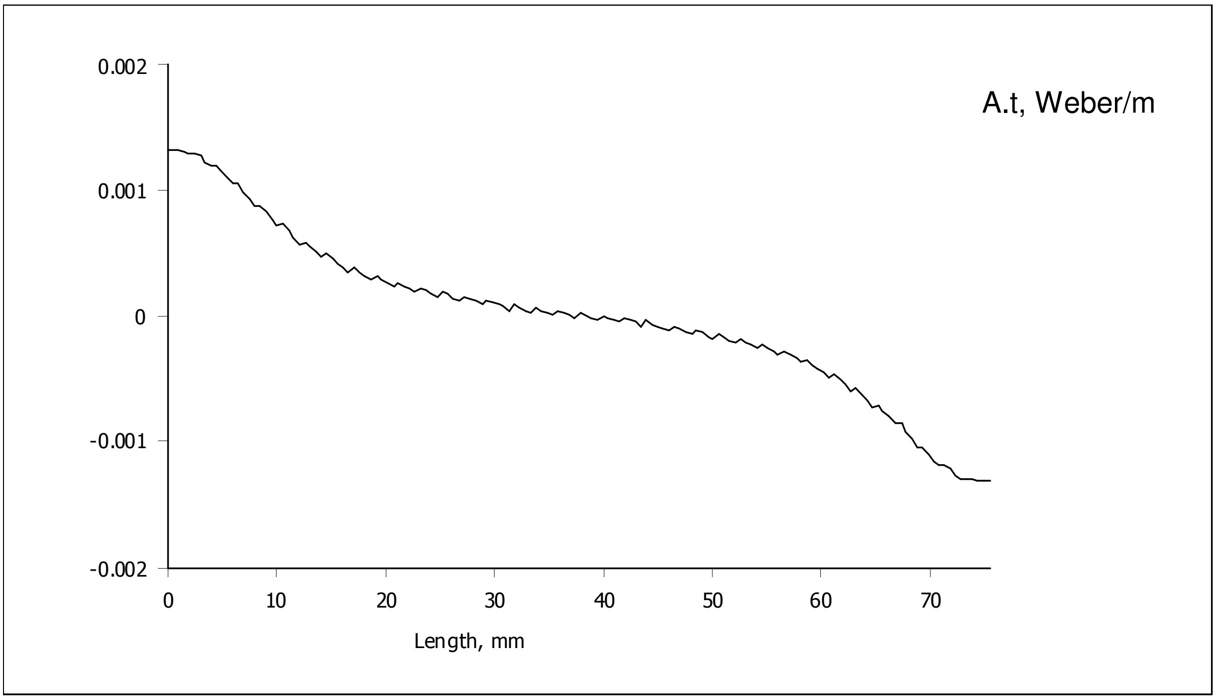

Figure 3. Tangential A component along contour for 1 Tesla core

If we multiply the tangential component A.t by the drift velocity of electrons we then have the scalar potential $\mathbf{v}\cdot\mathbf{A}$ which is what we set out to obtain. Thus figure 3 when multiplied by drift velocity is a plot of the potential induced into a semi-circle of wire placed at the contour in figure 1 when that wire is carrying a current. However current cannot flow in such a discontinuous wire, it needs a closed loop. Closing the loop creates exactly the opposite effect and the net induced potential capable of driving current around the loop is zero. This applies no matter what shape the loop possesses, so what benefit can be gained from this form of induction? Note that although the potentials induced into the loop equate to zero, that does not mean the potentials are everywhere zero around the loop, travelling around the loop there is a potential rise followed by a potential fall. Thus there will be a potential difference from one side of the loop to the other, from the position of maximum EKE to minimum EKE.

If the proposed induction is real, it is not necessary to have the non-uniform A field shown in figure 1. Induction would occur even in a uniform field simply due to the changing velocity direction along the wire. Hence the suggestion for using the Earth's A field which, although not completely uniform (if it were there would be no magnetic field) is much larger than can be easily produced in the laboratory, and can be considered to be nearly uniform.

4. Alternating Current Considerations.

It is of interest to note that it doesn't matter at which end you start the plot of tangential component, you get the same result in going from point A to point B in figure 1 as going from point B to point A. This suggests the possibility that alternating current within the semi-circular wire would benefit from voltage induction on both halves of the alternating cycle, and that insight opens the door to closing the loop not by conduction but by Maxwell's displacement current. Imagine conducting spheres placed at each end of the semi-circle at positions A and B, so that the bent wire forms an electric antenna. Now we have the situation where electrons do not travel around closed paths, they stop travelling when they reach each sphere and build up charge there. However that "stop travelling" is a deceleration, and, when the capacitance discharges, electrons start travelling in the reverse direction and that is a reverse acceleration. Those changes in velocity negate the induction around the semi-circle so again we see no benefit.

All is not lost though, there is no reason why electrons building up charge on a capacitor should be stationery. We can quite easily arrange that current flows continuously along one electrode leading to the situation shown in figure 4. Here we have a closed loop of current within the **A** field, and the electro-kinetic potentials induced create charge on the straight sections that couple via dielectric to other electrodes. If the current in the loop is alternating then we should see an alternating voltage across the resistor. *Note that we cannot simply connect capacitors to the top and bottom of the loop*. We must have current flowing along one electrode of each capacitor. If we wish to use classical techniques for obtaining high value capacity then we need specially constructed capacitors to allow this situation to occur.

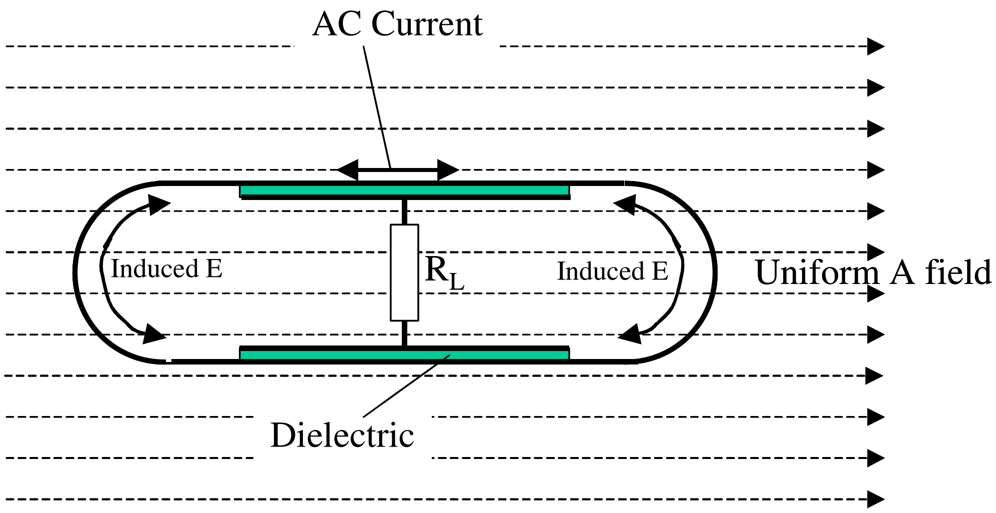

Figure 4. Closed loop experiment.

Figure 5 shows the situation at the peak of one half cycle of current with of course reversed polarity for the other half cycle.

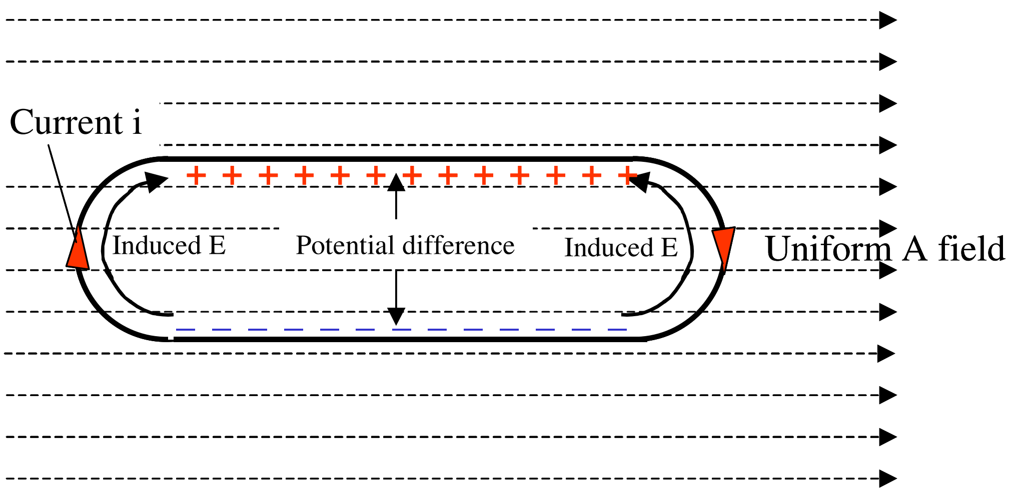

Figure 5. One half cycle peak

A practical realization of this experiment is shown in figure 6. The input has to drive current in the loop that will appear as a short circuit to the source, but we should see the voltage induced from the Earth's **A** field. The straight sections must lie on the E-W axis and reversing the whole system should create a 180° phase reversal of that induced voltage.

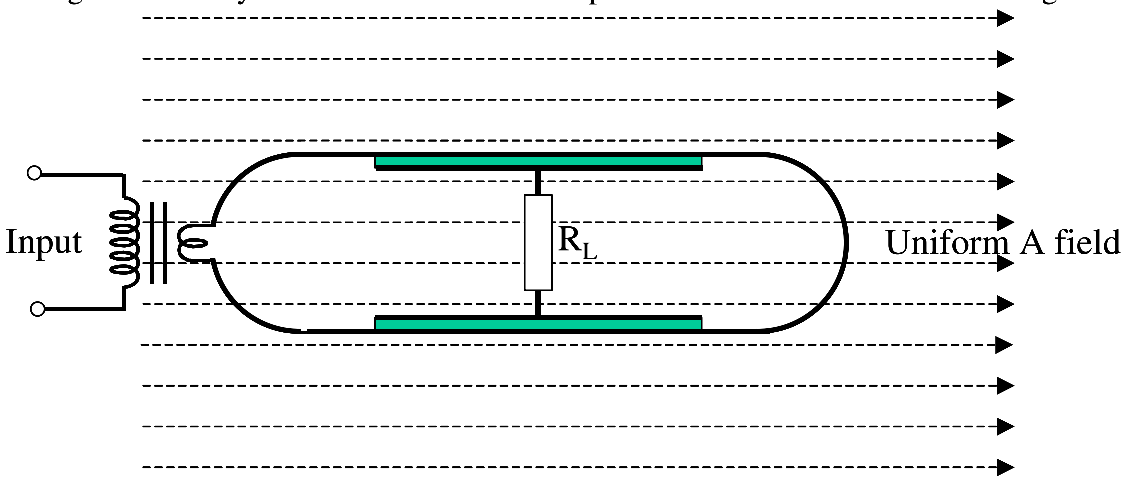

Figure 6. Practical experiment

If this experiment gives a positive result it will be the first conclusive evidence that energy can be extracted via the vector magnetic potential. Also it provides the first ever instrument for measuring that vector potential. And it offers potential as a means of creating an electronic compass.

Annex A.

Drift velocity considerations

The volumetric number density N_D of atoms in a material is given by

$$N_D = \frac{N_A \rho}{W_A} \text{ per cm}^3 \qquad \text{(A1)}$$

where N_A is Avagrado's number, ρ is the density and W_A the atomic weight. For copper ρ is 8.92 g/cm^3, W_A is 63.54 and Avagrado's number is 6.025×10^{23}. Thus 1 cm^3 of copper contains 8.46×10^{22} atoms. With one conduction electron per atom that same number applies to the electron density. For a wire of diameter d mm carrying a current i the drift velocity v_d is given by

$$v_d = \frac{4000i}{\pi e N_D d^2} \text{ mm/s} \qquad \text{(A2)}$$

where e is the electron charge 1.602×10^{-19} Coulombs.
Thus a copper wire 1mm diameter carrying 1 amp has a drift velocity of 9.39×10^{-2} mm/s, or 9.39×10^{-5} m/s in SI units. Using that SI value against the typical laboratory vector potential difference of 0.0026 in figure 3 we get an induced potential of only 0.244 μV. This tiny voltage is unlikely to be of any use. However in the Earth's equatorial vector field varying tangentially from +200 to –200 weber/m that same U shaped wire would have an induced potential of 37.6 millivolts which is much more respectable. Since that voltage is for a current of 1 amp the induction can be represented by a negative resistor of magnitude 37.6 milliohms but note this value applies to our 1mm diameter wire. In view of the simplicity of denoting the induction by a resistor value, enabling equivalent circuits to be created and solved, a useful formula for the induced resistance R_{ind} in U shaped copper wires of diameter d mm undergoing a 180 degree change from parallel to anti-parallel within a uniform vector potential field is

$$R_{ind} = \pm \frac{9 \cdot 4 \times 10^{-5} A}{d^2} \text{ Ohms}$$

where R_{ind} can be positive or negative depending on the initial parallel current direction relative to the **A** field. Note that this value may not apply to HF alternating current where skin effect will alter the drift velocity.

Figure A1 shows the equivalent circuit for the proposed experiment.

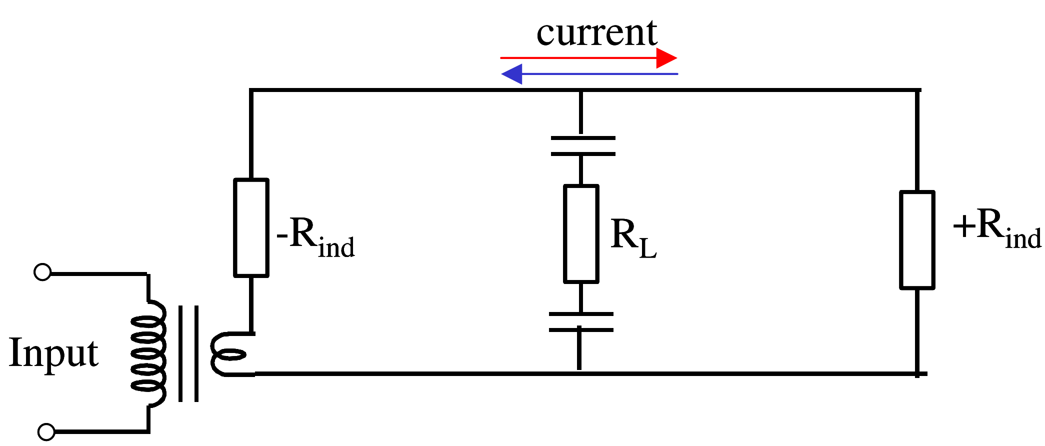

Figure A1. Equivalent Circuit

The driver sees induced $+R_{ind}$ and $–R_{ind}$ in series hence sees only a short circuit that does not consume any power (we have ignored actual wire resistance here). Solving this circuit gives some indication of how the system can obtain power in its load R_L. Current through R_L is a maximum when the induced potentials are passing through zero, which coincides with the loop current also passing through zero. The current through R_L therefore divides both ways on entering the loop and each flow is in the direction to gain energy from the **A** field. This then suggests a modification to the experiment where an input is applied at the position of R_L so as to drive current in those two directions, see figure A2.

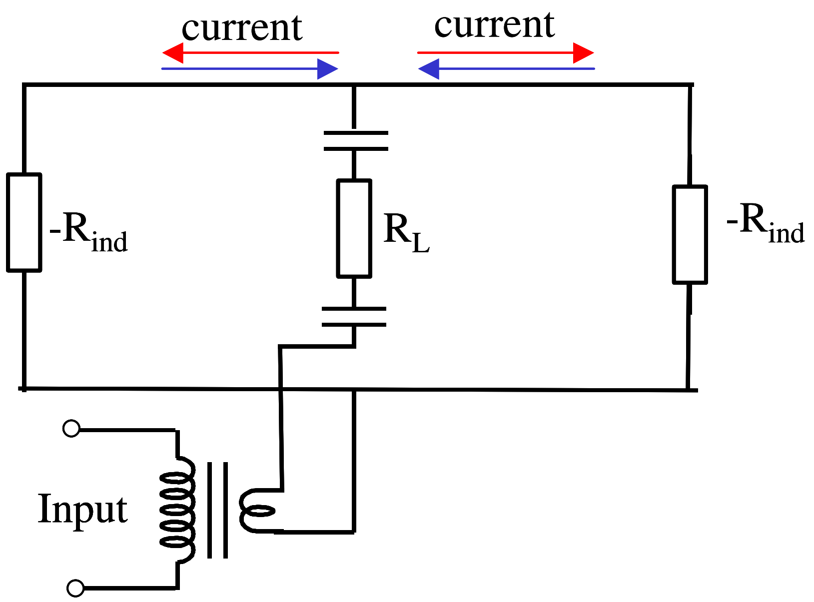

Figure A2. Alternative connection

Note that unlike capacitor components connected to the loop the load "current" arriving at the loop is not in the form of electron flow, but is Maxwell's displacement current. Thus that arriving "current" does not suffer the electro-kinetic potential change on turning through 90 degrees that would occur if the capacitors were actual components (as depicted in the equivalent circuit figure A2) that would otherwise negate the wanted induced potentials. If energy is gained from the **A** field then we should see OU with regard to the input power and the power dissipated in R_L.

References.

[1] J. P. Wesley:- The Marinov Motor, Notional Induction without a Magnetic B Field, APEIRON Vol. 5 Nr.3-4, July-October 1998
Wesley notes this type of motional induction has been used to account for the Aharonov-Bohm (1998) effect and the Hooper (1974)-Monstein (1997) experiment.

[2] Thomas E. Phipps Jr: Observations of the Marinov Motor, APEIRON Vol. 5 Nr. 3-4, July-October 1998

[3] Arnold Sommerfeld:- 1948 "Lectures on Theoretical Physics" vol. 3 "Electrodynamics" (translated by Edward G. Ramberg and published by Academic Press in 1964).

[4] Gianfranco Spavieri & Miguel Rodriguez: The effects of the Aharonov-Bohm type as tests of the relativistic interpretation of electrodynamics,
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