On using Complex Permeability Resonance to get OU.

1. Introduction

Examination of the frequency spectrum of complex permeability shows that certain ferromagnetic materials exhibit a rise in the real value μ' from its low frequency value reaching a peak value at a certain frequency beyond which μ' then falls and even goes negative. This is accompanied by a rise in the loss term μ" which goes from zero at low frequencies to reach a peak value at a frequency slightly beyond that of peak μ' and a fall thereafter. This behaviour is due to a ferromagnetic resonance in the core material at a frequency close to the peak value of μ". Figure 1 shows a characteristic for TDK PE22 material.

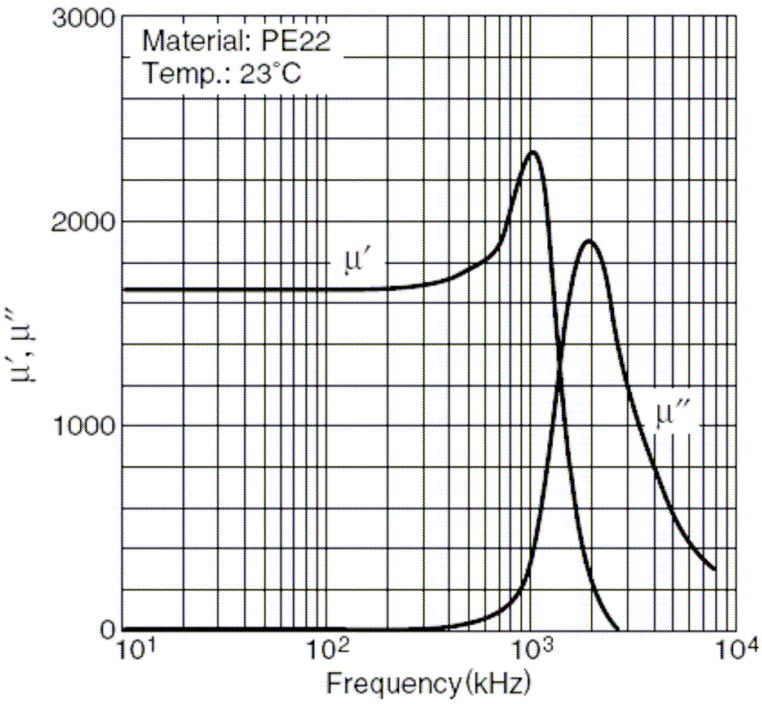

Figure 1. Complex Permeability

The LF value of μ' is 1680, rising to a peak of 2320 at 1MHz. At that frequency μ" has a value of 300. The intention is to use the change in μ' (which appears as a change in inductance value) to obtain OU by charging the inductor at low frequency then discharging it at high frequency.

2. Example calculation

The approach adopted here is to create a toroidal inductor using a ring core of suitable material. Then choose a capacitor that resonates with the inductor at a suitable low frequency, such as 100KHz. As an example a ring core of PE22 material having an inner diameter of 40mm, an outer diameter of 50mm and a height of 5mm, wound with 10 turns, has an inductance at 100KHz of 37.3μH which for resonance requires a capacitor of 0.068μF (68,000pF). Core losses, derived from μ", are represented by an effective series resistance of 0.14Ω. The circuit Q is 168.

The capacitor is charged to say 10V then switched on to the inductor. In a quarter cycle of the 100KHz resonance the voltage reduces to zero while the current in the inductor rises in a damped sinusoid to reach a peak value of 0.424A. At this point in time the capacitor value is changed to one that resonates at 1MHz. Because of the rise in μ' the inductance at this frequency is 51.56μH requiring a resonating capacitor of 491pF (hence the original 68,000pF would consist of 67,509pF in parallel with 491pF, and the 67,509pF is switched out of circuit). Because of the higher μ" value, core losses are now represented by an effective series resistance of 41.9Ω. The circuit Q is 7.73.

In a quarter cycle at 1MHz the current in the inductor reduces to zero while the voltage across the 491pF capacitor rises in a damped sinusoid to reach a peak value of 124.1V. This capacitor is then switched out of the circuit to be separately discharged into a load. Initial energy fed into the 68,000pF capacitor at 10V is 3.392μJ. Energy discharged from the 491pF capacitor at 124.1V is 3.788μJ. COP=1.117.

3. Math Analysis

Let L_{AIR} be the inductance of the toroidal coil if the permeability of the core were to be unity. Then the following equations apply.

$$L = \mu' L_{AIR}\,,\quad R_S = \omega\mu'' L_{AIR}\,,\quad Q = \frac{\omega L}{R_S} = \frac{\mu'}{\mu''} \qquad (1), (2), (3)$$

Using the suffix $_L$ to denote the low frequency charging phase, then for resonance at frequency f_L the capacitor value is given by

$$C_L = \frac{1}{4\pi^2 f_L{}^2 \mu_L' L_{AIR}} \qquad (4)$$

Charged to voltage V_{IN} this stores energy of value

$$W_{IN} = \frac{V_{IN}{}^2}{8\pi^2 f_L{}^2 \mu_L' L_{AIR}} \qquad (5)$$

When connected to the inductor, the undamped peak current that would appear a quarter cycle later is given by

$$i_{PK} = \omega_L C_L V_{IN} \qquad (6)$$

Losses cause an RF voltage or current envelope to decay with a time constant of $\tau = \frac{2Q}{\omega}$, which over a single quarter cycle yields an amplitude reduction given by $\exp\left(-\frac{\pi}{4Q}\right)$, hence the damped peak current becomes

$$i_{PK} = \omega_L C_L V_{IN} \exp\left(-\frac{\pi\mu_L''}{4\mu_L'}\right) \qquad (7)$$

At this point the capacitor is fully discharged. Using the suffix $_H$ to denote the high frequency at the peak value of μ', the capacitor is now reduced in value to one given by

$$C_H = \frac{1}{4\pi^2 f_H{}^2 \mu_H' L_{AIR}} \qquad (8)$$

Over a quarter cycle at frequency f_H this capacitor charges to a voltage V_{PK} given by

$$V_{PK} = i_{PK}\,\omega_H L_H \exp\left(-\frac{\pi\mu_H''}{4\mu_H'}\right) \qquad (9)$$

which includes the new damping factor at that frequency. Energy now stored in C_H as given by $W_{OUT} = \dfrac{C_H V_{PK}^{\;2}}{2}$ leads to the COP

$$COP = \frac{W_{OUT}}{W_{IN}} = \frac{\mu_H'}{\mu_L''}\exp\left(-\frac{\pi}{2}\left(\frac{\mu_L''}{\mu_L'}+\frac{\mu_H''}{\mu_H'}\right)\right) \qquad (10)$$