A Discussion on the Magnetic Vector Potential

Cyril Smith, December 2009

1. Introduction

The magnetic vector potential **A** is a valuable tool in electromagnetic theory, yet for static fields its detection has proved elusive. At the present time the only recognized detectors are Josephson Junctions which operate at cryogenic temperatures or experiments along the lines of the Aharonov-Bohm effect. This paper introduces a new term into the classical equation for an electric field that allows static **A** fields to be detected or measured. Examples are given of experiments that have been performed where the results can only be explained by this missing term.

2. Theory

If an **A** field is changing with time then its presence is easily detected as an electric field **E** producing force e**E** on electrons. As is well known the relationship is

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}. \tag{1}$$

Note this is a partial derivative for a point fixed in space. If an electron is moving through a *spatially non-uniform* **A** field, then it seems reasonable to assume the electron endures another time-changing **A** due to that movement. If that is true then we should replace the partial derivative (1) with a total derivative

$$\mathbf{E} = -\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} \tag{2}$$

which can then account for both components. For the case of the moving electron we must look for a vector function that has terms like $v_x \dfrac{\partial A_k}{\partial x}$ where k can be any of the x, y or z coordinates. The function that meets our requirement is $(\mathbf{v}\cdot\nabla)\mathbf{A}$ which in Cartesian coordinates has the following components.

$$\begin{aligned} & \mathbf{a}_x\left[v_x\frac{\partial A_x}{\partial x}+v_y\frac{\partial A_x}{\partial y}+v_z\frac{\partial A_x}{\partial z}\right] \\ (\mathbf{v}\cdot\nabla)\mathbf{A} = & \mathbf{a}_y\left[v_x\frac{\partial A_y}{\partial x}+v_y\frac{\partial A_y}{\partial y}+v_z\frac{\partial A_y}{\partial z}\right] \\ & \mathbf{a}_z\left[v_x\frac{\partial A_z}{\partial x}+v_y\frac{\partial A_z}{\partial y}+v_z\frac{\partial A_z}{\partial z}\right] \end{aligned} \tag{3}$$

The total derivative (2) then give the electric field as

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - (\mathbf{v}\cdot\nabla)\mathbf{A} \tag{4}$$

where the first term is the so-called transformer induction due to the time variation and the second term is an induction from movement at velocity **v**. It may be noted that (3) introduces both an electric field $E_{\Updownarrow}$ parallel to the velocity, which for electrons moving in a wire creates voltage induction, and an electric field $E_{\perp}$ at right angles to the velocity that produces a sideways force on the wire. Separating (3) into these two field components produces

$$E_{\Updownarrow} = -\begin{pmatrix} \mathbf{a}_x\left[v_x \dfrac{\partial A_x}{\partial x}\right] \\ \mathbf{a}_y\left[v_y \dfrac{\partial A_y}{\partial y}\right] \\ \mathbf{a}_z\left[v_z \dfrac{\partial A_z}{\partial z}\right] \end{pmatrix} \tag{5}$$

$$E_{\perp} = -\begin{pmatrix} \mathbf{a}_x\left[v_y \dfrac{\partial A_x}{\partial y} + v_z \dfrac{\partial A_x}{\partial z}\right] \\ \mathbf{a}_y\left[v_x \dfrac{\partial A_y}{\partial x} + v_z \dfrac{\partial A_y}{\partial z}\right] \\ \mathbf{a}_z\left[v_x \dfrac{\partial A_z}{\partial x} + v_y \dfrac{\partial A_z}{\partial y}\right] \end{pmatrix} \tag{6}$$

The transverse field (6) is seen to contain components that also occur in the classical $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ flux-cutting induction (see appendix), but accounts for only half the terms there. To bring $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ into the argument we need a vector function that can be added to $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ and supplies the longitudinal components (5) while at the same time supplies the $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ terms missing from (6). The appendix shows that those terms are supplied by $\nabla(\mathbf{v}\cdot\mathbf{A})$, but that function also supplies unwanted terms involving spatial derivatives of **v**. In the appendix the expression $\nabla_A(\mathbf{v}\cdot\mathbf{A})$ is used to denote that function with the velocity derivatives suppressed. It is therefore possible to create the vector identity

$$(\mathbf{v}\cdot\nabla)\mathbf{A} = -\mathbf{v}\times\mathbf{B} + \nabla_A(\mathbf{v}\cdot\mathbf{A}) \tag{7}$$

hence equation (4) becomes

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} - \nabla_A(\mathbf{v}\cdot\mathbf{A}) \tag{8}$$

Here we have the recognized transformer and flux-cutting induction terms, plus a new term that is seen to be a form of longitudinal induction i.e. along the electron velocity (current) direction. *This* $-\nabla_A(\mathbf{v}\cdot\mathbf{A})$ *term is missing from classical EM theory, and its inclusion opens the door to new discoveries.*

Adding the Coulomb term to (8) gives the full version for electric fields as

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} - \nabla_A(\mathbf{v}\cdot\mathbf{A}) \tag{9}$$

where ϕ is the scalar potential. Thus we have two **E** field components that are both the gradient of a scalar, $-\nabla\phi$ and $-\nabla_A(\mathbf{v}\cdot\mathbf{A})$. Bringing the two scalar terms together puts (9) into the form

$$\mathbf{E} = -\nabla(\phi + \mathbf{v}\cdot\mathbf{A}) - \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B} \tag{10}$$

Sommerfeld [6] points out that in 1903 Schwarzschild introduced his "electrokinetic potential" $L = (\phi - \mathbf{v}\cdot\mathbf{A})$, so it is over 100 years since the scalar product $\mathbf{v}\cdot\mathbf{A}$ was recognized as a potential. A Google search on "electrokinetic potential" plus Schwarzschild confirms that. Schwarzschild's electrokinetic potential L is really a *potential difference* (note the minus sign) which when multiplied by the charge density forms a relavistic invariant, which was important in the development of his

Principle of Least Action. The mathematical intricacies of that development is of little interest to engineers, they are interested in electric and magnetic forces that can do work. The **E** field (10) is just that, hence it would be more sensible if the electrokinetic potential capable of doing work in a fixed laboratory frame were the *sum* $(\phi + \mathbf{v} \,.\, \mathbf{A})$ and not the difference. Thus, ignoring the *electric* potential ϕ, an electron moving at velocity **v** through an **A** field can be considered to have a *kinetic* potential **v.A**, or a kinetic energy $e(\mathbf{v.A})$. Note this is a maximum when **v** is parallel to **A**, when the electron travels along the **A** field, and has a value evA.

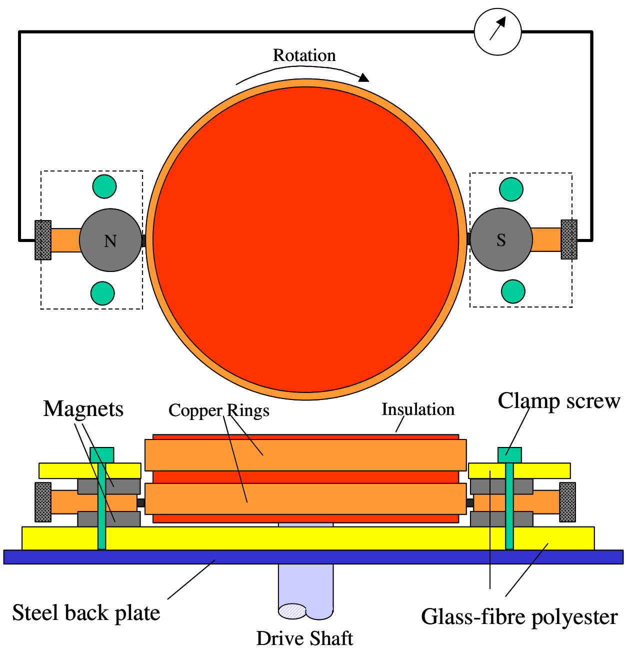
3. Closed Line-Integral

It is generally held that the closed line-integral of the gradient of a scalar is always zero, and such scalar fields are therefore conservative. In the case of the Coulomb potential ϕ this is true, it is impossible to extract energy from that field. But in the case of the effective scalar potential $(\mathbf{v} \cdot \mathbf{A})$ it is true only when, within the gradient function, the spatial derivatives of velocity are included. When those derivatives are excluded (because to the best of our knowledge a changing velocity does not create an immediate force) a change of velocity has the effect of regauging the potential. It is then quite easy to choose a forward path at a certain velocity through a non-uniform **A** field where the line-integral yields a voltage V, then follow a return path at a lower velocity where the line-integral yields a reversed value lower than V, hence the closed line-integral yields a non zero answer. It is not necessary for the **A** field to have curl, the induction occurs even when a **B** field is not present.

Examples where the voltage induction is known to occur are given in the next section. *Meanwhile it should be noted that when a field is the scalar product of two vectors and the spatial derivatives of only one of those vectors contribute to the gradient of that scalar product, then the resulting field is not necessarily conservative*. This insight could be valuable for other applications.

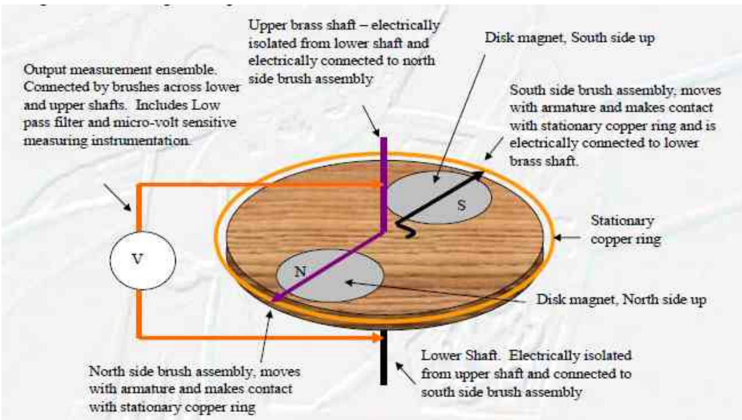
4. Examples

4.1 Marinov Generator, see Smith [2]



The generator version of the Marinov motor has drawn little attention, presumably because experiments on the *motor* have proved to be inconclusive. These experiments have looked for tiny values of torque, and there is little interest in motors yielding such low power. However recent generator experiments using slip-rings have produced readily detectable voltages. Here the forward path between diametrically opposing brushes follows the slip-ring contour where **v** is the slip-ring velocity, while the return path via the meter is through the connecting wires where **v** is the trivial electron drift velocity. The net result is a voltage induced into that closed circuit from movement through the **A** field. Although in these experiments the **B** field was also present, the voltage induction could not be explained by $\mathbf{v}\times\mathbf{B}$ flux cutting.

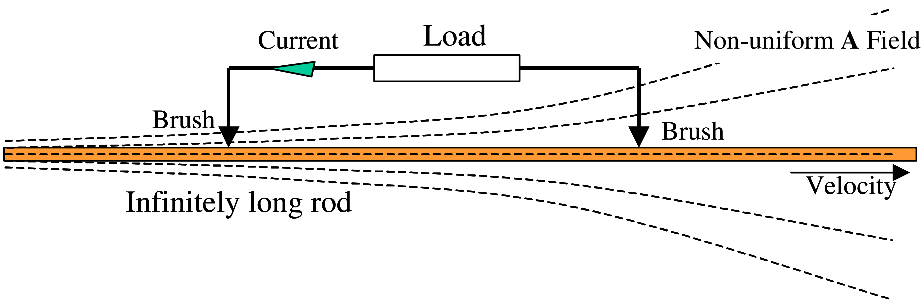
4.2 Distinti Paradox 2, see [3]



The Distinti Paradox 2 is essentially a more complex version of the Marinov generator. Whereas the Marinov version uses fixed magnets near a rotating slip-ring connected by two fixed brushes, the Distinti version uses magnets rotating within a fixed ring, with brushes rotating with the magnets, then requires two more brush connections to the voltmeter. Again, the induced voltage cannot be explained by flux cutting.

5. Discussion

The approach outlined in this paper has been put forward by others, notably Wesley [1] and Phipps [4]. Wesley and Phipps confine their discussion to the tiny torques offered by the Marinov motor, surprisingly no attention is given to the generator version which can yield more realistic results. Errede [5] starts by equating $\mathbf{E}=\mathbf{v}\times\mathbf{B}$ in the laboratory frame to $\mathbf{E}=-\dfrac{\partial\mathbf{A}}{\partial t}$ in the rest frame of the moving electron and derives a similar result. He shows a moving metal rod being electrically polarized by the longitudinal induction term but then incorrectly assumes that a load connected across the rod will receive current from the voltage induction. In fact, the load moving with the rod, will receive the same induction hence there is zero current through the load. However if an infinitely long rod is moving, and a stationary load connects to the rod via brushes, then current will flow and power can be extracted.



6. Conclusion

Contemporaneous EM theory has overlooked an important feature for voltage induction. Electric charge moving through a non-uniform magnetic vector potential will endure a force at right angles to the motion given by the $\mathbf{v}\times\mathbf{B}$ flux cutting rule, but in addition there is a force *parallel* to the motion given by a new $-\nabla_A\left(\mathbf{v}\cdot\mathbf{A}\right)$ term. The use of the scalar product $\mathbf{v}\cdot\mathbf{A}$ as a kinetic potential was recognized by Schwarzschild over 100 year ago, but only recently has the existence of this term been demonstrated by experiments. An interesting feature is that when taking a closed integral around an electric circuit the potential can be regauged by a change of velocity, so unlike normal scalars the closed integral yields a non-zero result. Therein lies not only a possible overunity machine, but also a method for detecting the **A** field.

Appendix.

The Cartesian components of $(\mathbf{v}\cdot\nabla)\mathbf{A}$ are:

$$(\mathbf{v}\cdot\nabla)\mathbf{A} = \begin{array}{l} \mathbf{a}_x\left[\overline{v_x\frac{\partial A_x}{\partial x}} + v_y\frac{\partial A_x}{\partial y} + v_z\frac{\partial A_x}{\partial z}\right] \\ \mathbf{a}_y\left[v_x\frac{\partial A_y}{\partial x} + \overline{v_y\frac{\partial A_y}{\partial y}} + v_z\frac{\partial A_y}{\partial z}\right] \\ \mathbf{a}_z\left[v_x\frac{\partial A_z}{\partial x} + v_y\frac{\partial A_z}{\partial y} + \overline{v_z\frac{\partial A_z}{\partial z}}\right] \end{array} \tag{A1}$$

where the overbars denote the longitudinal components parallel to the velocity.
The Cartesian components of $\mathbf{v}\times\mathbf{B}$ are:

$$\mathbf{v}\times\mathbf{B} = \begin{array}{l} \mathbf{a}_x\left[v_y\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) - v_z\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\right] \\ \mathbf{a}_y\left[v_z\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - v_x\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\right] \\ \mathbf{a}_z\left[v_x\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) - v_y\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\right] \end{array} \tag{A2}$$

where there are no longitudinal components.
The Cartesian components of $\nabla(\mathbf{v}\cdot\mathbf{A})$ are:

$$\nabla(\mathbf{v}\cdot\mathbf{A}) = \begin{array}{l} \mathbf{a}_x\left[v_x\frac{\partial A_x}{\partial x} + v_y\frac{\partial A_y}{\partial x} + v_z\frac{\partial A_z}{\partial x} + A_x\frac{\partial v_x}{\partial x} + A_y\frac{\partial v_y}{\partial x} + A_z\frac{\partial v_z}{\partial x}\right] \\ \mathbf{a}_y\left[v_x\frac{\partial A_x}{\partial y} + v_y\frac{\partial A_y}{\partial y} + v_z\frac{\partial A_z}{\partial y} + A_x\frac{\partial v_x}{\partial y} + A_y\frac{\partial v_y}{\partial y} + A_z\frac{\partial v_z}{\partial y}\right] \\ \mathbf{a}_z\left[v_x\frac{\partial A_x}{\partial z} + v_y\frac{\partial A_y}{\partial z} + v_z\frac{\partial A_z}{\partial z} + A_x\frac{\partial v_x}{\partial z} + A_y\frac{\partial v_y}{\partial z} + A_z\frac{\partial v_z}{\partial z}\right] \end{array} \tag{A3}$$

which include spatial derivatives of velocity. We are only interested in time variations of **A** since it is only these that induce an **E** field. If we denote (A3) with the derivatives of velocity suppressed as $\nabla_A(\mathbf{v}\cdot\mathbf{A})$ then its components are:

$$\nabla_A(\mathbf{v}\cdot\mathbf{A}) = \begin{array}{l} \mathbf{a}_x\left[\overline{v_x\frac{\partial A_x}{\partial x}} + v_y\frac{\partial A_y}{\partial x} + v_z\frac{\partial A_z}{\partial x}\right] \\ \mathbf{a}_y\left[v_x\frac{\partial A_x}{\partial y} + \overline{v_y\frac{\partial A_y}{\partial y}} + v_z\frac{\partial A_z}{\partial y}\right] \\ \mathbf{a}_z\left[v_x\frac{\partial A_x}{\partial z} + v_y\frac{\partial A_y}{\partial z} + \overline{v_z\frac{\partial A_z}{\partial z}}\right] \end{array} \tag{A4}$$

where as in (A1) the overbars represent the longitudinal components.
Examination of the terms in (A1), (A2) and (A4) shows that we can create the identity

$$(\mathbf{v}\cdot\nabla)\mathbf{A} = -\mathbf{v}\times\mathbf{B} + \nabla_A(\mathbf{v}\cdot\mathbf{A}) \tag{A5}$$

References.

[1] J. P. Wesley:- The Marinov Motor, Notional Induction without a Magnetic B Field, APEIRON Vol. 5 Nr.3-4, July-October 1998
Wesley notes this type of motional induction has been used to account for the Aharonov-Bohm (1998) effect and the Hooper (1974)-Monstein (1997) experiment.

[2] Cyril Smith:- The Marinov Generator

[3] Robert J. Distinti:- http://www.distinti.com/docs/pdx/paradox2.pdf

[4] Thomas E. Phipps Jr: Observations of the Marinov Motor, APEIRON Vol. 5 Nr. 3-4, July-October 1998

[5] Prof. Steven Errede:- Physics Phun with Motional Effects and the Magnetic Vector Potential *A*, Eh!!

[6] Arnold Sommerfeld:- 1948 "Lectures on Theoretical Physics" vol. 3 "Electrodynamics" (translated by Edward G. Ramberg and published by Academic Press in 1964).