

# Power-source element and its properties

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**Abstract:** A power-source element is defined and its properties are studied. The power source describes the output characteristics of the loss-free resistor, as well as high quality rectifiers, some types of switched-mode convertors, and other lossless two-ports. It is shown that controlled power sources are inherent in the large class of buffered power-conservative two-port networks. This approach has inspired some new realisations and applications of the loss-free resistor. A family of high frequency switched-mode convertors is shown to exhibit naturally power-source output characteristics.

## 1 Introduction

Power-conservative two-port networks are widely used in modelling of efficient power-processing systems. The transformer model, including the DC transformer [1], and the time-variable transformer [2, 3] has been widely applied in the attempt to model power-electronic circuits in a simple way. The gyrator has also found application [4, 5]. However, efficient power-processing systems exist which are not adequately described by these models. These systems perform nonlinear power-processing functions, yet are nonetheless power-conservative. Hence, a more general approach is needed which encompasses both linear and nonlinear two-port power-conservative networks. The generalised two port is denoted POPI ( $P_{out} = P_{in}$ ) [6].

The POPI approach was strengthened by the development of the loss-free resistor (LFR) [7]. This element is a two-port power-conservative network which emulates resistive characteristics at its input port, and transfers power to its output port. It was first realised based on continuous control of a time-variable transformer or gyrator such that resistive characteristics appear at the input terminals [7, 8]. Practical implementation was by means of a controlled switched-mode convertor. Later, it was found that switched-mode convertors exist which naturally (i.e. without additional control) exhibit LFR behaviour.

Hence, we proposed a simplified model of the LFR which includes a resistive element at the input terminals, and a new circuit element at the output terminals called the power source. We have later found that it has other

applications as well; in fact, any two-port power-conservative network which meets simple buffering conditions must contain a power-source element. Power-source definition, simplification of series and parallel combinations, and transferral by gyrators and transformers are given in Section 2. An interesting property is the equivalence of series- and parallel-connected power-sources, an attribute not shared by other known non-trivial one-port elements.

Unity power factor, low-harmonic rectification is becoming an important power-system issue, with standards being established. Such rectifiers must exhibit LFR characteristics, with power-source output characteristics driving energy storage and other elements [9]. We have found the LFR/power-source approach to be well suited for understanding these systems. The power-source output characteristic inherent in this application has been experimentally verified, and good agreement has been found.

## 2 Definition and some properties of the power source

The power source is defined as an element whose  $i-v$  curve obeys the following equation

$$vi = P = \text{constant} \quad |v| < \infty \quad |i| < \infty \quad (1)$$

Eqn. 1 is plotted in Fig. 1a for a positive value of  $P$ . Fig. 1b shows the proposed power-source symbol.

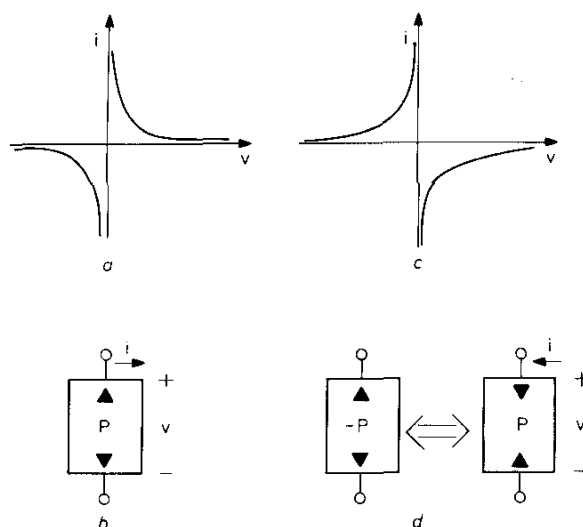


Fig. 1 The ideal power source element

a  $i-v$  characteristic  
b symbol  
c characteristic for negative  $P$   
d power sink symbol, and its equivalence to a negative power source  
The arrows indicate the direction of the power flow

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of  $P$  may be either positive or negative. In the case where  $P$  has a negative value (Fig. 1c), the element delivers negative power, i.e. is a power sink as shown in Fig. 1d. The direction of the arrows indicates whether power flows from the source to the external network (Fig. 1b) or vice versa (Fig. 1d). For simplicity of notation, the element is referred to here as a power source, regardless of whether  $P$  is positive or negative. Eqn. 1 also implies that, for nonzero  $P$ , the source can be terminated by neither a short circuit nor an open circuit. Similar to the voltage and current-source elements, the power-source element may be controlled by some other system quantity such as a voltage, current, or power signal.

As illustrated in Fig. 2, series, parallel, and other connections (provided the circuit forms a connected graph)

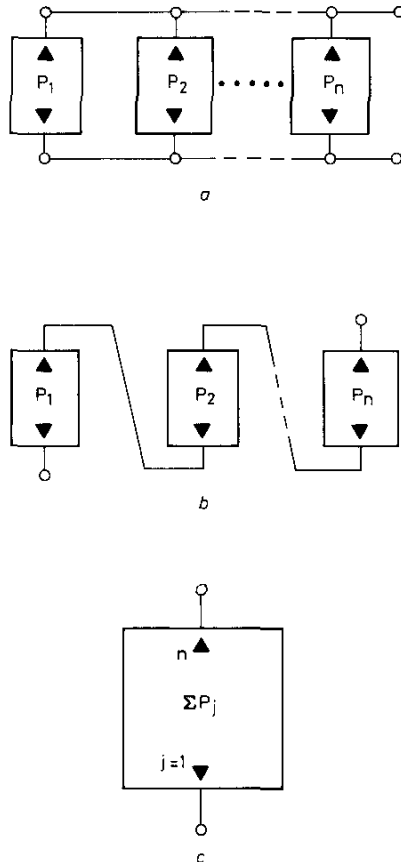


Fig. 2 Connections of power sources

- a parallel
- b series
- c equivalent single element

of power sources are equivalent to a single power-source element with value

$$P = \sum_{j=1}^n P_j \quad (2)$$

This implies that the reduction of complex structures of power sources to a single source is independent of topology. For example, in the simple case in which power sources  $P_1$ ,  $P_2$  are interconnected in series (denoted by  $\boxplus$ ) or parallel (denoted by  $\boxdot$ ) we have

$$P \boxplus P_2 = P_1 \boxdot P_2 \quad (3)$$

Eqn. 3 states that a network formed by series connection of power sources  $P_1$  and  $P_2$  is equivalent to a

network formed by parallel connection of the same power sources. This equivalence is not shared by any other known nontrivial circuit elements. The equivalent of series-connected resistors, for example, has a different resistive value from the same group of resistors in parallel connection.

Power-source elements are invariant to transfer by transformers and gyrators, as illustrated in Fig. 3, let  $P'$

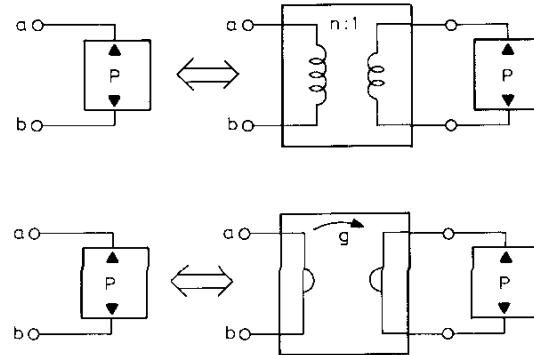


Fig. 3 Invariance of the power source element to transfer by gyrators and transformers

be a power source  $P$  transferred by a transformer or gyrator. Then

$$P' = P \quad (4)$$

for all nonzero  $n$  and  $g$ . This is true because the transformer and gyrator are both power-conservative two-port networks. The value  $P$  of the equivalent power source is independent of the gyration constant  $g$  or the transformer turns ratio  $n$ . This is also a unique property of the power source.

For  $P = 0$ , the power source can behave as both a short circuit and an open circuit. When it terminates a current source, it behaves as a short circuit, and it behaves as an open circuit when terminating a voltage source.

Interconnection of the power-source element and an arbitrary network does not necessarily imply real voltage and current solutions. An additional compatibility constraint must be satisfied: by Tellegen's theorem, the network must be capable of sinking or sourcing the power of the power-source element. Imaginary solutions indicate that the source and network power requirements cannot be balanced.

For example, in case of a power source  $P$  connected to a storage battery with open circuit voltage  $E$  and internal resistance  $r$ , the current flowing to the battery is given by

$$i = \frac{E}{2r} \left[ \sqrt{1 + \frac{PE^2}{4r}} - 1 \right] \quad \infty > P \geq -E^2/4r \quad (5)$$

For a positive value of  $P$  there is no limit (in principle) to the power which such kind of load can absorb. On the other hand, for a negative value of  $P$ , the limit is determined by the maximum power the battery can deliver, given by  $E^2/4r$ .

Another example is a power source connected to a capacitor  $C$  with initial voltage  $v_c$ . In this case we have

$$P = v_c i_c = v_c C \frac{dv_c}{dt} \quad (6)$$

and the solution is

$$\begin{aligned} v_c(t) &= \sqrt{[2(W_{C_o} + Pt)/C]} \\ W_{C_o} &> -Pt \\ W_{C_o} &= \frac{1}{2} C v_{C_o}^2 \end{aligned} \quad (7)$$

In the case in which a power source is connected in parallel to a voltage source  $E_1$ , its voltage  $v_s$  and current are given by

$$v_s = E \quad i_s = P/E \quad (8)$$

The same arguments apply in the case of the power source connected in series with a current source  $I$ , in this case

$$v_s = P/I \quad i_s = I \quad (9)$$

### 3 How power sources arise in two-port buffered POPI networks

The loss-free resistor is an example of a buffered network, in which the input waveforms are not influenced by the output port signals. It is shown in this section that whenever buffer conditions are satisfied in a power-conservative (POPI) two-port network, then a controlled power source must arise. Additional examples are discussed, including an ideal switching convertor with output voltage regulation.

In general, either the output port can be buffered from the input port, or vice versa. In the case when the output port is buffered, Fig. 4, there is a functional relationship between the output voltage and current, which is independent of the input voltage and current. In a controlled two-port, the voltage and current may additionally depend on the control signal  $u(t)$ . This condition can be formulated as follows

$$i_o = h(v_o, u(t)) \quad \forall v_i, i_i \quad (10)$$

In a power-conservative two-port, the output power must be derived from the input port. Finite and nonzero values

of the input signals  $v_i$  or  $i_i$  are therefore required, so the boundaries of  $v_i, i_i$  should be reduced; i.e.

$$i_o = h(v_o, u(t)) \quad v_i \neq 0 \quad i_i \neq 0 \quad (11)$$

Eqn. 11 can be written in the following form

$$0 = -i_o + h(v_o, u(t)) \quad v_i \neq 0 \quad i_i \neq 0 \quad (12)$$

or,

$$F(x_o, u(t)) = 0 \quad (13)$$

where

$$F(x_o, u(t)) \equiv -i_o + h(v_o, u(t))$$

$$x_i \equiv \begin{bmatrix} v_i \\ i_i \end{bmatrix} \quad x_o \equiv \begin{bmatrix} v_o \\ i_o \end{bmatrix}$$

$$x_i \notin \left\{ \begin{bmatrix} 0 \\ i_i \end{bmatrix}, \begin{bmatrix} v_i \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

Eqn. 13 indicates that the output port can be modelled by an equivalent one-port element, as in Fig. 4a.

This buffer condition, along with the power-conservative property of the two-port network, implies that the input port exhibits a power-source characteristic. By eqn. 6, the output power is given by

$$P_o = v_o i_o = v_o h(v_o, u(t)) \quad (14)$$

Thus, the output port buffer condition leads to output power  $P_o$  which is not a function of  $v_i$  and  $i_i$ . By use of the power-conservative property  $P_i = P_o$ , the input power is given by

$$P_i = v_o h(v_o, u(t)) \quad (15)$$

or,

$$v_i i_i = P(t) \quad (16)$$

where

$$P(t) = v_o h(v_o, u(t))$$

Eqn. 16 is the equation of the power source, as given in eqn. 1. Therefore, the buffered two-port POPI network

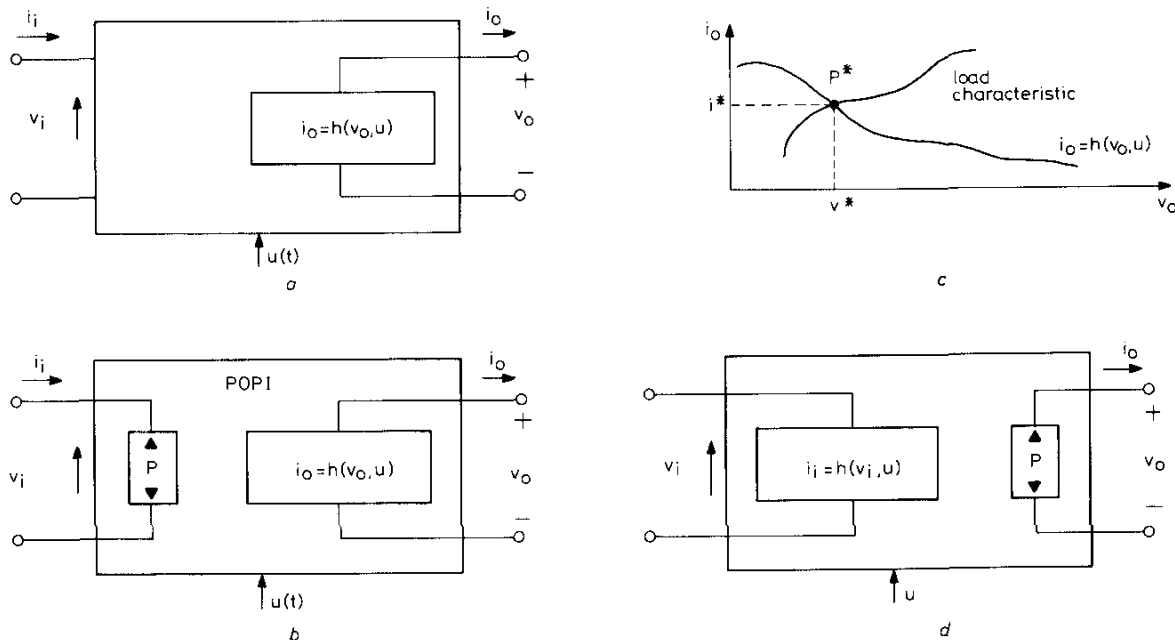
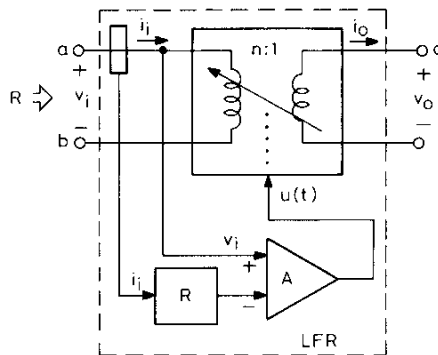


Fig. 4 Buffered two-port networks

- a buffered output port
- b embodiment of an input controlled power source in an output-buffered power-conservative two-port
- c operating point determined by intersection of output and arbitrary load characteristics
- d embodiment of an output-controlled power source in an input-buffered power-conservative two-port

can be represented as in Fig. 2, and must always contain a controlled power source at the non-buffered port.

For a given arbitrary load,  $P(t)$  is determined by the output port operating point. As in the case of any network composed of two elements, the operating point



**Fig. 5** Realisation of a loss-free resistor based on the controlled transformer

is determined by the intersection of the load characteristics and the output port characteristic, eqn. 16, as shown in Fig. 4c. This alone determines the power given in eqn. 10, which also determines the value of the power source  $P(t)$  according to eqn. 16.

Similar arguments can be used in the case where the input port is buffered from the output port. The result is depicted in Fig. 4d, in which the output port exhibits power-source characteristics. The loss-free resistor is an example of a network of this form.

Power sources may also arise in non-power-conservative buffered two-port networks operating at constant efficiency  $\eta$ . The arguments are the same as those for the power-conservative network. In case of positive power-source creation at the output terminals, the value of the power source is given by

$$P = \frac{P_{in}}{\eta} \quad (17)$$

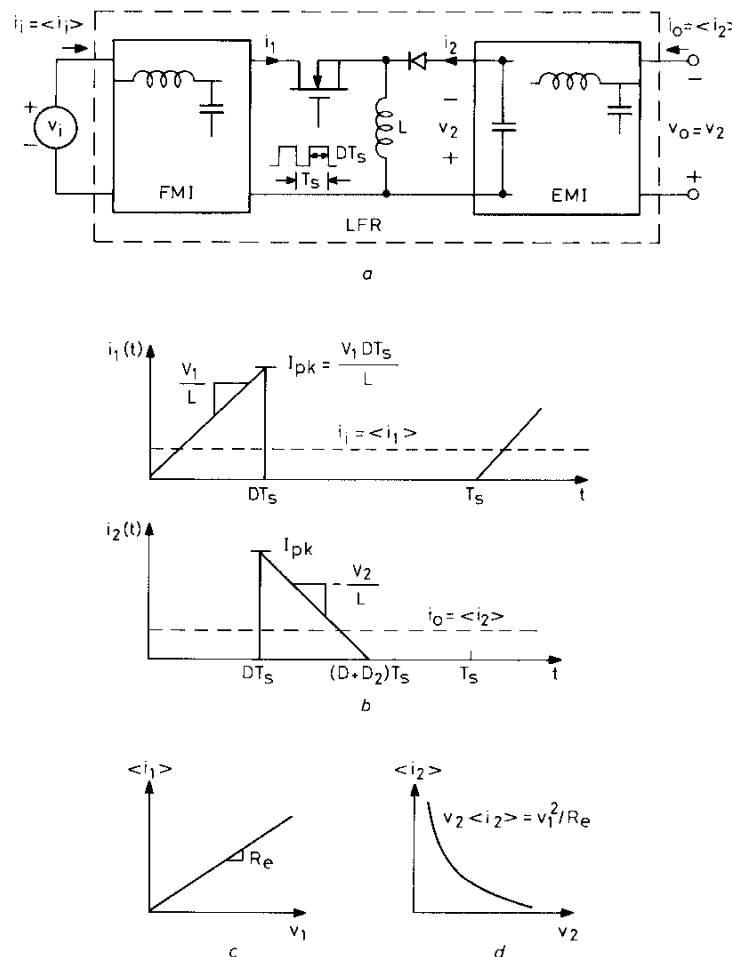
For the case of creation of a power sink at the input terminals, the power value is given by

$$P = \frac{P_{out}}{\eta} \quad (18)$$

#### 4 Some two-port networks with embodied power sources

##### 4.1 The loss-free resistor

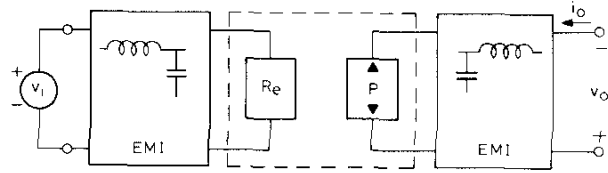
The loss-free resistor (LFR) was first developed to provide damping and waveshaping in a CO<sub>2</sub> laser system [7]. The objective was to obtain resistive characteristics (i.e. obeying Ohm's and Joule's laws) without the dis-



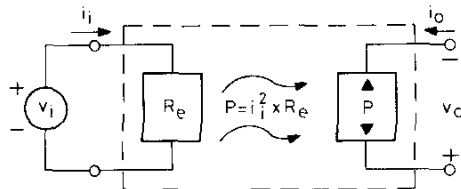
**Fig. 6** Discontinuous conduction mode buck-boost converter

a schematic b input and output current waveforms c effective resistive input characteristic d effective power-source output characteristic

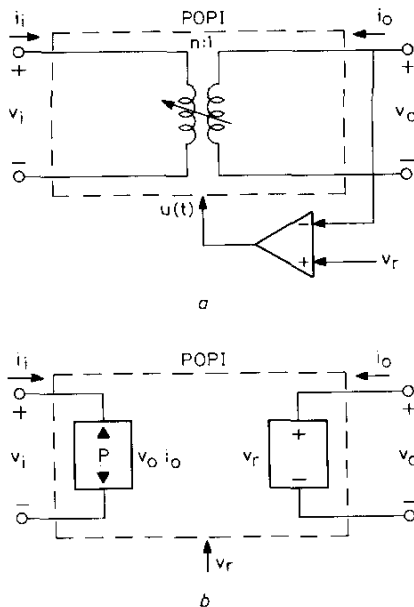
advantage of dissipation of the absorbed power. The realisation was based on the controlled transformer concept, as shown in Fig. 5. Through variation of the



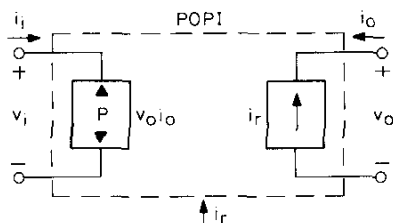
**Fig. 7** Equivalent circuit for the discontinuous conduction mode buck-boost converter



**Fig. 8** Loss-free resistor equivalent circuit model  
Based on the controlled power source



**Fig. 9** Modelling of output-voltage-stabilised converter  
a stabilisation of output voltage by control of transformer ratio  $n$   
b equivalent circuit containing controlled power sink



**Fig. 10** Equivalent circuit of average-current-programmed converter

transfer ratio by a controller, the input port was made to emulate resistive characteristics. The absorbed power was returned to the system power supply. It is well known that a large family of switched-mode converters obeys the equation of the controlled transformer [1], and one of these was used in the practical circuit.

It has been shown [6, 9] that switching converters exist which naturally exhibit LFR characteristics, with no controller at all. An example is the buck-boost converter of Fig. 6, operating in the discontinuous conduction mode (DCM). Any well-designed converter incorporates EMI filtering with a low-pass characteristic, such that the input current  $i_i(t)$  is well approximated by the average value of  $i_i(t)$  [1]. Use of the state-space averaging approach [1] yields the following expression for  $\langle i_1(t) \rangle$

$$\langle i_1 \rangle = v_1 \frac{D^2 T_s}{2L} \quad (19)$$

where  $\langle \cdot \rangle$  denotes that  $\cdot$  has been averaged over one switching period  $T_s$  and  $D$  is the transistor duty cycle as shown in Fig. 6. Note that this equation is of the form

$$\langle i_1 \rangle = \frac{v_1}{R_e} \quad (20)$$

where

$$R_e = \frac{2L}{D^2 T_s} \quad (21)$$

Hence, the input port emulates a resistor, as depicted in Fig. 6c. The average output current is given by

$$\langle i_2 \rangle = \frac{v_1^2 D^2 T}{v_2 2L} \quad (22)$$

Substitution of eqn. 21 into eqn. 22 and rearrangement of terms yields the expression for the output power

$$v_2 \langle i_2 \rangle = \frac{v_1^2}{R_e} = P \quad (23)$$

Hence, the output power  $P$  is equal to the input power absorbed by  $R_e$ , and is not influenced by the output quantities  $v_2$  and  $i_2$ . The output port characteristic is shown in Fig. 6d. Thus the DCM buck-boost exhibits LFR behaviour, and contains a power-source output characteristic. Other DCM switched-mode converters exist, such as the flyback and Cuk converters [6, 9], which exhibit similar properties. Hence, there is a family of converters which naturally exhibit LFR characteristics. In the practical cases where the input variations occur at frequencies sufficiently lower than the EMI filter cutoff frequency, the filter dynamics can be ignored. The equivalent circuit model can then be simplified as shown in Fig. 8. This model directly describes the resistive characteristic of the input port, the power source behaviour of the output port, and the lossless transfer of power from input to output. It is not based on the realisation method, so it describes the principal features of any LFR.

#### 4.2 Output voltage stabilised DC-DC converters

It is known that a large family of DC-DC converters, such as the buck, boost, Cuk, half-bridge, forward, etc., converters are well modelled as controlled DC transformers [1]. When operated open-loop, the outputs of these converters are directly influenced by input variations. In many applications, a stabilised output voltage is required, and hence a feedback loop is applied as shown in Fig. 9a. In a well-designed system, the output tends to exhibit voltage source characteristics, regardless of the values of the input voltage and current. Therefore, the voltage regulator satisfies the buffer conditions, eqn. 10. Since the control process does not change the power conservative nature of the converter (it is assumed that the

controller power consumption is negligible), the input should exhibit power-source characteristics, as given by eqn. 16.

#### 4.3 Current-programmed converters

Other similar examples are the average-current-programmed buck, bridge, and forward converters, which can be modelled by power-conservative two-port networks whose output currents are proportional to a control signal [10]. Hence, the output ports of the converters also satisfy the buffer conditions, eqn. (10). The input ports must therefore behave as controlled power sinks, dependent on the output power. The model of Fig. 10 can be applied.

### 5 Experimental verification

A practical high quality rectifier based on a natural LFR was constructed, and is described in Reference 9. The loss-free resistor was realised using a DCM flyback converter, as shown in Fig. 11a. An equivalent circuit is shown in Fig. 11b. The converter operated from the 120 VAC 60 Hz line. The measured DC output characteristics are plotted in Fig. 11c. It can be seen that the converter output port does indeed behave very nearly as a source of constant DC power. The actual measured power varied between 11.9 W and 12.4 W for load resistances varying from 3.9  $\Omega$  to 88  $\Omega$ . Thus, operation of the DCM flyback converter as a loss-free resistor, in a high quality rectification application, has been verified experimentally. Output port power-source characteristics have been demonstrated.

### 6 Conclusions

It has been observed that a number of power-processing networks exhibit constant power output characteristics. This is the motivation for the definition of the power-source element, and the study of its properties. It was

observed that the loss-free resistor can be simply modelled by the combination of resistive and controlled power-source elements. It was later realised that the power-source concept has much wider application. Systems in which one port is not influenced by the other include a majority of power electronics applications. Therefore, this buffer property is formalised in this paper. A wider class of power-conservative networks which incorporate this property has been defined. It has been found that a power source is embodied in each power-conservative two-port network which satisfies the buffering condition.

Some interesting properties of the power-source element follow from its definition. Attributes not shared by other known nontrivial one-port elements are the equivalence of series- and parallel-connected power sources, and their invariance to their transfer by gyrators and transformers.

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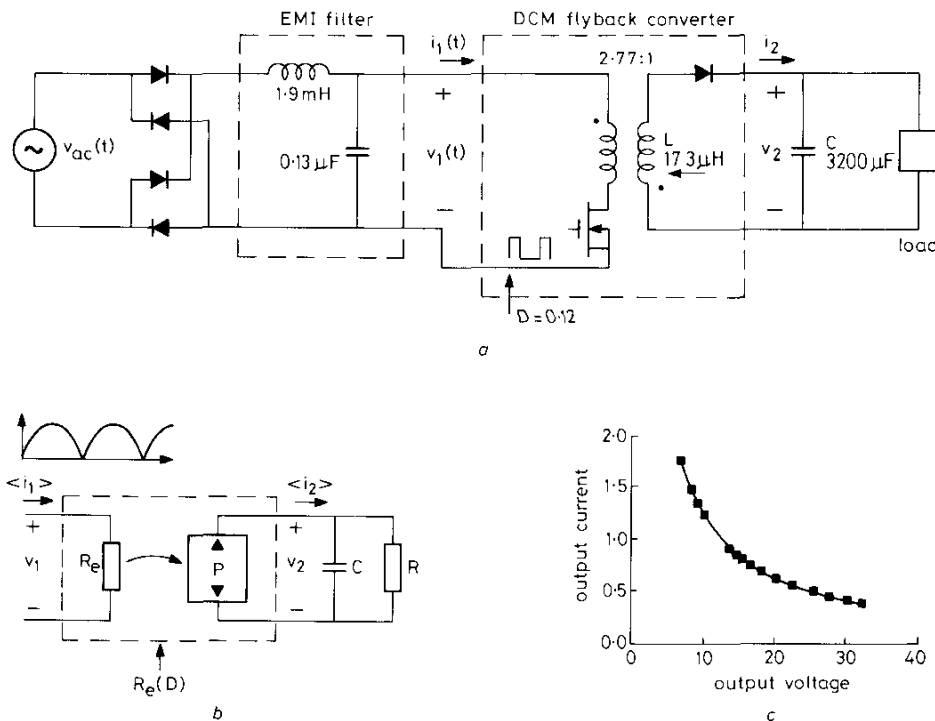


Fig. 11 Experimental high quality rectifier/loss-free resistor

a power stage schematic b equivalent circuit c measured output characteristic

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