Mathias states

a. In Lemma VII, Newton states that at the limit (when the interval between two points goes to zero), the arc, the chord and the tangent are all equal.

b. But if this is true, then both his diagonal and the versine must be zero.

c. According to Lemma VII, everything goes to either equality or to zero at the limit, which is not helpful in calculating a solution.

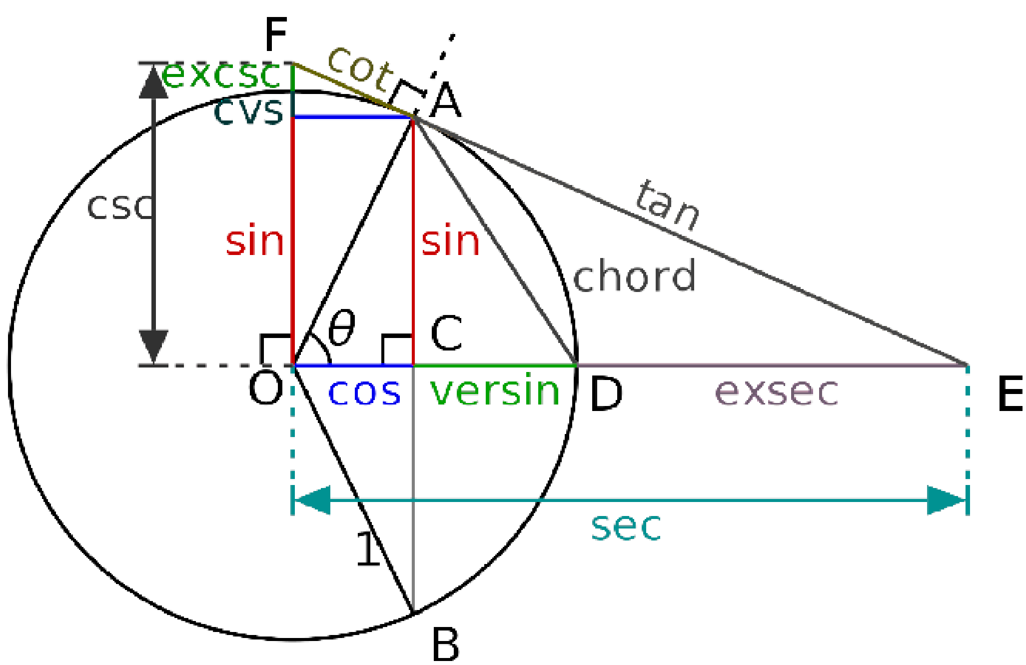
d. Neither the versine equation nor the Pythagorean theorem apply when we go to a limit by Newton's definition.

e. I will show below, with a very simple analysis, that the tangent must be allowed to remain greater than the chord at the limit; only then can the problem be solved without contradiction."

First and foremost we can clarify a lot of the confusion and mystery over the math if we place Newton's circle within a unit circle. Now I will address each point.

a. In Newton's circle at the limit the arc, chord, which is also a secant, and the tangent are indeed all equal. They go to trigonometric zero, not algebraic zero, and become indistinguishable from one another.
b. At the circumference the versine does indeed go to zero. But note that the versine or sagitta is defined as simply sagitta=versine=1-cosine. (More on Mathias' definition later). When the versine goes to zero then the cosine goes to 1. 1 is the full radius of a unit circle. Now given the way the paper describes the movement of the particle, b, it appears that b is released from the rotational forces of the orbit at point A. It then proceeds rectilinearly to point B whereupon it is then subjected to some acceleration ,a, forcing it to intersect the circle at C. This is equivalent to rotating b from point A to point C. Or traveling in a straight line AC. This rectilinear path forms the right triangle ACv.
c. Newton once again is correct in his proposal that everything does reach an equality or zero at the limit, as it must. Note that line AC forms the hypotenuse of a right triangle ACv but it also forms the secant AC. Taking the secant to the limit is the classic method of finding the limit. So I disagree with Mathias here. The Pythagorean theorem is indispensible here. It is the foundation of our calculations since we are using the unit circle. The use of the versine is also indispensible. So it is quite helpful in forming our calculations.
d. I do not see much utility in Newton's versine equation. The standard definition of the versine, i.e., versine=1-cosine is the equation we want and proves to be invaluable. Of course if we are to take the secant, AC, which forms one of the sides of triangle ACv, to its limit, then both the Pythagorean theorem and the limit can and does apply. Must apply.

e. Here is where I differ from Mathias. Keep in mind line segments AB and BC form opposing tangents to the unit circle. Points A and C both form points of tangency. To find our limit we shrink the secant, line AC, until it coincides with point C. At point C the tangent and the secant become a single point, coexisting as one, one indistinguishable from the other. The most simple method is to rotate radius OA until it coincides with radius OC. Note as you rotate OA counterclockwise the secant, AC necessarily shrinks. In rotating OA we can still maintain its right angle partner (conjugate), line Ac. This preserves the sense of the tangential velocity vector c. Thus the double point, or ICR, AC forms the point of tangency and line Ac is our shifted tangent line. Note I said the double point AC, not line segment or secant AC. Again they coincide and become indistinguishable . This is the classic definition of a limit. Note the only thing that changed was the orientation of line segment Ac and that is as it should be since this represents the tangential velocity, which is constant for all points lying along the circumference. Given one's interpretation you can start b's rotation form either point A or point b. If it's at point A then the secant is AC. If from point b then its .5AC since ABCorthocenter forms an equilateral. The paper has a, the acceleration , projecting radially away from the circle. However, given the description in the paper its obvious that a was the consequence of some force being exerted that caused the rotation of b.



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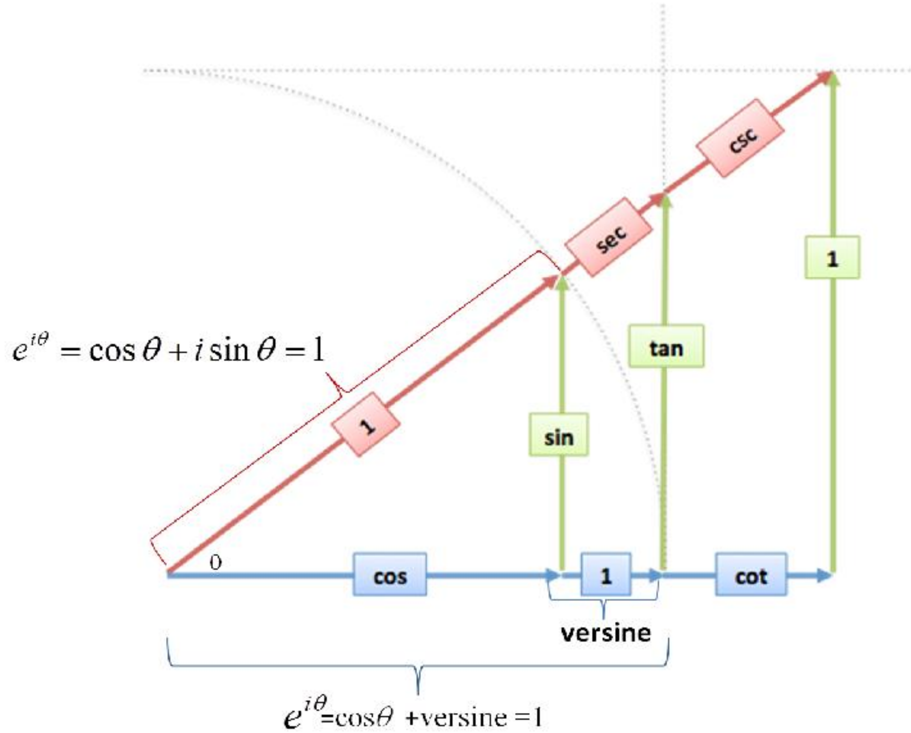

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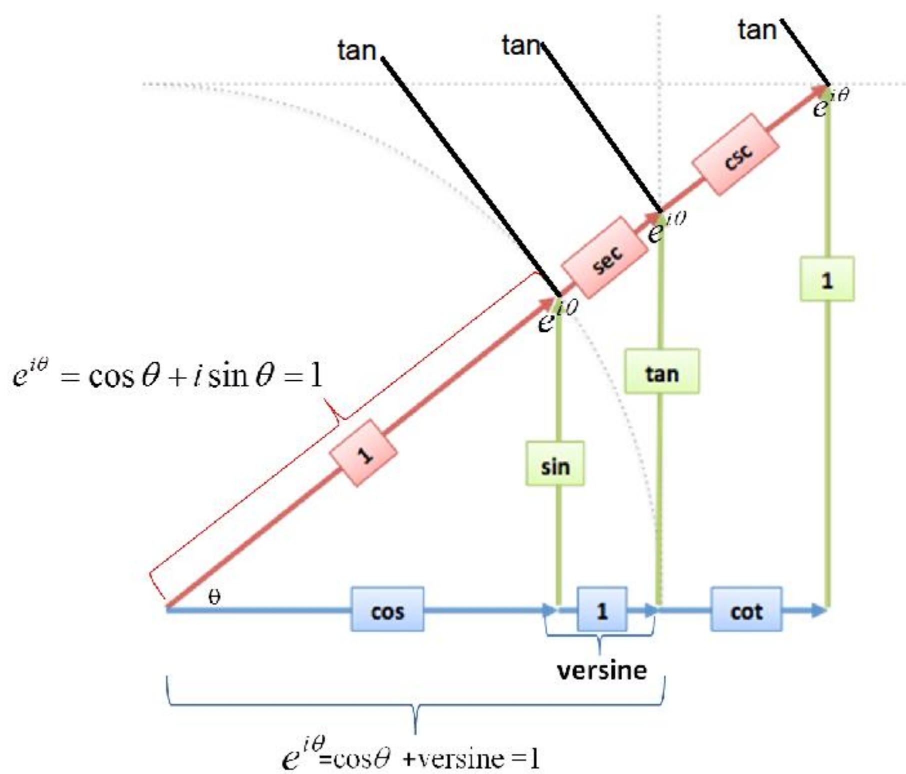

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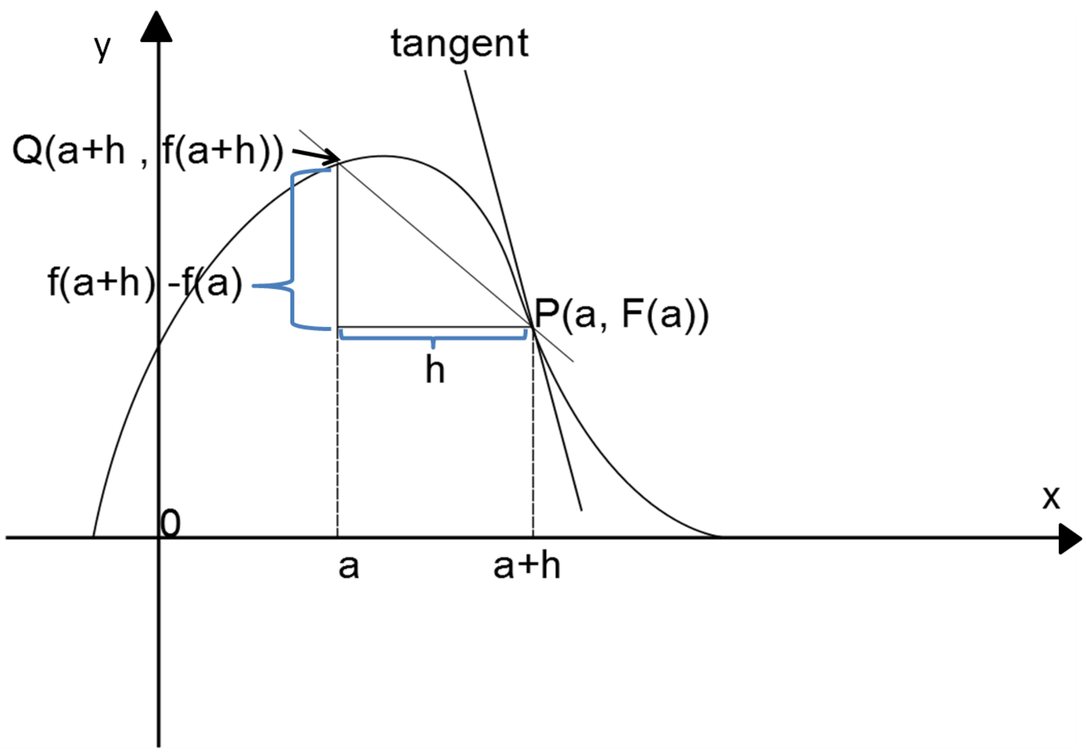
$e^{i\theta}$=cosθ +versine =1

f. Newton is once again perfectly correct. He states "These forces tend to the centers of the circles and are one to another as the versed sines of the least arcs described in equal times; that is, as the squares of the same arcs applied to the diameters of the circles."
I see this as meaning simply the versine is inwardly directed. And it is. The versine is zero at the circumference and the cosine is one. As the versine gets larger and larger it reaches a maximum of 1 at the origin and the cosine reaches a minimum of zero. Thus every point of tangency is a double point and the origin is always a double point of zero and 1. The square of the arcs is simply a utilization of the Pythagorean theorem. He uses arcs, I use the hypotenuse. Note also that as the secant AC shrinks we necessarily shrink v the versine. But again versine=sagitta=1-cosine. As we shrink v the cosine goes to 1. Thus in Newton's difference of quotient formula for finding the limit where , $\frac{f(x-h)-f(x)}{h}$ we have h=v. But as v goes to zero its conjugate the cosine goes to 1, thus there is neither ostensible nor actual division by zero. There is only division by 1 since at this point of tangency zero and 1 form a double point, an ICR and become indistinguishable. Again we must distinguish between algebraic zero and trigonometric zero. In this case we are dealing with trigonometric zero.



3.8 Johnson's Eulerian FTOC

We now have most of the major tools locked securely in place, compactly packed within our tool "box" sphere. Now having such a disparate collection tools locked up in such a confined space requires rules of interaction, otherwise sheer chaos and tool damage will definitely occur. Without such rules it would be akin to throwing a running power saw into a box of chisels. Disaster. Rules are needed to avoid such disasters and the ultimate rule is is the Law of Dimensions. But the next invaluable rule for computing change is Johnson's Eulerian FTOC. Again, change is life and calculus calculates change so calculus calculates life. The sum total of calculus is distilled within $e^{i\theta}$. But operationally it is summed up in this previous quote:

> *We may now alter our terminology and state, as h goes to zero the conjugate of h goes to F. The letter F is the Greek letter Wau or Digamma, which represents the double point ICR of $\pm$zero and ± 1.*
>
> $$\lim_{\frac{h \to 0}{F \leftarrow -h}} f(x) = L$$
>
> where
>
> $$-h = \frac{1}{h} = the\ conjugate\ of\ h$$

This "recalibration" of calculus is invaluable. It is this reformulation of the limit which allows us to finally dispense with the probabilistic basis of QM and place it firmly within

g. Now given this analysis we see that at every point of tangency there is a radius lying at right angles to a tangent and that radius is and must be composed of the versine and cosine. If the particle is rotating then again it must be the case that the sagitta or versine represents the inward directed acceleration or the centripetal force and the cosine represents the outward acceleration of centrifugal force.

h. I don't know if d=2a. I know these relationships are true: secant=1+a, versine=1-cosine, I don't know if we really need to invoke d. d is useful in establishing that rotation from A to b is equal to rotation from b to C. The notion of there being a 2r or a diameter across which the force is acting I don't quite see. Now it is true that the secant can be represented as a double distance as shown in the lower diagram. Note that this diagram is very similar to that expressed by Newton accept here we have everything placed within the unit circle and therefore a would equal the sagitta and the entire cosine =1 would be the sum of $\frac{e^{ix} - e^{-ix}}{2}$. Thus Newton's use of a double distance begins to make sense. In fact all of his computations make sense and are true. His depiction, as given in the paper, is however a bit wanting.